O‘ZBEKISTON RESPUBLIKASI
OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

TOSHKENT MOLIYA INSTITUTI

“OLIY VA AMALIY MATEMATIKA”
KAFEDRASI

IQTISODCHILAR UCHUN MATEMATIKA
(1-semestr. 2-modul. Amaliy mashg‘ulotlar
Matematik analiz asoslari va uning tatbiqlari)

Bilim sohasi:	100000	– Gumanitar
	200000	– Ijtimoiy soha, iqtisod va huquq
Ta’lim sohasi:	110000	– Pedagogika
	230000	– Iqtisod
Ta’lim yo‘nalishlari:	5111000	– Kasb ta’limi (5230600 – Moliya, 5230700 – Bank ishi, 5230900 – Buxgalteriya hisobi va audit, 5231200 – Sug‘urta ishi)
	5230200	– Menejment (xizmatlar sohasi)
	5230600	– Moliya
	5230700	– Bank ishi
	5230800	– Soliqlar va soliqqa tortish
	5230900	– Buxgalteriya hisobi va audit (tarmoqlar bo‘yicha)
	5231200	– Sug`urta ishi
	5231300	– Pensiya ishi
	5231500	– Baholash ishi
	5232000	– Davlat budjetining g‘azna ijrosi

Toshkent – 2018

O‘zbekiston Respublikasi Oliy va o‘rta maxsus ta’lim vazirligining 201__ yil “__”.___dagi “__”-sonli buyrug‘ining __-ilovasi bilan fan dasturi ro‘yxati tasdiqlangan.

Oliy va o‘rta maxsus, kasb-hunar ta’limi yo‘nalishlari bo‘yicha O‘quv-uslubiy birlashmalar faoliyatini Muvofiqlashtiruvchi Kengashining 201__ yil “___”__________ dagi “____”-son bayonnomasi bilan ma’qullangan fan dasturi asosida O‘UM ishlab chiqilgan.

O‘UM Toshkent moliya institutida ishlab chiqildi.

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O‘UM Toshkent moliya instituti Kengashida ko‘rib chiqilgan va tavsiya qilingan (201__ yil __ _________ ___-sonli bayonnoma).

19-amaliy mashg‘ulot. $\mathbb{R}^n$ fazoda nuqtalarning o‘zaro joylashishi. Sonli ketma-ketlik

Quyidagi sohalar bilan chegaralangan to‘plamlar qavariq to‘plam bo‘ladimi?

19.1. $\begin{cases} 3x_1 + 3x_2 = 8 \\ 6x_1 + 6x_2 = 17 \end{cases}$

19.2. $\begin{cases} x_1 + x_2 \le 2 \\ -x_1 + x_2 \le 2 \end{cases}$

19.3. $\begin{cases} 2x_1 + 5x_2 \le 20 \\ 5x_1 + 6x_2 \le 30 \\ x_1 \ge 0,\ x_2 \ge 0 \end{cases}$

19.4. $\begin{cases} -2x_1 + x_2 \le 1 \\ x_1 - 2x_2 \ge 1 \\ x_1 \ge 0,\ x_2 \ge 0 \end{cases}$

19.5. $\begin{cases} x_1^2 + x_2^2 \le 6 \\ x_1 - \frac{1}{3}x_2^2 + 1 \le 0 \end{cases}$

19.6. $x_1^2 + 3x_2^2 \le 5$

Quyidagi to‘plamlar qavariqmi?

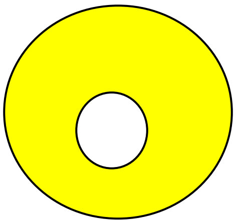
19.7. Markazsiz doira. Markazsiz shar.

19.8. Kesma va bu kesmada yotmaydigan nuqta.

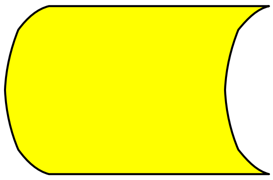
19.9. R_3- fazoda umumiy nuqtaga ega bo‘lgan 2 ta tetraedr.

19.10. R_2- fazoda umumiy tomonga ega bo‘lgan 2 ta uchburchak.

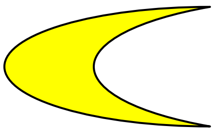
19.11.



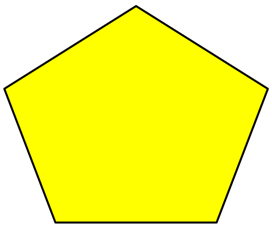
19.12.



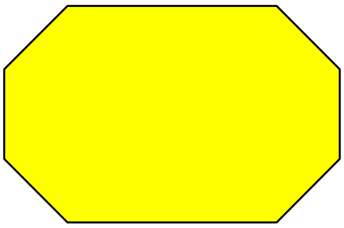
19.13.



19.14.



19.15.



19.16. Sonli ketma-ketlik chegaralanganligini isbotlang: $x_n = \dfrac{n^2+1}{n^2+2}$

Isbot. $\dfrac{n^2+1}{n^2+2} = 1 - \dfrac{1}{n^2+2}$ va $0 < \dfrac{1}{n^2+2} \le \dfrac{1}{2}$ shuning uchun $\dfrac{1}{2} < x_n < 1$.

Sonli ketma-ketlik chegaralanganligini isbotlang.

19.17. $x_n = \dfrac{(-1)^n\ n+1}{\sqrt{n^2+2}}$

19.18. $x_n = Sin\ n$

19.19. $x_n = (1-(-1)^n)$

19.20. Sonli ketma-ketlik monotonligini isbotlang: $x_n = \lg n - \lg(n-1),\ (n>1)$

Isbot. $x_n = \lg n - \lg(n-1) = \lg\frac{n}{n-1}$ $x_{n+1} - x_n = \lg\frac{n+1}{n} - \lg\frac{n}{n-1} = \lg\frac{n^2-1}{n^2} = \lg(1-\frac{1}{n^2}) < 0$.

Demak, $x_{n+1} - x_n < 0,\ x_{n+1} < x_n$, shuning uchun bu ketma-ketlik monoton kamayuvchi.

Sonli ketma-ketlik monotonligini isbotlang:

19.21. $x_n = 3^n - 2^n$ $x_n = \sqrt{n^2-1}$ **19.22.** $x_n = \sum_{k=1}^{n} k$

19.23. Ketma-ketlik limiti ta'rifidan foydalanib, quyidagilarni isbotlang: $\lim_{n\to\infty}\frac{n}{n+1} = 1$

Isbot. ixtiyoriy $\varepsilon > 0$ son olamiz, $|x_n - 1| = \left|\frac{n}{n+1} - 1\right| = \frac{1}{n+1}$; $|x_n - 1| < \varepsilon$ tengsizlikni qanoatlantiruvchi n larni topish uchun $\frac{1}{n+1} < \varepsilon$ tengsizlikni yechamiz. $n > \frac{1-\varepsilon}{\varepsilon}$.

Shunday qilib, $\frac{1-\varepsilon}{\varepsilon}$ sonining butun qismi $N = \left[\frac{1-\varepsilon}{\varepsilon}\right]$ bo'ladi, u holda $|x_n - 1| < \varepsilon$ tengsizlik barcha $n > N$ larda bajariladi. ε-ixtiyoriy son bo'lgani uchun $\lim_{n\to\infty}\frac{n}{n+1} = 1$.

Agar $\varepsilon = 0.01$ bo'lsa, $N = \left[\frac{1-0.01}{0.01}\right] = 99,$ $n > 99$ larda $|x_n - 1| < 0.01$ bo'ladi.

Ketma-ketlik limiti ta'rifidan foydalanib, quyidagilarni isbotlang:

19.24. $\lim_{n\to\infty}\frac{4n-1}{2n+1} = 2$ **19.25.** $\lim_{n\to\infty}\frac{3n+1}{5n-1} = \frac{3}{5}$

19.26. $\lim_{n\to\infty}\frac{2n-1}{2-3n} = -\frac{2}{3}$ qaysi n dan boshlab, $\left|\frac{2n-1}{2-3n} - \left(-\frac{2}{3}\right)\right| < 0.0001$ tengsizlik o'rinli bo'ladi?

Umumiy hadi orqali berilgan ketma-ketlikning birinchi beshta hadini yozing:

19.27. $x_n = \frac{1}{2n+1}$ **19.28.** $x_n = \frac{n+2}{n^3+1}$ **19.29.** $x_n = (-1)^n\ \frac{n+1}{n^2}$

Ketma-ketlikning berilgan hadlari orqali umumiy hadining formulasini yozing:

19.30. $1; \frac{1}{2}; \frac{1}{6}; \frac{1}{24}; \ldots$ **19.31.** $1;\ 2\frac{1}{4};\ 2\frac{7}{9};\ 3\frac{1}{16};\ 3\frac{6}{25}; \ldots$

19.32. 2; 10; 26; 82; 242; 730; …

Quyidagi limitlarni toping:

19.33. $\lim_{n\to\infty}\frac{3n^3+2}{4n^3-1}$ **19.34.** $\lim_{n\to\infty}\frac{2n^3+3}{n^3+n-1}$

19.35. $\lim_{n\to\infty}\frac{(n+1)^3}{5n^3+1}$

19.36. $\lim_{n\to\infty}\frac{\sqrt[3]{n^2+n}}{4+n}$

19.37. $\lim_{n\to\infty}\frac{\sqrt{2n^4+n^2+1}}{2n+n^2-1}$

19.38. $\lim_{n\to\infty}\frac{n!}{(n+1)!-n!}$

19.39. $\lim_{n\to\infty}\frac{1+\frac{1}{2}+...+\frac{1}{2^n}}{1+\frac{1}{3}+...+\frac{1}{3^n}}$

19.40. $\lim_{n\to\infty}\frac{3+6+9+...+3n}{n^2+4}$

19.41. $\lim_{n\to\infty}\sqrt{n}(\sqrt{n+2}-\sqrt{n-3})$

19.42. $\lim_{n\to\infty}\frac{(2n+1)!+(2n+2)!}{(2n+3)!-(2n+2)!}$

19.43. $\lim_{n\to\infty}\sqrt{n^3+8}\left(\sqrt{n^3+2}-\sqrt{n^3-1}\right)$

19.44. $\lim_{n\to\infty}\left(\frac{1}{3}+\frac{1}{15}+...+\frac{1}{4n^2-1}\right)$

19.45. $\lim_{n\to\infty}\left(\frac{1}{2n}Cos\,n^3-\frac{3n}{6n+1}\right)$

19.46. $\lim_{n\to\infty}\left(\frac{n}{n^2+1}Sin\,n!+\frac{2n^2}{1-9n^2}\right)$

20-amaliy mashg'ulot. Sonli qatorlar

20.1. Ushbu qatorni yaqinlashishga tekshiring:

$$\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}+\cdots.$$

Yechish. Berilgan qatorning n-xususiy yig'indisi

$$S_n=\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}.$$

Bu yig'indini soddalashtirish maqsadida qatorning n-hadini quyidagi $\frac{1}{n(n+2)}=\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+2}\right)$ ko'rinishda yozib olamiz. U holda

$$S_n=\frac{1}{2}\left(\frac{1}{1}-\frac{1}{3}\right)+\frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right)+\frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots+\frac{1}{2}\left(\frac{1}{n-1}-\frac{1}{n+1}\right)+\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+2}\right)=$$

$$=\frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n+1}-\frac{1}{n+2}\right)$$

bo'ladi. Ravshanki, $\{S_n\}$ ketma-ketlik limiti mavjud va $\frac{3}{4}$ ga teng. Demak, berilgan qator yaqinlashuvchi bo'lib, uni

$$\frac{3}{4}=\frac{1}{1\cdot 3}+\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}+\cdots+\frac{1}{n(n+2)}+\cdots \quad \text{yoki} \quad \frac{3}{4}=\sum_{n=1}^{\infty}\frac{1}{n(n+2)}$$

kabi yozish mumkin ekan.

20.2. Ushbu qatorning n-xususiy yig‘indisi uchun ifoda toping va yaqinlashishga tekshiring:

a) $\sum_{n=1}^{\infty} \frac{2}{(2n-1)(2n+1)}$; b) $\sum_{n=1}^{\infty} \frac{2^n+3^n}{5^n}$.

20.3. $\sum_{n=1}^{\infty} \frac{2n+1}{5n-3} = \frac{3}{2}+\frac{5}{7}+\frac{7}{12}+...+\frac{2n+1}{5n-3}+...$ sonli qatorning yaqinlashuvchi bo‘lishining zaruriy sharti bajarilishini tekshiring.

Yechish. $a_n = \frac{2n+1}{5n-3}$ $\lim\limits_{n\to\infty} \frac{2n+3}{5n-3} = \frac{2}{5}$; $\lim\limits_{n\to\infty} a_n = \frac{2}{5} \neq 0$. Demak yaqinlashuvchi bo‘lishining zaruriy sharti bajarilmaydi, qator uzoqlashuvchi.

20.4. Qatorlar uchun yaqinlashuvchi bo‘lishining zaruriy sharti bajarilishini tekshiring:

a) $\sum_{n=1}^{\infty} \frac{n+1}{2n+1}$. b) $\sum_{n=1}^{\infty} \frac{n+2}{\ln(n+1)}$.

20.5. Birinchi taqqoslash alomatidan foydalanib $\frac{2}{3}+\frac{1}{2}\left(\frac{2}{3}\right)^2+\frac{1}{3}\left(\frac{2}{3}\right)^3+...+\frac{1}{n}\left(\frac{2}{3}\right)^n+...$ qatorni yaqinlashishga tekshiring.

Yechish. Ushbu qatorni qaraymiz: $\frac{2}{3}+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+...+\left(\frac{2}{3}\right)^n+....$

Ravshanki, $a_n = \frac{1}{n}\left(\frac{2}{3}\right)^n \le \left(\frac{2}{3}\right)^n = b_n$. Mahraji $q = \frac{2}{3}$ bo‘lgan $\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ geometrik qator yaqinlashuvchi, demak yuqoridagi teoremaga ko‘ra berilgan $\sum_{n=1}^{\infty}\frac{1}{n}\cdot\left(\frac{2}{3}\right)^n$ qator ham yaqinlashuvchi bo‘ladi.

Taqqoslash alomatini qo‘llab berilgan qatorlarni yaqinlashishga tekshiring:

20.6. $\sum_{n=1}^{\infty} \frac{2n}{3n^2-5}$. **20.7.** $\sum_{n=1}^{\infty} \frac{2n+7}{3n^3+11}$. **20.8.** $\sum_{n=1}^{\infty} \frac{\sqrt{n^2+7}}{n^5+12}$.

20.9. $\sum_{n=1}^{\infty} \frac{1}{\ln(n+3)}$. **20.10.** $\sum_{n=1}^{\infty} \frac{3^n}{n(3^n-4)}$. **20.11.** $\sum_{n=1}^{\infty} \frac{\ln(n+3)}{n^2}$.

20.12. Qatorni yaqinlashishga tekshiring:

$$\frac{2}{1^2}+\frac{2^2}{2^2}+\frac{2^3}{3^2}+...+\frac{2^n}{n^2}+...$$

Yechish. Ravshanki, $a_n = \frac{2^n}{n^2}$, $a_{n+1} = \frac{2^{n+1}}{(n+1)^2}$.

$$\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{\dfrac{2^{n+1}}{(n+1)^2}}{\dfrac{2^n}{n^2}}=2\lim_{n\to\infty}\frac{n^2}{(n+1)^2}=2>1.$$

Demak, qator uzoqlashuvchi.

Dalamber alomati yordamida qatorni yaqinlashishga tekshirish mumkinmi?

20.13. $\sum_{n=1}^{\infty}\frac{2^n}{3^n+7}$ **20.14.** $\sum_{n=1}^{\infty}\frac{n\cdot 2^n}{5^n+12}$

20.15. $\sum_{n=1}^{\infty}\frac{n!}{3^n(n+1)}$ **20.16.** $\sum_{n=1}^{\infty}\frac{n^2+1}{n^3-5}$

20.17. $\sum_{n=1}^{\infty}\frac{2^n}{2^n+n}$ **20.18.** $\sum_{n=1}^{\infty}\frac{n!\cdot 3^n}{n^n}$

20.19. $\sum_{n=1}^{\infty}\frac{n!}{5^n+n^2}$ **20.20.** $\sum_{n=1}^{\infty}\frac{(n+1)!}{n^n+2^n}$

20.21. Berilgan qatorni yaqinlashishga tekshiring:

$$\frac{1}{ln\,2}+\frac{1}{ln^2\,3}+...+\frac{1}{ln^n(n+1)}+...$$

Yechish. $\lim_{n\to\infty}\sqrt[n]{a_n}=\lim_{n\to\infty}\sqrt[n]{\frac{1}{\ln^n(n+1)}}=\lim_{n\to\infty}\frac{1}{\ln(n+1)}=0<1.$ Demak, qator yaqinlashuvchi.

Koshining radikal alomati yordamida qatorlarning yaqinlashishishni tekshiring:

20.22. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2}.$ **20.23.** $\sum_{n=1}^{\infty}\left(\frac{n-1}{2n+1}\right)^n.$

20.24. $\sum_{n=1}^{\infty}\left(\frac{2n-1}{3n+1}\right)^{\frac{n}{2}}.$ **20.25.** $\sum_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^{n(n-1)}.$

20.26. $\sum_{n=1}^{\infty}\left(\arcsin\frac{1}{n}\right)^n.$ **20.27.** $\sum_{n=1}^{\infty}n\cdot\left(\frac{3n+2}{2n+1}\right)^n.$

20.28. Umumlashgan garmonik qator deb ataluvchi

$$1+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+...+\frac{1}{n^p}+...$$

qatorni yaqinlashishga tekshiring.

Yechish. $a_1=f(1)=1,\ a_2=f(2)=\frac{1}{2^p},\ ...,\ a_n=f(n)=\frac{1}{n^p},\ ...$ va $f(x)=\frac{1}{x^p}$ ekanligi ravshan, bu yerda p-haqiqiy son. Ushbu

$$\int_1^\infty \frac{1}{x^p}dx = \frac{x^{-p+1}}{-p+1}\Bigg|_1^\infty = \frac{1}{1-p}\lim_{n\to\infty} x^{-p+1}\Big|_1^\infty = \frac{1}{1-p}\lim_{n\to\infty}(n^{1-p}-1) \qquad (p\neq 1)$$

xosmas integralni hisoblaymiz.

Agar $p>1$ bo‘lsa, u holda $\lim_{n\to\infty} n^{1-p}=0$ ва $\int_1^\infty \frac{1}{x^p}dx = \frac{1}{1-p}$ yaqinlashuvchi;

Agar $p<1$ bo‘lsa, u holda $\lim_{n\to\infty} n^{1-p}=\infty$ ва $\int_1^\infty \frac{1}{x^p}dx$ uzoqlashuvchi;

Agar $p=1$ bo‘lsa, u holda $\int_1^\infty \frac{1}{x}dx = ln x\Big|_1^\infty = \infty$ uzoqlashuvchi.

Shu sababli umumlashgan garmonik qator $p>1$ bo‘lsa yaqinlashuvchi, $p\leq 1$ bo‘lsa uzoqlashuvchi bo‘ladi.

Integral alomati yordamida qatorlarni yaqinlashishga tekshiring:

20.29. $\sum_{n=1}^\infty \frac{1}{(3n+2)\ln(3n+2)}$ **20.30.** $\sum_{n=2}^\infty \frac{1}{n\sqrt[3]{\ln^5 n}}$ **20.31.** $\sum_{n=2}^\infty \frac{\ln n}{n}$

20.32. $\frac{1}{2^2}-\frac{1}{3^2}+\frac{1}{4^2}-...+(-1)^{n+1}\frac{1}{(n+1)^2}+...$ qatorni yaqinlashishga tekshiring.

Yechish. $\frac{1}{2^2}>\frac{1}{3^2}>\frac{1}{4^2}>...>\frac{1}{(n+1)^2}>...$ va $\lim_{n\to\infty} u_n = \lim_{n\to\infty}\frac{1}{(n+1)^2}=0$.

Demak, yuqoridagi teoremaga asosan qator yaqinlashuvchi.

20.33. Quyidagi qatorni qaraylik: $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+...+(-1)^{n-1}\frac{1}{n}+...$

Leybnits alomatiga ko‘ra bu qator yaqinlashuvchi, lekin qator hadlarining absolyut qiymatlaridan tuzilgan $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+...+\frac{1}{n}+...$ qator uzoqlashuvchi. Demak, $\sum_{n=1}^\infty \frac{(-1)^{n-1}}{n}$ qator shartli yaqinlashuvchi.

Quyidagi ishoralari navbatlashuvchi qatorlarni yaqinlashishga tekshiring:

20.34. $\sum_{n=1}^\infty (-1)^{n+1}\frac{1}{n\sqrt{n}}$. **20.35.** $\sum_{n=1}^\infty (-1)^{n+1}\frac{n+3}{n+1}$.

20.36. $\sum_{n=1}^\infty (-1)^{n+1}\frac{1}{(2n-1)^3}$. **20.37.** $\sum_{n=1}^\infty (-1)^{n}\frac{1}{\sqrt[4]{n}}$.

20.38. $\sum_{n=1}^\infty (-1)^{n}\frac{1\cdot 3\cdot 5\cdots(2n+1)}{2\cdot 5\cdot 8\cdots(3n-1)}$. **20.39.** $\sum_{n=1}^\infty (-1)^{n}\frac{\sin\frac{\pi}{n}}{n}$.

21-amaliy mashg‘ulot. Bir va ko‘p o‘zgaruvchili funksiyalar. Kobb-Duglass funksiyasi

21.1. $X = Y = R$ da $y = x^2$ funksiyaning aniqlanish sohasini toping.

Yechish. $X = Y = R$ da $y = x^2$ funksiyaning aniqlanish sohasini $D(y) = (-\infty; +\infty)$ bo‘ladi.

21.2. $f(x) = \sqrt{1-2x} + 3\arcsin\frac{3x-1}{2}$ funksiyaning aniqlanish sohasini toping.

Yechish. Birinchi qo‘shiluvchi $1-2x \ge 0$ da haqiqiy qiymatlarni qabul qiladi, ikkinchi qo‘shiluvchi esa $-1 \le \frac{3x-1}{2} \le 1$ bo‘lganda. Shunday qilib, funksiyaning aniqlanish sohasini topish uchun

$$\begin{cases} 1-2x \ge 0 \\ \frac{3x-1}{2} \le 1 \\ \frac{3x-1}{2} \ge -1 \end{cases}$$

tengsizliklar sistemasini yechib topamiz

$$x \le \frac{1}{2}, \quad x \le 1, \quad x \ge -\frac{1}{3}$$

Demak, $D(f) = \left[-\frac{1}{3}; \frac{1}{2}\right]$.

Quyidagi funksiyalarning aniqlanish sohasini toping:

21.3. $y = \lg(x^2 - 4x + 3)$ **21.4.** $y = \arcsin(3x-4)$

21.5. $X = Y = R$ da $y = x^2$ funksiyaning qiymatlar to‘plamini toping.

Yechish. $X = Y = R$ da $y = x^2$ funksiyaning qiymatlar to‘plami $E(y) = [0; +\infty)$ bo‘ladi.

21.6. $y = 1 + 2^{x+1}$ funksiyaning qiymatlar to‘plamini toping.

Yechish. $y = 2^{x+1} = 2 \cdot 2^x$ ko‘rsatkichli funksiya, uning qiymatlar to‘plami $y \in (0; +\infty)$, demak berilgan funksiyaning qiymatlar to‘plami $(1; +\infty)$ bo‘ladi yoki berilgan funksiyaning qiymatlar to‘plami uning teskari funksiyasi $x = \log_2(y-1) - 1$ ning aniqlanish sohasi $y > 1$ bilan ustma-ust tushadi, shuning uchun $E(y) = (1; +\infty)$.

Quyidagi funksiyalarning qiymatlar to‘plamini toping:

21.7. $y = x + \frac{1}{x}$ **21.8.** $y = \sqrt{-x^2 - x + 2}$

21.9. $y = 2x + 1$ funksiya $(-\infty; +\infty)$ da monotonligini ko‘rsating.

Yechish. $y=2x+1$ funksiya $(-\infty;+\infty)$ da o‘suvchi, chunki $x_1<x_2$ bo‘lsa, u holda $f(x_2)-f(x_1)=2x_2+1-(2x_1+1)=2(x_2-x_1)>0$ bo‘ladi va $f(x_1)<f(x_2)$ tengsizlik kelib chiqadi.

Quyidagi funksiyalarning har biri ko‘rsatilgan oraliqlarda monoton ekanligini ko‘rsating:

21.10. $f(x)=2x-1;\quad x\in(-\infty;+\infty)$, **21.11.** $f(x)=2-3x;\quad x\in(-\infty;+\infty)$,

21.12. $f(x)=\lg\left(x+\sqrt{x^2+1}\right)$ funksiyaning juft yoki toqligini tekshiring.

Yechish. $f(-x)=\lg\left(-x+\sqrt{(-x)^2+1}\right)=\lg\dfrac{\left(x+\sqrt{x^2+1}\right)\left(-x+\sqrt{x^2+1}\right)}{\left(x+\sqrt{x^2+1}\right)}=$

$$=\lg\frac{1}{\left(x+\sqrt{x^2+1}\right)}=\lg\left(x+\sqrt{x^2+1}\right)^{-1}=-\lg\left(x+\sqrt{x^2+1}\right)=-f(x).$$

Demak, $f(x)=\lg\left(x+\sqrt{x^2+1}\right)$ toq funksiya ekan.

Quyidagi funksiyalarning juft yoki toqligini tekshiring:

21.13. $y=\dfrac{1}{x+\dfrac{1}{x+\dfrac{1}{x}}}$ **21.14.** $y=x^2\cos x$

21.15. $y=Sin\,6x+tg\,4x$ funksiyaning asosiy davrini toping.

Yechish. Birinchi qo‘shiluvchi uchun asosiy davr $\frac{2\pi}{6}=\frac{\pi}{3}$ bo‘ladi, ikkinchi qo‘shiluvchi uchun davr $\frac{\pi}{4}$ bo‘ladi. $\frac{\pi}{3}$ va $\frac{\pi}{4}$ sonlarining eng kichik umumiy karralisi bo‘lgan π funksiyaning asosiy davri bo‘ladi. $f(x+T)=f(x); T=\pi$.

21.16. $y=\cos\left(\sqrt{x}\right)^2$ funksiyaning asosiy davrini toping.

Yechish. $y=\cos\left(\sqrt{x}\right)^2$ funksiya davriy emas, chunki uning aniqlanish sohasi bo‘lgan $D(y)=[0;+\infty)$ to‘plam quyidan chegaralanganligi uchun davriy to‘plam emas.

Funksiyalarning asosiy davrini toping:

21.17. $y=2$ **21.18.** $y=Sin\frac{3}{2}x+1$

21.19. $f(x)=x^3-3x^2+3x$ funksiyaga teskari funksiyani toping:

Yechish. $y=(x-1)^3+1$, $(x-1)^3=y-1$ va $x=\sqrt[3]{y-1}+1$. x ni y ga almashtirib quyidagi teskari funksiyani olamiz: $f^{-1}(x)=\sqrt[3]{x-1}+1$.

Quyidagi funksiyalarning teskari funksiyalarini quring:

21.20. $y=2-5x$ **21.21.** $y=\dfrac{2}{x+1}-3$

21.22. $f(x,y)=\arcsin(x+y)$ funksiyaning aniqlanish sohasini toping:

Yechish. $f(x,y)=\arcsin(x+y)$ funksiya ma'noga ega bo'lishi uchun x va y lar ushbu $-1\le x+y\le 1$ munosabatda bo'lishi lozim. Bu tengsizliklarni tekislikning $x+y+1=0$ va $x+y-1=0$ to'g'ri chiziqlar orasidagi nuqtalarning koordinatalari qanoatlantiradi. Demak, $D(f)=\{\,(x,y)\in R^2;\,|x+y|\le 1\}$.

Quyidagi funksiyalarning aniqlanish sohasini toping va koordinatalar tekisligida tasvirlang:

21.23. $f(x,y)=\sqrt{x+y}$ **21.24.** $f(x,y)=\sqrt{-x}+\sqrt{y}$

Quyidagi funksiyalarning aniqlanish sohasini toping:

21.25. $y=\dfrac{1}{\lg(1-x)}+\sqrt{x+2}$ **21.26.** $y=\sqrt{Sin(x)}-\sqrt{9-x^2}$

21.27. $y=\arcsin\dfrac{1+x^2}{2x}$ **21.28.** $y=x^{\frac{1}{\lg x}}$

21.29. $y=tg\sqrt{16-x^2}$ **21.30.** $y=\dfrac{1}{\sqrt{|x|-2|x-1|}}$

21.31. $f(x)=\ln Cos(x)$. **21.32.** $y=\sqrt{x^2+x-2}$

21.33. $y=\log_2(-x)$ **21.34.** $y=\arccos\left(\dfrac{x}{2}-1\right)$

21.35. $y=\sqrt{3^x-5^x}$ **21.36.** $y=\dfrac{1}{xe^x}$

Quyidagi funksiyalarning qiymatlar to'plamini toping:

21.37. $y=\dfrac{x+1}{x-2}$ **21.38.** $y=\dfrac{1}{x^2+1}+1$

21.39. $y=\dfrac{1}{\arcsin(1-x)}$ **21.40.** $y=Sin^2x-Cos^2x$

21.41. $y=-x^2-5x+6$ **21.42.** $y=1-|x|$

21.43. $y=\dfrac{2-x}{x+3}$ **21.44.** $y=\lg\dfrac{2}{\sqrt{4-x^2}}$

21.45. $y=\sqrt{Sin\,x-1}$

Quyidagi funksiyalarning har biri ko‘rsatilgan oraliqlarda monoton ekanligini ko‘rsating:

21.46. $f(x)=x^2+2x+5; \quad x\in(-\infty;-1)\cup(-1;+\infty)$,

21.47. $f(x)=\cos x; \quad x\in[0;\pi]$,

21.48. $f(x)=\sin x; \quad x\in\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$,

21.49. $f(x)=|x|-x; \quad x\in(-\infty;+\infty)$,

21.50. $f(x)=|x|+x; \quad x\in(-\infty;+\infty)$.

Quyidagi funksiyalarning juft yoki toqligini tekshiring:

21.51. $y=|x|-5e^{x^2}$

21.52. $y=\frac{|Sinx|}{1-Cosx}$

21.53. $y=Sin(arcCos\,x)$

21.54. $y=\left|\frac{10^x+1}{10^x-1}\right|$

21.55. $y=\lg\frac{x+3}{x-3}$

21.56. $f(x)=\begin{cases}1, & agar\ \ x-ratsional\ son\ bo`lsa\\ -1, & agar\ \ x-irratsional\ son\ bo`lsa\end{cases}$

21.57. $y=x^2Sin\frac{1}{x}$

21.58. $y=\lg Cos\,x$

21.59. $y=\frac{16^x-1}{4^x}$

21.60. $y=x^4Sin7x$

Funksiyalarning asosiy davrini toping:

21.61. $y=Sin\,x-Cos\,x$

21.62. $y=Sin2x-2tg\left(\frac{x}{2}\right)$

21.63. $y=Cos^2x$

21.64. $y=x-[x]$

21.65. $y=-2Cos\frac{x}{3}+1$

21.66. $y=Sin\frac{x}{3}+ctg\frac{x}{4}$

21.67. $y=2^{Sin2x}\cdot2^{Cos2x}$

21.68. $y=\log_2 Sin\ x$

Quyidagi funksiyalarning teskari funksiyalarini quring:

21.70. $y=\frac{x+2}{3-2x}$

21.71. $y=\frac{1}{2-x}+4$

Quyidagi funksiyalarning aniqlanish sohasini toping va koordinatalar tekisligida tasvirlang:

21.72. $f(x,y)=\arccos\frac{x^2+y^2}{9}$

21.73. $f(x,y)=\sqrt{x-\sqrt{y}}$

21.74. $f(x,y)=1+\sqrt{-(x-y)^2}$

21.75. $f(x,y)=\frac{1}{x-1}+\frac{1}{y}$

21.76. $f(x,y)=\dfrac{\sqrt{4x-y^2}}{\lg(1-x^2-y^2)}$

21.77. $f(x,y)=\sqrt{x^2-4}+\sqrt{1-y^2}$

21.78. $f(x,y)=\sqrt{x}+\sqrt{y}$

21.79. $f(x,y)=\sqrt{1-x^2-y^2}$

21.80. $f(x,y)=\sqrt{y\,Sin\,x}$

21.81. $f(x,y)=\ln(x+y)$

21.81. $f(x,y)=\ln\left(\dfrac{x^2}{9}-\dfrac{y^2}{4}-1\right)$

21.82. $f(x,y)=\sqrt{Cos(x^2+y^2)}$

21.83. $f(x,y)=\sqrt{x+y}-\sqrt{x-y}$

21.84. $f(x,y)=\sqrt{25-x^2-y^2}$

21.85. $f(x,y)=\ln(y^2-4x+8)$

21.86. $f(x,y)=\dfrac{1}{2-x^2-y^2}$

21.87. $f(x;y;z)=\dfrac{z}{x-y}$

21.88. $f(x;y)=\arcsin\dfrac{y-1}{x}$

21.89. $f(x;y)=\arcsin\dfrac{x}{2}+\arccos\dfrac{y}{2}$

21.90. $f(x;y;z)=\dfrac{1}{\sqrt{x}}+\dfrac{1}{\sqrt{y}}+\dfrac{1}{\sqrt{z}}$

22-amaliy mashg‘ulot. Funksiya limiti

Funksiya limiti ta’rifidan foydalanib quyidagilarni isbotlang:

22.1. $\lim\limits_{x\to 3}(3x-5)=4$

Ixtiyoriy $\varepsilon>0$ son uchun shungay $\delta>0$ topilib, $|x-3|<\delta$ tengsizlikni qanoatlantiruvchi barcha x lar uchun $|(3x-5)-4|<\varepsilon$ tengsizlik o‘rinli bo‘lishini ko‘rsatishimiz kerak. Ixtiyoriy $\varepsilon>0$ son olaylik. $|(3x-5)-4|=|3x-9|=|3(x-3)|=3|x-3|<\varepsilon$ $|x-3|<\dfrac{\varepsilon}{3}$. Agar $\delta<\dfrac{\varepsilon}{3}$ deb olsak, $|x-3|<\delta$ tengsizlikni qanoatlantiruvchi x lar uchun $|(3x-5)-4|<\varepsilon$ tengsizlik o‘rinli bo‘ladi. Shu bilan $\lim\limits_{x\to 3}(3x-5)=4$ ekanligi isbotlandi.

22.2. $\lim\limits_{x\to 1}(4x-1)=3$

22.3. $\lim\limits_{x\to\frac{\pi}{2}}Sinx=1$

22.4. $\lim\limits_{x\to 2}(x^2-1)=3$, δ ning qanday qiymatlarida $0<|x-2|<\delta$ tengsizlikdan $|(x^2-1)-3|<0.001$ tengsizlik kelib chiqadi?

$\dfrac{0}{0},\dfrac{\infty}{\infty}$ ko‘rinishidagi aniqmasliklarni oching:

22.5. $\lim\limits_{x\to 2}\dfrac{x-2}{x^2-3x+2}=\lim\limits_{x\to 2}\dfrac{x-2}{(x-2)(x-1)}=\lim\limits_{x\to 2}\dfrac{1}{x-1}=1$

22.6. $\lim\limits_{x\to3}\frac{x^2-9}{x^2-2x-3}$

22.7. $\lim\limits_{x\to\pi}\frac{tgx}{\sin 2x}$

22.8. $\lim\limits_{x\to\frac{\pi}{4}}\frac{\sin x-\cos x}{\cos 2x}$

22.9. $\lim\limits_{x\to0}\frac{x}{\sqrt{1+3x}-1}$

22.10. $\lim\limits_{x\to1}\frac{\sqrt[3]{x}-1}{\sqrt{x}-1}$

22.11. $\lim\limits_{x\to0}\frac{\sqrt[3]{1+mx}-1}{x}$

22.12. $\lim\limits_{x\to0}\frac{\sqrt{1+x}-\sqrt{1-x}}{x}$

22.13. $\lim\limits_{x\to\pi}\frac{\sqrt{1-tgx}-\sqrt{1+tgx}}{Sin2x}$

22.14. $\lim\limits_{x\to\infty}\frac{5x^3-7x}{1-2x^3}$

22.15. $\lim\limits_{x\to\infty}\frac{x^3-1}{x^2+1}$

22.16. $\lim\limits_{x\to\infty}\frac{\sqrt{x}-6x}{3x+1}$

22.17. $\lim\limits_{x\to-2}\frac{3x+6}{x^3+8}$

22.18. $\lim\limits_{x\to-1}\frac{x^2-x-2}{x^3+1}$

22.19. $\lim\limits_{x\to\pi+0}\frac{\sqrt{1+Cosx}}{Sinx}$

22.20. $\lim\limits_{x\to7}\frac{2-\sqrt{x-3}}{x^2-49}$

22.21. $\lim\limits_{x\to1}\frac{x^4-2x^3+2x^2-2x+1}{3x^4-5x^3+2x^2-x+1}$

22.22. $\lim\limits_{x\to5}\frac{\sqrt{6-x}-1}{3-\sqrt{4+x}}$

22.23. $\lim\limits_{x\to1}\frac{3x-2-\sqrt{4x^2-x-2}}{x^2-3x+2}$

22.24. $\lim\limits_{x\to0}\frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{x^2-x}$

$\lim\limits_{x\to0}\frac{Sinx}{x}=1$ ajoyib limitdan foydalanib quyidagi limitlarni toping:

22.25. $\lim\limits_{x\to0}\frac{Sin6x}{Sin7x}=\lim\limits_{x\to0}\frac{\frac{Sin6x}{6x}6x}{\frac{Sin7x}{7x}7x}=\lim\limits_{x\to0}\frac{6x}{7x}=\frac{6}{7}$

22.26. $\lim\limits_{x\to0}\frac{Sin4x}{x}$

22.27. $\lim\limits_{x\to0}\frac{1-Cos2x}{xSinx}$

22.28. $\lim\limits_{h\to0}\frac{Sin(x+h)-Sin(x-h)}{h}$

22.29. $\lim\limits_{x\to0}\frac{1-Cosx}{x^2}$

22.30. $\lim\limits_{x\to0}\frac{Sin4x}{\sqrt{x+1}-1}$

22.31. $\lim\limits_{x\to0-0}\frac{\sqrt{1-Cos2x}}{x}$

22.32. $\lim\limits_{x\to0}\frac{2x\cdot Sinx}{\sec x-1}$

22.33. $\lim\limits_{x\to0}\frac{1-Cos2x+tg^2x}{xSinx}$

22.34. $\lim\limits_{x\to-2}\frac{\arcsin(x+2)}{x^2+2x}$

22.35. $\lim\limits_{x\to0}\frac{1-Cos5x}{1-Cos3x}$

22.36. $\lim\limits_{x\to\frac{\pi}{2}}\frac{Cos5x}{Sin4x}$

22.37. $\lim\limits_{x\to 0}\frac{\sqrt{1+Cosx}-\sqrt{2Cosx}}{\sqrt{3+Cosx}-2\sqrt{Cosx}}$

22.38. $\lim\limits_{x\to\frac{\pi}{2}}\frac{1-Sinx}{(\frac{\pi}{2}-x)}$

22.39. $\lim\limits_{x\to\frac{\pi}{2}}\frac{Cos\frac{x}{2}-Sin\frac{x}{2}}{Cosx}$

Limitlarni toping:

22.40. $\lim\limits_{x\to 1}\frac{x^2-x-2}{x^3+1}$

22.41. $\lim\limits_{x\to 3}\frac{9-x^2}{\sqrt{3x}-3}$

22.42. $\lim\limits_{x\to a}\frac{\sqrt{ax}-x}{x-a}$

22.43. $\lim\limits_{x\to\infty}\frac{5x^2-3x+2}{2x^2+4x+1}$

22.44. $\lim\limits_{x\to\infty}\frac{5x^2+2^{\frac{1}{x}}}{1-x^2}$

22.45. $\lim\limits_{x\to\frac{\pi}{4}}\frac{Sin2x-Cos2x-1}{Cosx-Sinx}$

22.46. $\lim\limits_{x\to 0}\frac{Sin^2\frac{x}{2}}{x^2}$

22.47. $\lim\limits_{x\to 0}\frac{Sin3x}{\sqrt{x+2}-\sqrt{2}}$

22.48. $\lim\limits_{x\to\frac{1}{5}}\frac{\arcsin(1-2x)}{4x^2-1}$

22.49. $\lim\limits_{x\to 0}\frac{1-Cosmx}{x^2}$

22.50. $\lim\limits_{x\to 2}\left[\frac{Sin(x-2)}{x^2-4}+2^{\frac{-1}{(x-2)^2}}\right]$

22.51. $\lim\limits_{x\to 0}\frac{tgx-Sinx}{x^3}$

22.52. $\lim\limits_{h\to 0}\frac{Cos(x+h)-Cos(x-h)}{h}$

22.53. $\lim\limits_{x\to x_0}\frac{tgx-tgx_0}{x-x_0}$

22.54. $\lim\limits_{x\to\frac{\pi}{4}}\frac{Sinx-Cosx}{\pi-4x}$

22.55. $\lim\limits_{x\to\infty}\frac{2x^4+3x^2+5x-6}{x^3+3x^2+7x-1}$

22.56. $\lim\limits_{x\to\infty}\frac{(2x^3+4x+5)(x^2+x+1)}{(x+2)(x^4+2x^3+7x^2+x-1)}$

22.57. $\lim\limits_{x\to+\infty}(\sqrt{x^2+3x}-x)=\lim\limits_{x\to+\infty}\frac{\left(\sqrt{x^2+3x}-x\right)\left(\sqrt{x^2+3x}+x\right)}{\sqrt{x^2+3x}+x}=\lim\limits_{x\to+\infty}\frac{x^2+3x-x^2}{\sqrt{x^2+3x}+x}=\frac{3}{2}$

22.58. $\lim\limits_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$

22.59. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+x+1}-\sqrt{x^2-x}\right)$

22.60. $\lim\limits_{x\to 2}\left(\frac{1}{x-2}-\frac{12}{x^3-8}\right)$

22.61. $\lim\limits_{x\to 0}\left(\frac{1}{Sin^2x}-\frac{1}{4Sin^2\frac{x}{2}}\right)$

22.62. $\lim\limits_{x\to-\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-4x}\right)$

22.63. $\lim\limits_{x\to-2}\left(\frac{1}{x+2}+\frac{4}{x^2-4}\right)$

22.64. $\lim\limits_{x\to\pi} Sin\,2x\cdot ctgx$

22.65. $\lim\limits_{x\to\infty}\left(1-\frac{1}{x^2}\right)^x$; Bu limitni hisoblashda $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$ ajoyib limitdan foydalanamiz:

$$\lim_{x\to\infty}\left(1-\frac{1}{x^2}\right)^x=\lim_{x\to\infty}\left[\left(1+\frac{1}{-x^2}\right)^{-x^2}\right]^{\frac{x}{-x^2}}=e^{\lim\limits_{x\to\infty}\left(\frac{-x}{x^2}\right)}=e^{-\lim\limits_{x\to\infty}\frac{1}{x}}=e^0=1$$

22.66. $\lim\limits_{x\to\infty}\left(1+\frac{5}{x}\right)^{2x}$

22.67. $\lim\limits_{x\to\infty}\left(\frac{2x+1}{2x-1}\right)^x$

22.68. $\lim\limits_{x\to 0}(1-4x)^{\frac{1-x}{x}}$

22.69. $\lim\limits_{x\to\infty}\left(\frac{x+8}{x-2}\right)^x$

22.70. $\lim\limits_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+1}\right)^x$

22.71. $\lim\limits_{x\to\infty}\left(\frac{3x+1}{3x-1}\right)^{3x+1}$

22.72. $\lim\limits_{x\to\infty}\left(\frac{2x}{2x-3}\right)^{2-5x}$

Ko‘p o‘zgaruvchili funksiyalarning limitini toping:

22.73. $\lim\limits_{\substack{x\to 0\\ y\to 3}}\frac{Sin(x^2\cdot y)}{x^2}$

$$\lim_{\substack{x\to 0\\ y\to 3}}\frac{\sin(x^2\cdot y)}{x^2}=\lim_{\substack{x\to 0\\ y\to 3}}\frac{\sin(x^2\cdot y)}{x^2\cdot y}\cdot y=\lim_{\substack{x\to 0\\ y\to 3}}y=3$$

22.74. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{a-\sqrt{a^2-xy}}{xy},\quad a\neq 0$

22.75. $\lim\limits_{\substack{x\to 3\\ y\to 0}}\frac{tg(xy)}{y}$

22.76. $\lim\limits_{\substack{x\to 0\\ y\to 0}}(x^2+y^2)\cdot Sin\frac{1}{xy}$

22.77. $\lim\limits_{\substack{x\to\infty\\ y\to a}}\left(1+\frac{y}{x}\right)^x$

22.78. $\lim\limits_{\substack{x\to 0\\ y\to 0}}\frac{\sqrt{1+x^2y^2}-1}{x^2+y^2}$

22.79. $\lim\limits_{\substack{x\to 0\\ y\to 0}}(1+x^2+y^2)^{\frac{1}{x^2+y^2}}$

23-amaliy mashg‘ulot. Funksiya uzluksizligi

23.1. Quyidagi fuksiyalarning ko‘rsatilgan nuqtalarida bir tomonli limitlarini toping:

a) $f(x)=\begin{cases}x+1, & agar\ \ 0<x<1\\ 3x+1, & agar\ \ 1\le x\le 3\end{cases}$ x=1 nuqtasida

$$f(1-0)=\lim_{x\to 1-0}f(x)=\lim_{x\to 1-0}(x+1)=2$$

$$f(1+0)=\lim_{x\to 1+0} f(x)=\lim_{x\to 1+0}(3x+1)=4$$

b) $f(x)=\begin{cases} 3x, & agar\ -1\le x\le 1 \\ 2x, & agar\ 1<x\le 3 \end{cases}$ x=1, x=2 nuqtalarda

c) $y=\{x\},\quad \{x\}-x$ ning kasr qismi; x=1, x=2, x=3 nuqtalarda

d) $f(x)=\dfrac{3x+1}{x-1},\ x=1$ nuqtada

23.2. Quyidagi funksiyalarning uzluksizligini ta'rifga binoan isbotlang.

a) $f(x)=x^2+x-2$ barcha $x\in(-\infty;+\infty)$ larda

$$\lim_{\Delta x\to 0}(f(x+\Delta x)-f(x))=\lim_{\Delta x\to 0}((x+\Delta x)^2+(x+\Delta x)-2-(x^2+x-2))=$$

$$=\lim_{\Delta x\to 0}(2x\cdot\Delta x+\Delta x^2+\Delta x)=0$$

Demak, *f(x)* barcha $x\in(-\infty;+\infty)$ larda uzluksiz.

b) $f(x)=\sin(3x+2)$, barcha $x\in(-\infty;+\infty)$ larda

c) $f(x)=\dfrac{1}{x+1}$, barcha $(-1;+\infty)$ larda

Quyidagi funksiyalarning uzilish nuqtalari va ularning turlarini aniqlang. Grafiklarini yasang:

23.3. $f(x)=\begin{cases} x^2, & agar\ -\infty<x\le 0 \\ (x-1)^2, & agar\ 0<x\le 2 \\ 5-x, & agar\ 2<x<\infty \end{cases}$

f(x) funksiya $(-\infty;0)$, $(0;2)$ va $(2;+\infty)$ intervallarda aniqlangan va uzluksiz bo'lgan elementar funksiyalar bilan berilgan. Demak, faqat $x_1=0\ va\ x_2=2$ nuqtalarda uzulishga ega bo'lishi mumkin.

$x_1=0$ nuqta uchun chap va o'ng limitlarni hisoblaymiz:

$$\lim_{x\to 0-0} f(x)=\lim_{x\to 0-0} x^2=0;$$

$$\lim_{x\to 0+0} f(x)=\lim_{x\to 0+0}(x-1)^2=1;\quad f(0)=0$$

Bu esa $x_1=0$ nuqtada *f(x)* fuksiya birinchi tur uzilishga ega bo'lishini bildiradi.

$x_2=2$ nuqta uchun:

$$\lim_{x\to 2-0} f(x)=\lim_{x\to 2-0}(x-1)^2=1$$

$$\lim_{x\to 2+0} f(x)=\lim_{x\to 2+0}(5-x)=3\quad f(2)=1$$

bo'ladi.

$x_2=2$ nuqtada funksiya 1-tur uzilishga ega bo'ladi.

23.4. $y=\dfrac{4}{x-2}$

23.5. $f(x)=\begin{cases} \dfrac{x}{2}, & x\ne 2 \\ 0, & x=2 \end{cases}$

23.6. $y=1+\frac{|x+1|}{x+1}$ **23.7.** $y=2^{\frac{1}{x}}$

23.8. $f(x)=\begin{cases}0.5x^2, & agar\ |x|<2\\ 2.5, & agar\ |x|=2\\ 3, & agar\ |x|>2\end{cases}$ **23.9.** $f(x)=\begin{cases}Sinx, & x<0\\ x, & 0\le x\le 2\\ 0, & x>2\end{cases}$

23.10. $f(x)=\begin{cases}-1, & x<0\\ Cosx, & 0\le x\le \pi\\ 1-x, & x>\pi\end{cases}$ **23.11.** $f(x)=\begin{cases}-x, & x\le 1\\ \frac{2}{x-1}, & x>1\end{cases}$

23.12. $f(x)=\begin{cases}-\frac{1}{x}, & x<0\\ 1, & 0\le x<1\\ x, & 1\le x\le 2\\ 3, & 2<x\le 3\end{cases}$

Funksiyalarning uzilish nuqtalarini toping va uzilish turlarini aniqlang:

23.13. $y=\frac{4}{4-x^2}$ **23.14.** $y=5^{\frac{4}{1-x}}+1$

23.15. $y=\frac{x+5}{x-3}$ **23.16.** $f(x)=9^{\frac{1}{2-x}}$

Quyidagi tenglamalar ko‘rsatilgan kesmalarda yechimga ega ekanligini ko‘rsating:

23.17. a) $x^3+3x+1=0;\ [-1;0]$ kesmada. Bu funksiya $[-1;0]$ da uzluksiz. Kesmaning uchlaridagi qiymatlari $f(1)=-3,\quad f(0)=1$ bo‘lib, turli ishorali. Boltsano-Koshi teoremasiga binoan (-1; 0) da biror *C* nuqta topib $f(x)=c^3+3c+1=0$ bo‘lib, *c* berilgan tenglamalarning yechimi bo‘ladi.

b) $x^5-6x^2+3x-7=0;\quad [0;2]$

c) $3Sin^3x-5Sinx+1=0;\ \left[0;\frac{\pi}{2}\right]$

d) $Cos^4x+3Cosx+1=0;\quad [0;\pi]$

23.18. Quyidagi funksiyalar ko‘rsatilgan kesmalarda chegaralangan ekanligini isbotlang:

a) $f(x)=Sinx\cdot Cos^2x-\sqrt{x+1},\quad [0;10]$

$y=Sinx,\ y=Cos^2x\ va\ y=\sqrt{x+1}$ funksiyalarning har biri $[0;10]$ da uzluksiz bo‘lganligi uchun, $f(x)=Sinx\cdot Cos^2x-\sqrt{x+1}$ funksiya ham [0;10] da uzluksiz. Shuning uchun Veyershtrass teoremasiga binoan *f(x)* funksiya [0;10] da chegaralangan.

b) $f(x)=\frac{\sqrt{x^2+3x+1}}{x-1}$, [2;7]

c) $f(x)=\sqrt{x^2+3x+1}\cdot Cos^7 x,\ \ [0;2\pi]$

23.19. Bir tomonlama limitlarini toping:

a) $f(x)=\begin{cases} x^2, & agar\ \ -1<x\le 2 \\ 2x+1, & agar\ \ 2<x<3 \end{cases}$ $\quad x=2$ nuqtada

b) *y=E(x), E(x) – x* ning butun qismi *x*=-2, *x*=0, *x*=1 nuqtalarda

c) $f(x)=\dfrac{1}{x-2},\ x=2$ nuqtada.

23.20. Funksiyalarning uzluksizligini ta'rifga binoan izbotlang:

a) $f(x)=x^3-3$ $\qquad$ b) $f(x)=Cos(2x+1)$

Quyidagi funksiyalarning uzilish nuqtalari va uzilish turlarini aniqlang:

23.21. $y=\dfrac{x}{x+2}$ $\qquad$ **23.22.** $y=2-\dfrac{|x|}{x}$ $\qquad$ **23.23.** $y=\dfrac{1}{1+2^{\frac{1}{x}}}$

23.24. $y=2^{\frac{1}{x-2}}$ $\qquad$ **23.25.** $f(x)=\begin{cases} 2, & agar\ x=0\ va\ x=\pm 2 \\ 4-x^2, & agar\ |x|<2 \\ 4, & agar\ |x|>2 \end{cases}$

24-amaliy mashg'ulot. Bir o'zgaruvchili funksiya hosilasi va differensiali

24.1. Hosila ta'rifidan foydalanib, $y=2x^3+5x^2-7x-4$ funksiya uchun y' hosilasini toping.

Yechish. $y=2x^3+5x^2-7x-4$ funksiya orttirmasini topamiz:

$$\Delta y=(2(x+\Delta x)^3+5(x+\Delta x)^2-7(x+\Delta x)-4)-(2x^3+5x^2-7x-4)=$$
$$=6x^2\Delta x+6x\Delta x^2+2\Delta x^3+10x\Delta x+5\Delta x^2-7\Delta x$$

$\Delta x\to 0$ da quyidagi limitni topamiz:

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}(6x^2+6x\Delta x+2\Delta x^2+10x+5\Delta x-7)=6x^2+10x-7$$

Shunday qilib, ta'rifga ko'ra hosila $y'=6x^2+10x-7$.

Ta'rifdan foydalanib, $y=f(x)$ funksiyalar uchun y' hosilani toping:

24.2. a) $y=5x-2$ b) $y=x^3$ $\qquad$ c) $y=\sqrt{x}$ $\qquad$ d) $y=\dfrac{1}{x}$

24.3. Differensiallash qoida va formulalaridan foydalanib, $y=\left(x^4-x\right)(3tgx-1)$ funksiyaning hosilasini toping.

Yechish. Ko'paytmaning hosilasi uchun formuladan foydalanamiz:

$$y' = \left[\left(x^4 - x\right)\left(3tgx - 1\right)\right]' = \left(x^4 - x\right)'\left(3tgx - 1\right) + \left(x^4 - x\right)\left(3tgx - 1\right)' =$$
$$= \left(4x^3 - 1\right)\left(3tgx - 1\right) + \left(x^4 - x\right)\cdot\frac{3}{\cos^2 x}.$$

Quyidagi funksiyalarning hosilasini toping:

24.4. $y = x + \frac{1}{x^2} - \frac{1}{5x^5}$

24.5. $y = 3x^2 + 5\sqrt[3]{x^5} - \frac{4}{x^3}$

24.6. $y = x^3 Sinx$

24.7. $y = Sinx \cdot \ln x$

24.8. $y = \frac{x^4 + 1}{x^4 - 1}$

24.9. $y = x^2 ctgx$

24.10. *y=x arccos x*

24.11. $y = e^x arctgx$

24.12. $y = \frac{Cosx}{x^2}$

24.1.13. $y = \frac{x^2}{x^2 + 1}$

24.14. $y = 3x^3 \ln x - x^3$

24.15. $y = \frac{Cosx}{1 - Sinx}$

24.16. $y = \frac{\sqrt{x}}{\sqrt{x} + 1}$

24.17. $y = x^2 \log_3 x$

24.18. $y = \frac{\ln x}{Sinx} + x\,ctgx$

24.19. $y = \frac{x\,tgx}{1 + x^2}$

24.20. $y = arctgx - arcctgx$

24.21. a) $y = |\ln x|$ funksiya x=1 da hosilaga egami? Tekshiring.

b) $y = |x|$ funksiyani x=0 da bir tomonli hosilalarini toping. Bu funksiya x=0 da hosilaga egami?

24.22. Murakkab funksiyani differensiallash qoidalarini qo'llab y funksiyaning hosilasini toping:

Yechish. $y = \frac{1}{3}Sin^3\sqrt{x} - \frac{2}{5}Sin^5\sqrt{x} + \frac{1}{7}Sin^7\sqrt{x}$

$$y` = \frac{1}{3}\cdot 3Sin^2\sqrt{x}Cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}} - \frac{2}{5}\cdot 5Sin^4\sqrt{x}\cdot Cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}} + \frac{1}{7}\cdot 7Sin^6\sqrt{x}\cdot Cos\sqrt{x}\cdot\frac{1}{2\sqrt{x}} =$$
$$= \frac{1}{2\sqrt{x}}Sin^2\sqrt{x}\cdot Cos\sqrt{x}\cdot\left(1 - 2Sin^2\sqrt{x} + Sin^4\sqrt{x}\right) = \frac{1}{2\sqrt{x}}\cdot Sin^2\sqrt{x}\cdot Cos^5\sqrt{x}$$

Funksiya hosilasini toping:

24.23. $y = \ln\left(2x^3 + 3x^2\right)$

24.24. $y = \sqrt{4x + Sin4x}$

24.25. $y = \sqrt{\frac{x}{2} - Sin\frac{x}{2}}$

24.26. $y = e^{\frac{x}{a}}\cdot Cos\frac{x}{a}$

24.27. $y = \sqrt{1 - 3x^2}$

24.28. $y = Cos^3\left(\frac{x}{3}\right)$

24.29. $y = -ctg^2\frac{x}{2} - 2\ln\left(Sin\frac{x}{2}\right)$

24.30. $y = \arccos\frac{9 - x^2}{9 + x^2}$

24.31. $y = 1 - e^{Sin^2 3x} \cdot Cos^2 3x$

24.32. $y = e^{\sqrt{2x}}\left(\sqrt{2x} - 1\right)$

24.33. $y = -\cos ec^2\left(\frac{x}{2}\right)$

24.34. $y = \arcsin\sqrt{1 - 0.2x^2}$

24.35. $y = \frac{1}{\sqrt{1 - mx^2}}$

24.36. $y = \frac{Sinx}{1 + \ln Sinx}$

24.37. $y = arctg(x+1) + \frac{x+1}{x^2 + 2x + 2}$

24.38. $y = \ln tg\frac{x}{2} + Cosx + \frac{1}{3}Cos^3 x$

24.39. $y = \ln\left(1 - \frac{1}{x}\right) + \frac{1}{x}$

24.40. $y = \ln\ln x(\ln\ln\ln x - 1)$

24.41. $y = tg^3 tgx + 3tgtgx$

24.42. $y = 2^{Cos3x - 3Cosx}$

24.43. $y = \frac{x^2 e^{x^2}}{x^2 + 1}$

24.44. $y = arctg\frac{2x^4}{1 - x^2}$

24.45. $y = \arccos\sqrt{1 - 2^x}$

24.46. $y = \log_2 Sin^2 x$

24.47. $y = xe^x(Sinx - Cosx) + e^x Cosx$

24.48. $y = \log_x e$

24.49. $y = x^x$

24.50. $y = \sqrt[3]{x\sqrt{x\sqrt{x}}}$

24.51. $y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)}}$

24.52. Oshkormas ko'rinishda berilgan $x^3 + \ln y - x^2 e^y = 0$ funksiyaning hosilasini toping.

Yechish. $x^3 + \ln y - x^2 e^y = 0$

$$3x^2 + \frac{y`}{y} - x^2 e^y y` - 2xe^y = 0, \quad ya'ni \quad y` = \frac{\left(2xe^y - 3x^2\right)y}{1 - x^2 ye^y}$$

Oshkormas ko'rinishda berilgan funksiyalar hosilasini toping:

24.53. $Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0.$

24.54. $x^4 - 6x^2 y^2 + 9y^4 - 5x^2 + 15y^2 - 100 = 0$

24.55. $x^y - y^x = 0$

24.56. $xSiny + ySinx = 0$

24.57. $e^x + e^y - 2^{xy} - 1 = 0$

24.58. $Sin\left(y - x^2\right) - \ln\left(y - x^2\right) + 2\sqrt{y - x^2} - 3 = 0$

24.59. $\frac{y}{x} + e^{\frac{y}{x}} - 3\sqrt{\frac{y}{x}} = 0$

24.60. Parametrik ko'rinishda berilgan $\begin{cases} x = 2\cos t, \\ y = 3\sin t. \end{cases}$ funksiyaning hosilasini toping:

Yechish. Funksiya hosilasini $y' = \frac{y'(t)}{x'(t)}$ formuladan topamiz

$$y'(x) = \frac{(3\sin t)'}{(2\cos t)'} = -\frac{3\cos t}{2\sin t} = -1{,}5ctgt.$$

Parametrik ko‘rinishda berilgan quyidagi funksiyalarning hosilasini toping:

24.61. $\begin{cases} x = aCost \\ y = aS\text{int} \end{cases}$ **24.62.** $\begin{cases} x = aCos^3 t \\ y = aSin^3 t \end{cases}$

24.63. $\begin{cases} x = \dfrac{3at}{1+t^2} \\ y = \dfrac{3at^2}{1+t^3} \end{cases}$ **24.64.** $\begin{cases} x = cht \\ y = sht \end{cases}$

24.65. $x^2 + 2xy^2 + 3y^4 = 6$ egri chiziqqa M(1; -1) nuqtada o‘tkazilgan urinma va normal tenglamalari yozilsin.

Yechish. Egri chiziq tenglamasidan *y’* hosilani topamiz:

$$2x + 2y^2 + 4xyy` + 12y^3 y` = 0, \quad \text{ya’ni} \quad y` = -\frac{x + y^2}{2xy + 6y^3}.$$

Demak $y'(1;-1) = -\dfrac{1+(-1)^2}{2\cdot 1\cdot(-1) + 6\cdot(-1)^3} = -\dfrac{1}{4}.$

Urinma tenglamasi:

$$y + 1 = \frac{1}{4}(x-1), \qquad x - 4y - 5 = 0$$

Normal tenglamasi:

$$y + 1 = -4(x-1), \qquad 4x + y - 3 = 0.$$

Quyidagi masalalarda egri chiziqlarga o‘tkazilgan urinmalarning tenglamalari yozilsin va egri chiziqlar hamda urinmalari yasalsin:

24.66. $y = \dfrac{x^3}{3}$ $x = -1$ nuqtada;

24.67. $y = \dfrac{8}{4 + x^2}$ $x = 2$ nuqtada;

24.68. $y = Sinx$ $x = \pi$ nuqtada;

24.69. $y = x^2 + 2x - 1$ parabolaning $y = 2x^2$ parabola bilan kesishgan nuqtasida o‘tkazilgan urinma va normal tenglamalarini yozing.

24.70. $y = x^2$ *va* $y^2 = x$ parabolalar qanday burchak ostida kesishadi?

24.71. $y = Sinx$ sinusoida *Ox* o‘qini qanday burchak ostida kesib o‘tadi?

24.72. $y = x - x^3$ *va* $y = 5x$ chiziqlar orasidagi burchakni toping.

24.73. $y = 1 + Sinx,\ y = 1$ chiziqlar orasidagi burchakni toping.

24.74. $y = \sin 3x$ funksiya uchun y''' ni toping.

Yechish. Birinchi tartibli hosilani topamiz:

$$y' = (\sin 3x)' = 3\cos 3x.$$

Ikkinchi tartibli hosilani topamiz:

$$y'' = (3\cos 3x)' = -9\sin 3x.$$

Uchinchi tartibli hosilani topamiz:

$$y''' = (-9\sin 3x)' = -27\cos 3x.$$

24.75. Parametrik ko‘rinishda berilgan $\begin{cases} x = t^2, \\ y = t^3. \end{cases}$ funksiya uchun y''_{xx} ni topamiz.

Yechish. $y''_{xx} = \dfrac{x'_t \cdot y''_{tt} - y'_t \cdot x''_{tt}}{(x'_t)^3}$ formuladan foydalanamiz.

Bu yerda $y''_{xx} = \dfrac{(t^2)' \cdot (t^3)'' - (t^3)' \cdot (t^2)''}{\left[(t^2)'\right]^3} = \dfrac{2t \cdot 6t - 3t^2 \cdot 2}{(2t)^3} = \dfrac{6t^2}{8t^3} = \dfrac{3}{4t}.$

24.76. $y = \ln x$ funksiyaning n-tartibli hosilasi topilsin:

Yechish.

$$y' = \frac{1}{x} = x^{-1};\ y'' = -1 \cdot x^{-2};\ y''' = -1 \cdot (-2) \cdot x^{-3};\ y^{(IV)} = -1 \cdot 2 \cdot 3 \cdot x^{-4};$$

$$y^{(n)} = (-1)^{n-1} \cdot 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot x^{-n} = (-1)^{n-1} \cdot \frac{(n-1)!}{x^n}.$$

Quyidagi funksiyalarning ko‘rsatilgan tartibli hosilalarini toping.

24.77. Funksiyalarning 2-tartibli hosilalari topilsin:

1) $y = Sin^2 x;$ 2) $y = tgx;$ 3) $y = \sqrt{1 + x^2}.$

24.78. Funksiyalarning 3-tartibli hosilalari topilsin:

1) $y = x\ln x;$ 2) $s = t \cdot e^{-t};$ 3) $y = arctg\dfrac{x}{a}.$

Quyidagi funksiyalarning n-tartibli hosilalari topilsin:

24.79. $y = e^{-\frac{x}{a}};$

24.80. $y = \sqrt{x};$

24.81. $y = x^n;$

24.82. $y = Sinx;$

24.83. $y = Cos^2 x;$

24.84. $\begin{cases} x = \ln t \\ y = 1/t \end{cases}$

24.85. $y = e^{x^3}$ funksiya differensialini toping.

Yechish. $dy = y'dx,$ berilgan holatda $dy = \left(e^{x^3}\right)' dx = 3x^2 \cdot e^{x^3} dx.$

24.86. Quyidagi funksiyalarning differensialini toping:

a) $y = \dfrac{3}{4}x\sqrt[3]{x}$ b) $y = (x^2 + 2x + 2)e^{-x}$

c) $y = x^2 Sinx + 2xCosx - 2Sinx$ d) $y = \frac{2^{3x}}{3^{2x}}$

24.87. $y = arctg\frac{x}{a}$; $dy - ?$ **24.88.** $y = \frac{(x+1)^2}{(x+2)^3(x+3)^4}$ $dy - ?$

24.89. Taqribiy hisoblang: a) $\ln 1{,}02$; b) $\sqrt{24}$

Yechish. $f\left(x_0 + \Delta x\right) \approx f\left(x_0\right) + f'\left(x_0\right)\Delta x$ formuladan foydalanamiz.

a) $f\left(x\right) = \ln x, \quad \ln\left(x_0 + \Delta x\right) \approx \ln x_0 + \frac{1}{x_0}\cdot\Delta x.$

$$x_0 = 1,\ \Delta x = 0{,}02.\ \ln 1{,}02 \approx \ln 1 + \frac{1}{1}\cdot 0{,}02 = 0{,}02.$$

Shunday qilib, $\ln 1{,}02 \approx 0{,}02.$

b) $f\left(x\right) = \sqrt{x},\ x_0 = 25,\ \Delta x = -1$

$$\sqrt{x_0 + \Delta x} \approx \sqrt{x_0} + \frac{1}{2\sqrt{x_0}}\cdot\Delta x,$$

$$\sqrt{24} \approx \sqrt{25} + \frac{1}{2\sqrt{25}}\cdot\left(-1\right) = 4{,}9.\ \ \sqrt{24} \approx 4{,}9.$$

Quyidagilarni taqribiy hisoblang:

24.90. $\sqrt[3]{1.02}$ **24.91.** $\sqrt[5]{33}$

24.92. $Sin29^o$ **24.93.** $arctg1.05$

24.94. $tg46^o$ **24.95.** $Cos31^o$

24.96. $\ln tg47^o15`$ **24.97.** $\sqrt[4]{15.8}$

24.98. $y = \sqrt[3]{x}$ funksiya uchun $dy,\ d^2y$ va d^3y ni toping.

Yechish.

$$dy = y'dx = \left(\sqrt[3]{x}\right)' dx = \frac{1}{3}x^{-\frac{2}{3}}dx = \frac{dx}{3\sqrt[3]{x^2}},$$

$$d^2y = d\left(dy\right) = d\left(\frac{dx}{3\sqrt[3]{x^2}}\right) = \left(\frac{dx}{3\sqrt[3]{x^2}}\right)' dx =$$

$$= \frac{1}{3}\left(x^{-\frac{2}{3}}\right)'\left(dx\right)^2 = -\frac{2}{9}x^{-\frac{5}{3}}dx^2 = -\frac{2dx^2}{9x\sqrt[3]{x^2}}.$$

$$d^3y = d\left(d^2y\right) = d\left(-\frac{2dx^2}{9x^{\frac{5}{3}}}\right) = -\frac{2}{9}\left(x^{-\frac{5}{3}}\right)' dx^3 = \frac{10}{27}x^{-\frac{8}{3}}dx^3 = \frac{10dx^3}{27x^2\sqrt[3]{x^2}}.$$

24.99. $y = \frac{x-1}{x+1}$ funksiya uchun $dy,\ d^2y$ ni toping.

24.100. $y = x(\ln x - 1)$ funksiya uchun dy, d^2y ni toping.

24.101. $y = x^n$ funksiya uchun dy, d^2y va d^3y ni toping.

25-amaliy mashg‘ulot. Differensiallanuvchi funksiyalar va ular uchun asosiy teoremalar

25.1. $f(x) = \dfrac{x^4}{4}$ funksiya uchun [-1;1] segmentda Roll teoremasini tatbiq etish mumkinmi?

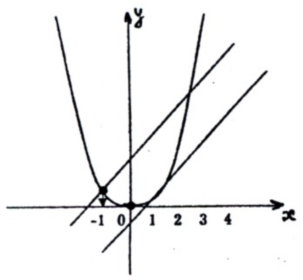
a) *f(x)* funksiya uchun Roll teoremasining birinchi sharti bajariladi: *f(x)* funksiya [-1;1]da uzluksiz.

b) *f(x)* funksiya uchun ikkinchi shart ham bajariladi. $f`(x) = x^3$ hosila mavjud.

a) *f(x)* funksiya uchun $f(-1)=f(1)=1/4$ tenglik o‘rinli. Demak, $f\ `(c)=0$ bo‘ladigan nuqta mavjud: $f`(x)=x^3=0,\ x=c=0$ da o‘rinli.

$$f\ ‘(c)=f\ ‘(0)=0$$

25.2. $y=x^2$ parabolaning qaysi nuqtasiga o‘tkazilgan urinma *A*(-1;1) va *B*(3;9) nuqtalarnui birlashtiruvchi vatarga parallel bo‘ladi?



***a*= -1; *b*=3**

AB vatarning burchak koeffisiyenti $K = \dfrac{9-1}{3+1} = 2$.

$f‘(x)=2x;\ 2x=2$ tenglik faqat $x=1$ bo‘lganda o‘rinli, demak $x=1$ nuqtaga o‘tkazilgan urinma vatarga parallel.

25.3. Roll teoremasini $f(x)=\sqrt[3]{x^2}$ funksiyaga [-1;1] segmentda tatbiq qilish mumkinmi?

25.4. $f(x)=x^2-6x+100$ funksiya uchun Roll teoremasi shartlari *x* ning qanday qiymatlarida qanoatlantiradi?

$$a=1,\ b=5.$$

25.5. [*a, b*] segmentda $f(x)=x^2$ funksiya uchun Lagranj formulasi yozilsin va *C(x,y)* nuqta topilsin

25.6. [1;4] segmentda $f(x) = \sqrt{x}$ funksiya uchun Lagranj formulasi yozilsin va *C(x,y)* nuqta topilsin.

25.7. $y=2x-x^2$ egri chiziqning *AB* yoyi ustida shunday *M(x,y)* nuqtani topingki, bu nuqtaga o‘tkazilgan urinma *AB* vatarga parallel bo‘lsin: *A*(1;1) *B*(3;-3).

25.8. $f(x)=x^3$ va $\varphi(x)=x^2$ funksiyalar uchun Koshining $\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)}=\frac{f`(c)}{\varphi`(c)}$ formulasi yozilsin hamda *c* topilsin.

Quyidagi funksiyalarni Makloren formulasi bo‘yicha yozing:

25.9. $f(x) = e^x$ **25.10.** $f(x) = Sinx$ **25.11.** $f(x)=ln(1+x)$

25.12. $\sqrt{e^x}$ ni Makloren formulasiga ko‘ra x= 1 da hisoblang (4 ta hadini olib).

Lopital qoidasi

1) $\lim\limits_{x\to 0}\frac{f(x)}{\varphi(x)}=\left\{\frac{0}{0}\right\}=\lim\limits_{x\to 0}\frac{f`(x)}{\varphi`(x)}=\left\{\frac{0}{0}\right\}=\lim\limits_{x\to 0}\frac{f``(x)}{\varphi``(x)}=...$

2) $\lim\limits_{x\to \infty}\frac{f(x)}{\varphi(x)}=\left\{\frac{\infty}{\infty}\right\}=\lim\limits_{x\to \infty}\frac{f`(x)}{\varphi`(x)}=\left\{\frac{\infty}{\infty}\right\}=\lim\limits_{x\to \infty}\frac{f``(x)}{\varphi``(x)}=...$

Quyidagi funksiyalar limitini hisoblang:

25.13. $\lim\limits_{x\to 0}\frac{Sinax}{Sinbx}=\left\{\frac{0}{0}\right\}=\lim\limits_{x\to 0}\frac{aCosax}{bCosax}=\frac{a}{b}$

25.14. $\lim\limits_{x\to 4}\frac{x^2-16}{x^2-6x+8}$ **25.15.** $\lim\limits_{x\to a}\frac{e^x-e^a}{x-a}$

25.16. $\lim\limits_{x\to 0}\frac{tgx-x}{x-Sinx}$ **25.17.** $\lim\limits_{x\to 1}\frac{1-4Sin^2\frac{\pi x}{6}}{1-x^2}$

25.18. $\lim\limits_{x\to 0}\frac{x-Sinx}{x^3}$ **25.19.** $\lim\limits_{x\to 1}\left(\frac{1}{x-1}-\frac{1}{\ln x}\right)$

25.20. $\lim\limits_{x\to\infty}(1-x)^{\frac{1}{x}}$ $y=(1-x)^{\frac{1}{x}}$ deb belgilab, tenglikning ikkala qismini lagorifmlaymiz $lny=\frac{1}{x}\ln|1-\mathrm{x}|=\frac{\ln|1-x|}{x}$. Endi limitga o‘tamiz

$$\lim_{x\to\infty}\ln y=\lim_{x\to\infty}\frac{\ln|1-x|}{x}=\left\{\frac{\infty}{\infty}\right\}=\lim_{x\to\infty}\frac{-1/|1-x|}{1}=0.\ \ln y=0,\quad y=1$$

25.21. $\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^{tgx}$ **25.22.** $\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^{Sinx}$

25.23. $\lim\limits_{x\to\frac{\pi}{2}}(tgx)^{2Cosx}$ **25.24.** $\lim\limits_{x\to 0}(Sinx)^{Sinx}$ **25.25.** $\lim\limits_{x\to 0}x\ln x$

25.26. $f(x)=\sqrt[3]{8x-x^2}$ funksiya uchun *a*=0, *b*=8 bo‘lganda Roll teoremasi shartlari *x* ning qanday qiymatlarida bajariladi?

25.27. [-1;2] segmentda 4/*x* va $1-\sqrt[3]{x^2}$ funksiyalariga Lagranj teoremasini tatbiq qilish mumkin emasligi ko‘rsatilsin.

25.28. Tenglamalari $x=t^2$, $y=t^3$ parametrik ko'rinishda berilgan egri chiziqning *AB* yoyida shunday *M* nuqtani topingki, bu nuqtada o'tkazilgan urinma *AB* vatarga parallel bo'lsin, *A* va *B* nuqtalarga *t=1, t=3* qiymatlar mos keladi.

25.29. $y=x^3-3x$ egri chiziqning *AB* yoyining qaysi nuqtasida o'tkazilgan urinma *AB* vatarga parallel bo'ladi: *A*(0;0), *B*(3;8)

25.30. Quyidagi funksiyalar uchun Lagranj formulasi yozilsin va *C(x,y)* nuqta topilsin.

1) [0;1] segmentda $f(x) = arctgx$
2) [0;1] segmentda $f(x) = arcSinx$
3) [1;2] segmentda $f(x) = lnx$

25.31. Quyidagi funksiyalar uchun Koshi formulasi yozilsin va *C* nuqta topilsin:

1) [0; π/2] segmentda *Sinx* va *Cosx*
2) [1;4] segmentda x^2 va $\sqrt{x}$

26-amaliy mashg'ulot. Teylor formulasi va qatori. Lopital qoidasi

26.1. $f(x) = \frac{1}{x}$ funksiyani $x=4$ nuqtaning atrofida Teylor qatoriga yoying.

Yechish. yuqorida keltirilgan tartibda ish bajaramiz:

1) $f'(x) = -\frac{1}{x^2},\ f''(x) = \frac{2!}{x^3}, ...,\ f^{(n)}(x) = (-1)^n \frac{n!}{x^{n+1}}, ...;$

2) $f(4) = \frac{1}{4},\ f'(4) = -\frac{1}{4^2},\ f''(4) = \frac{2!}{4^3}, ...,\ f^{(n)}(4) = (-1)^n \frac{n!}{4^{n+1}}, ...;$

3) $\frac{1}{4} - \frac{1}{4^2}(x-4) + \frac{1}{4^3}(x-4)^2 - ... + (-1)^n \frac{1}{4^{n+1}}(x-4)^n + ...;$

4) tuzilgan qatorning yaqinlashish radiusi $r=4$, yaqinlashish intervali (0;8) dan iborat;

5) $$R_n(x) = \frac{f^{(n+1)}(a+\theta(x-a))}{(n+1)!}(x-a)^{n+1} =$$
$$= \frac{(-1)^{n+1} n!}{(n+1)! \cdot 4^n \cdot (4+\theta(x-4))^n}(x-4)^n =$$
$$= \frac{(-1)^{n+1}}{(n+1)\cdot(4+\theta(x-4))^n} \cdot \frac{(x-4)^n}{4^n}$$

6) (0;8) dan olingan ixtiyoriy x uchun $\lim\limits_{n\to\infty}\left|\frac{(-1)^{n+1}}{(n+1)\cdot(4+\theta(x-4))^n} \cdot \frac{(x-4)^n}{4^n}\right| = 0$ bundan $\lim\limits_{n\to\infty} R_n(x) = 0$, demak,

$$\frac{1}{x}=\frac{1}{4}-\frac{1}{4^2}(x-4)+\frac{1}{4^3}(x-4)^2-...+(-1)^n\frac{1}{4^{n+1}}(x-4)^n+...$$

formulaning o'rinli ekanligi kelib chiqadi.

26.2. $\sum_{n=0}^{\infty}\frac{x^n}{n3^{n+1}}$ qatorning yaqinlashish radiusi, yaqinlashish intervali va sohasini toping.

Yechish. Berilgan qator uchun $a_n=\frac{1}{n3^{n+1}}$. $\lim_{n\to\infty}\sqrt[n]{|a_n|}=l$ ni hisoblaymiz: $l=\lim_{n\to\infty}\sqrt[n]{\frac{1}{n3^{n+1}}}=\frac{1}{3}\sqrt[n]{\frac{1}{n3}}=\frac{1}{3}$, demak, qatorning yaqinlashish radiusi r=3, yaqinlashish intervali (–3;3). Berilgan qatorni yaqinlashish intervali uchlarida yaqinlashishga tekshiramiz: x=3 da $\sum_{n=0}^{\infty}\frac{3^n}{n3^{n+1}}=\frac{1}{3}\cdot\sum_{n=0}^{\infty}\frac{1}{n}$. Bu esa garmonik qator, demak berilgan qator x=3 nuqtada uzoqlashuvchi.

x= –3 da $\sum_{n=0}^{\infty}\frac{(-3)^n}{n3^{n+1}}=\frac{1}{3}\cdot\sum_{n=0}^{\infty}\frac{(-1)^n}{n}$. Bu Leybnits qatori, yaqinlashuvchi.

Demak, berilgan darajali qatorning yaqinlashish sohasi [–3;3) to'plamdan iborat.

Quyidagi funksiyalarni Makloren formulasi bo'yicha yozing:

26.3. $f(x) = e^x$ **26.4.** $f(x) = Sinx$ **26.5.** $f(x)=ln(1+x)$

26.6. $\sqrt{e^x}$ ni Makloren formulasiga ko'ra x= 1 da hisoblang (4 ta hadini olib).

Quyidagi funksiyalarni Makloren formulasi bo'yicha yozing:

26.7. $f(x) =cosx$ **26.8.** $f(x) =(1+x)^a$

Quyidagi funksiyalarni $x-x_0$ ning darajalari bo'yicha Teylor qatoriga yoying, yaqinlashish intervalini ko'rsating:

26.9. $f(x)=\sqrt{x},\ x_0=4.$ **26.10.** $f(x)=e^{x+1},\ x_0=-2.$

26.11. $f(x)=\frac{1}{x^2-5x+6},\ x_0=1.$ **26.12.** $f(x)=\frac{1}{x^2},\ x_0=-1.$

26.13. $sin2x$ funksiya uchun Lagranj qoldiq hadli Makloren formulasini yozing.

26.14. $y=e^x$ funksiyaning x_0=1 nuqta atrofidagi Teylor formulasini yozing.

26.15. $\lim_{x\to-1}\frac{\sqrt{2+x}+x}{\ln(2+x)}$ ni toping.

Yechish. Bu holatda aniqmaslik $\left[\frac{0}{0}\right]$ ko'rinishda bo'lganligi sababli, Lopital qoidasini qo'llash mumkin:

$$\lim_{x\to-1}\frac{\sqrt{2+x}+x}{\ln(2+x)}=\lim_{x\to-1}\frac{(\sqrt{2+x}+x)'}{(\ln(2+x))'}=\lim_{x\to-1}\frac{\dfrac{1}{2\sqrt{2+x}}+1}{\dfrac{1}{2+x}}=\frac{3}{2}.$$

26.16. $\lim\limits_{x\to\infty}\dfrac{4^x-3^x}{x^2}$ ni toping.

Yechish. Aniqmaslik $\left[\dfrac{\infty}{\infty}\right]$ ko‘rinishda. Lopital qoidasini qo‘llab, quyidagiga ega bo‘lamiz:

$$\lim_{x\to\infty}\frac{4^x-3^x}{x^2}=\lim_{x\to\infty}\frac{4^x\ln 4-3^x\ln 3}{2x}$$

Ko‘rib turganimizdek aniqmaslik $\left[\dfrac{\infty}{\infty}\right]$ ko‘rinishda qoladi. Lopital qoidasini yana bir bor qo‘llaymiz:

$$\lim_{x\to\infty}\frac{4^x\ln 4-3^x\ln 3}{2x}=\lim_{x\to\infty}\frac{4^x\ln^2 4-3^x\ln^2 3}{2}=\infty.$$

26.17. $\lim\limits_{x\to\infty}x^{\frac{1}{\sqrt{x}}}$ ni toping.

Yechish. $\left[\infty^0\right]$ ko‘rinishdagi aniqmaslikka egamiz. $\lim\limits_{x\to\infty}\left(\ln x^{\frac{1}{\sqrt{x}}}\right)$ ni topamiz:

$$\lim_{x\to\infty}\left(\ln x^{\frac{1}{\sqrt{x}}}\right)=\lim_{x\to\infty}\frac{1}{\sqrt{x}}\ln x=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{(\ln x)'}{\left(\sqrt{x}\right)'}=\lim_{x\to\infty}\frac{\left(\dfrac{1}{x}\right)}{\left(\dfrac{1}{2\sqrt{x}}\right)}=\lim_{x\to\infty}\frac{2}{\sqrt{x}}=0$$

Demak, $\lim\limits_{x\to\infty}x^{\frac{1}{\sqrt{x}}}=e^{\lim\limits_{x\to\infty}\ln\left(x^{\frac{1}{\sqrt{x}}}\right)}=e^0=1.$

26.18. $\lim\limits_{x\to1}\left[\left(x-\sqrt{x}\right)\ln\ln x\right]$ ni toping.

Yechish. $x\to1$ da $\ln x\to0$ bo‘lganligi sababli $\ln\ln x=\ln(\ln)x\to\infty$. Shunday qilib, $[0\cdot\infty]$ ko‘rinishidagi aniqmaslikka ega bo‘lamiz. Uni aniqmaslikning $\left[\dfrac{\infty}{\infty}\right]$ ko‘rinishiga keltiramiz va Lopital qoidasini qo‘llaymiz.

$$\lim_{x\to 0}\left(x-\sqrt{x}\right)\ln\ln x=\lim_{x\to 1}\frac{\ln\ln x}{\frac{1}{x-\sqrt{x}}}=\lim_{x\to 1}\frac{\frac{1}{\ln x}\cdot\frac{1}{x}}{1-\frac{1}{2\sqrt{x}}\Big/-\frac{\cdot}{\left(x-\sqrt{x}\right)^2}}=\lim_{x\to 1}\frac{\left(\sqrt{x}-1\right)^2 2\sqrt{x}}{\left(\ln x\right)\left(2\sqrt{x}-1\right)}=$$

$$=\lim_{x\to 1}\frac{\left(\sqrt{x}-1\right)^2}{\ln x}\lim_{x\to 1}\frac{2\sqrt{x}}{2\sqrt{x}-1}=\lim_{x\to 1}\frac{\left[\frac{2\left(\sqrt{x}-1\right)}{2\sqrt{x}}\right]}{\left[\frac{1}{x}\right]}\cdot 1=0$$

26.19. $\lim\limits_{x\to\infty}\left(x\ln^2 x-\sqrt{1+x+x^2}\right)$ ni toping.

Yechish. $[\infty-\infty]$ ko‘rinishdagi aniqmaslikka egamiz. Shakl almashtiramiz:

$$\lim_{x\to\infty}\left(x\ln^2 x-\sqrt{1+x+x^2}\right)=\lim_{x\to\infty}x\ln x^2\left(1-\frac{\sqrt{1+x+x^2}}{x\ln x^2}\right).$$

Lopital qoidasini qo‘llab alohida

$$\lim_{x\to\infty}\frac{\sqrt{1+x+x^2}}{x\ln x^2}=\lim_{x\to\infty}\frac{\frac{1+2x}{2\sqrt{1+x+x^2}}}{\ln^2 x+2\ln x}=\lim_{x\to\infty}\frac{\frac{1}{x}+2}{2\sqrt{\frac{1}{x^2}+\frac{1}{x}+1}\left(\ln^2 x+2\ln x\right)}=0$$

ni topamiz. Shunday qilib, $\lim\limits_{x\to\infty}\left(x\ln^2 x-\sqrt{1+x+x^2}\right)=\lim\limits_{x\to\infty}x\ln^2 x=\infty.$

Lopital qoidasini qo‘llab, limitlarni toping.

26.20. $\lim\limits_{x\to 0}\dfrac{\ln^2(1+x)}{x}.$

26.21. $\lim\limits_{x\to 0}\dfrac{e^x+e^{-x}-2}{x^2}.$

26.22. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}+\ln(1+x)}{x}.$

26.23. $\lim\limits_{x\to\infty}\dfrac{x^2}{\ln\left(e^{x^2}+1\right)}.$

26.24. $\lim\limits_{x\to 0}x\ln^3 x.$

26.25. $\lim\limits_{x\to 0}\dfrac{e^{3x}-e^{2x}-x}{x^2}.$

26.26. $\lim\limits_{x\to\infty}x^2\left(1-e^{\frac{1}{x}}\right).$

26.27. $\lim\limits_{x\to\infty}\left(x\ln x-\sqrt{1+x^2}\right).$

26.28. $\lim\limits_{x\to -\frac{\pi}{4}}\dfrac{\sin x+\cos x}{x+\frac{\pi}{4}}.$

26.29. $\lim\limits_{x\to 0}\dfrac{arctgx}{tgx}.$

26.30. $\lim\limits_{x\to 1}\dfrac{\ln(1+x)-\ln 2}{\sqrt{1+2x}-3^{x-\frac{1}{2}}}$.

26.31. $\lim\limits_{x\to 0}\dfrac{\sin(\sin x)}{\sqrt{1+x}-1}$.

26.32. $\lim\limits_{x\to\infty}\left(x+\sqrt{x}\right)^{\frac{1}{x}}$.

26.33. $\lim\limits_{x\to 2}\left(\dfrac{x-3}{x-2}\right)^{\sqrt{2x}-2}$.

26.34. $\lim\limits_{x\to 2}\dfrac{\ln\ln\left(x^2-3\right)}{\ln(x-2)}$.

26.35. $\lim\limits_{x\to 1}\dfrac{1}{x-1}-\dfrac{1}{e^x-e}$.

26.36. $\lim\limits_{x\to 0}\left(\sin x-\sqrt{x}\right)^{x}$.

26.37. $\lim\limits_{x\to\sqrt{\pi}}\dfrac{\ln\dfrac{x^2}{\pi}}{\sin^2 x}$.

26.38. $\lim\limits_{x\to 0}\dfrac{\arcsin x}{\sin x}$.

26.39. $\lim\limits_{x\to -1}\left(\dfrac{1}{\sin(x+1)}+\ln(1+x)\right)$.

27-amaliy mashg'ulot. Bir o'zgaruvchili funksiya ekstremumlari

27.1. Ushbu $f(x)=2x^2-\ln x$ funksiyaning o'sish va kamayish intervallarini toping.

Yechish. Funksiya $(0;+\infty)$ intervalda aniqlangan. Uning hosilasi $f'(x)=4x-\dfrac{1}{x}$ ga teng. Agar $4x-\dfrac{1}{x}>0$ bo'lsa ya'ni $x>\dfrac{1}{2}$ da o'suvchi, agar $4x-\dfrac{1}{x}<0$ bo'lsa ya'ni $x<\dfrac{1}{2}$ da funksiya kamayuvchi bo'ladi. Demak funksiya $\left(0;\dfrac{1}{2}\right)$ intervalda kamayuvchi, $\left(\dfrac{1}{2};+\infty\right)$ intervalda o'suvchi bo'ladi.

Funksiyaning o'sish va kamayish oraliqlarini toping:

27.2. $f(x)=(x-2)^2\cdot(x+2)$.

27.3. $f(x)=\ln\left(x^2-2x+4\right)$.

27.4. $f(x)=x+e^{-x}$.

27.5. $f(x)=x\ln x$.

27.6. $f(x)=\dfrac{1}{1-x^2}$.

27.7. $S(t)=t+\cos t$.

27.8. Ushbu $f(x)=x^3-9x^2+15x$ funksiyaning ekstremumlarini toping.

Yechish. Funksiya sonlar o'qining barcha nuqtalarida aniqlangan va differensiallanuvchi $f'(x)=3x^2-18x+15=3(x-1)(x-5)$. $x_1=1,\ x_2=5$ kritik nuqtalar. Ekstremumning ikkinchi yetarli shartidan foydalanamiz

$$f''(x)=6x-18 \Rightarrow f''(1)=-12,\ f''(5)=12.$$

$f'(1)=0,\ f''(1)<0$ demak $x=1-$lokal maksimum nuqta, $f(1)=7.$

$f'(5)=0,\ f''(5)>0$ demak $x=5-$lokal minimum nuqta, $f(5)=-25.$

Funksiya ekstremumlarini toping.

27.9. $f(x)=\dfrac{\ln x}{x}.$ **27.10.** $f(x)=\dfrac{x}{1+x^2}.$

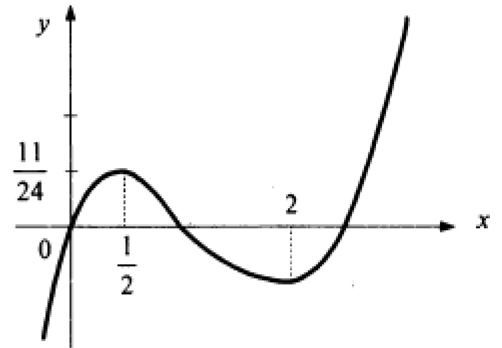
27.11. $f(x)=x^3-3x+1.$ **27.12.** $f(x)=e^{x^2-4x+5}.$

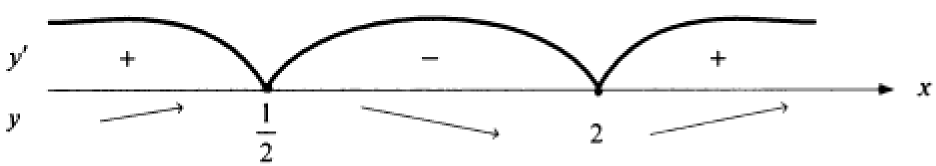
27.13. $f(x)=x-arctgx.$ **27.14.** $r=\sqrt{5-2\varphi}+\varphi.$

27.15. Funksiyaning ekstremumlari va monotonlik intervallarini toping.

$$y=\frac{2}{3}x^3-\frac{5}{2}x^2+2x$$

Yechish. Tekshirish sxemasiga muvofiq y' ni topamiz: $y'=2x^2-5x+2$. Ko‘rinib turibdiki, x ning barcha qiymatlarida hosila mavjud. Hosilani nolga tenglab, $2x^2-5x+2=0$ tenglamani olamiz, bu yerdan $x_1=\frac{1}{2}$ va $x_2=2$ kabi kritik nuqtalarni topamiz. Hosila ishoralari quyidagi chizmada ko‘rsatilgan:





$\left(-\infty;\frac{1}{2}\right)$ va $(2;+\infty)$ orliqlarda hosila $f'(x)>0$ va funksiya o‘suvchi, $\left(\frac{1}{2};2\right)$ oraliqda hosila $f'(x)<0$ ya’ni funksiya kamayuvchi. $x=\frac{1}{2}$ – maksimum nuqta va $f_{\max}\left(\frac{1}{2}\right)=\frac{11}{24}$, $x=2$ – minimum nuqta va $f_{min}(2)=-\frac{2}{3}$. Chunki hosila bu nuqtalardan o‘tishda o‘z ishorasini ($x=\frac{1}{2}$ da) «+» dan «-» ga va ($x=2$ da) «-» dan «+» ga o‘zgartiradi.

Izoh. $x=\frac{1}{2}$ va $x=2$ kritik nuqtalarda ekstremum mavjudligini ikkinchi tartibli hosila yordamida aniqlasa bo'ladi: $f''(x)=4x-5$, $f''\left(\frac{1}{2}\right)=-3<0$ va $f''(2)=3>0$ bo'lganligi uchun $x=\frac{1}{2}$ – maksimum nuqta va $x=2$ – minimum nuqta.

Funksiyaning o'sish va kamayish intervallarini va ekstremumlarini toping.

27.16. $f(x)=\frac{1}{4}x^4+\frac{1}{3}x^3-x^2.$

27.17. $f(x)=\frac{x}{\ln x}.$

27.18. $f(x)=(2x+1)e^{-\frac{x}{2}}.$

27.19. $f(x)=\frac{x^3}{1+x}.$

27.20. $f(x)=x^3(x-1).$

27.21. $f(x)=\frac{x^3}{1+x^2}.$

27.22. $f(x)=\frac{e^{2x}}{1+x}.$

27.23. $f(x)=\left(1+x^2\right)e^{-\frac{4x}{5}}.$

27.24. $f(x)=x^3e^{-\frac{3x^2}{2}}.$

27.25. $f(x)=\sqrt{xe^{3x}+1}.$

27.26. $f(x)=\sqrt[4]{x^4-4x^3}.$

27.27. $f(x)=x\ln x-3x.$

27.28. $f(x)=\ln(1+2\cos x).$

27.29. $f(x)=arctg\frac{\ln x}{x}.$

27.30. $f(x)=2x^2\ln x.$

27.31. $f(x)=\frac{x^2}{\ln x}.$

27.32. $f(x)=\frac{1+\sqrt{x}}{3+x}.$

27.33. $f(x)=\cos(\ln x).$

27.34. $f(x)=\frac{1}{\ln x}-\frac{1}{\ln^2 x}.$

27.35. $y=3x-x^3$ funksiyaning $[-2;4]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

Yechish. $y'=3-3x^2$ funksiya hosilasi $x=\pm1$ nuqtalarda nolga teng. Bu nuqtalarda va kesma oxirlarida funksiya qiymatlarini topamiz: $f(-2)=2, f(-1)=-2,$ $f(1)=2,$ $f(4)=-52.$ Shunday qilib,

$$f_{eng\,katta}=f(-2)=f(1)=2,\ f_{eng\,kichik}=f(4)=-52.$$

$y=f(x)$ funksiyaning $[a;b]$ kesmadagi eng katta va eng kichik qiymatlarini toping.

27.36. $f(x)=x^3-3x^2;\ [-1;4].$

27.37. $f(x)=x\ln x;\ [0,1;\ 1].$

27.38. $f(x)=\frac{x}{2+x^3};\ [0;3].$

27.39. $f(x)=3\sin x+4\cos^3 x;\ \left[0;\frac{\pi}{4}\right].$

27.40. $f(x)=\frac{x+1}{e^x};\ [-1;1].$

27.41. $f(x)=\frac{2x-1}{2+x^2};\ [-2;0].$

27.42. $f(x)=tg^2x;\ \left[-1;\frac{\pi}{2}\right].$

27.43. $f(x)=\frac{2x}{1+x^4};\ [-2;0,5].$

27.44. $f(x)=\frac{1+x}{3+x^2};\ [0;2].$

27.45. $f(x)=-3x^4+6x^2;\ \left[-\sqrt{2};\sqrt{2}\right].$

27.46. $f(x)=\frac{2+x^2}{1-x^2};\ \left[-\frac{1}{2};1\right].$

27.47. $f(x)=2\sin 2x+3\cos 2x;\ \left[0;\frac{\pi}{4}\right].$

27.48. Ushbu $f(x)=\frac{1}{x^2+1}$ funksiya grafigining qavariqlik oraliqlarini va burilish nuqtasini toping.

Yechish. Funksiya haqiqiy sonlar o‘qida aniqlangan va ikki marta differensiallanuvchi. Funksiyaning ikkinchi tartibli hosilasini topamiz

$$f''(x)=\frac{6\left(x^2-\frac{1}{3}\right)}{\left(x^2+1\right)^3}.$$

$f''(x)<0$ da funksiya yuqoriga qavariq $x^2-\frac{1}{3}<0$ yoki $|x|<\frac{1}{\sqrt{3}}$. $f''(x)>0$ da funksiya quyiga qavariq $x^2-\frac{1}{3}>0,\ x\in\left(-\infty;-\frac{1}{\sqrt{3}}\right)\cup\left(\frac{1}{\sqrt{3}};+\infty\right).$

Shunday qilib funksiya grafigi $\left(-\frac{1}{\sqrt{3}};\frac{1}{\sqrt{3}}\right)$ da yuqoriga qavariq, $\left(-\infty;-\frac{1}{\sqrt{3}}\right)$ va $\left(\frac{1}{\sqrt{3}};+\infty\right)$ da quyiga qavariq bo‘ladi. Demak $x_1=-\frac{1}{\sqrt{3}}$ va $x_2=\frac{1}{\sqrt{3}}$ nuqtalar funksiyaning burilish nuqtalari bo‘ladi.

Funksiyalar grafigining qavariqlik oraliqlarini va burilish nuqtalarini toping.

27.49. $f(x)=\frac{x^3}{4-x^2}.$

27.50. $f(x)=x^4-4x^3-48x^2+6x-9.$

27.51. $f(x)=e^{-x^2}.$

27.52. $f(x)=x^5-10x^2+7x-9.$

27.53. $f(x)=\cos x.$

27.54. $x=t\cdot arctgt.$

27.55. $f(x)=\frac{1}{6}x^3\left(x^2-5\right).$

27.56. $f(x)=\frac{x}{\sqrt{x^2+1}}.$

27.57. $f(x)=\sqrt[3]{x^2-2x}.$

27.58. $f(x)=\sqrt[3]{1-x^3}.$

27.59. $f(x)=(x+1)arctgx.$

27.60. $f(x)=x^3e^{-\frac{x^2}{2}}.$

27.61. $f(x)=\frac{\ln x}{x^2}.$

27.62. $f(x)=x^2e^{\frac{2}{x}}.$

27.63. $f(x)=x^3\ln x+1.$

27.64. $f(x)=\frac{x^2}{\sqrt{x^2+1}}.$

27.65. $f(x)=\frac{x^3}{3\sqrt[3]{x^3+2}}.$

27.66. $f(x)=\left(\frac{x^3}{6}-x^2\right)\ln x-\frac{5x^3}{36}+\frac{3x^2}{2}.$

27.67. Q mahsulot miqdoriga bog‘liq bo‘lgan daromad funksiyasi $R(Q)=100Q-Q^2$ formula bilan, mahsulotni ishlab chiqarishga ketgan harajatlar funksiyasi esa $C(Q)=Q^3-37Q^2+169Q+4000$ formula bilan aniqlansin. Maksimal foydani toping.

Yechish. Foyda $F(Q)=R(Q)-C(Q)$ formula bilan aniqlanadi. Bu yerdan $F(Q)=-Q^3+36Q^2-69Q-4000.$ Foyda funksiyasining hosilasini nolga tenglashtirib $Q^2-24Q+23=0$ tenglamani hosil qilamiz. Bu tenglamaning ildizlari $Q=1$, $Q=23$. Tekshirish shuni ko‘rsatadiki, maksimal foydaga $Q=23$da erishiladi. $F_{max}=1290.$

27.68. Q sotilgan mahsulot miqdoriga bog‘liq bo‘lgan daromad funksiyasi $R(Q)=\frac{Q^3}{3}+2000000Q$ formula bilan, mahsulotni ishlab chiqarishga ketgan harajatlar funksiyasi esa $C(Q)=1500Q^2$ formula bilan ifodalanadi. Maksimal foydani toping.

28-amaliy mashg‘ulot. Bir o‘zgaruvchili funksiyani tekshirish

28.1. $f(x)=e^{\frac{1}{x^2(1-x)}}$ funksiya grafigining vertikal asimptotasini toping.

Yechish. $x=0$ va $x=1$- uzilish nuqtalari,

$$\lim_{x\to 0}e^{\frac{1}{x^2(1-x)}}=+\infty,\qquad \lim_{x\to 1-0}e^{\frac{1}{x^2(1-x)}}=+\infty,\qquad \lim_{x\to 1+0}e^{\frac{1}{x^2(1-x)}}=-\infty.$$

$x=0,\ \ x=1$ ikkinchi tur uzilish nuqtalari, $x=0,\ \ x=1$ to‘g‘ri chiziqlar vertikal asimptotalar.

28.2. $f(x)=\dfrac{x^3+3x^2}{x^2-2}$ funksiya grafigining og‘ma asimptotasini toping.

Yechish. $k=\lim\limits_{x\to\infty}\dfrac{f(x)}{x}=\lim\limits_{x\to\infty}\dfrac{x^3+3x^2-1}{x\left(x^2-2\right)}=1,\ b=\lim\limits_{x\to\infty}\left(f(x)-kx\right)=\lim\limits_{x\to\infty}\dfrac{3x^2+2x-1}{x^2-2}=3.$

$y=x+3-$og‘ma asimptota.

28.3. $f(x)=x+\sqrt{x}$ funksiya grafigining og‘ma asimptotasini toping.

Yechish. Funksiya faqat $x\geq 0$ da aniqlangan.

$$k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\left(1+\frac{1}{\sqrt{x}}\right)=1,\qquad b=\lim_{x\to+\infty}\sqrt{x}=+\infty.$$

bo‘lgani uchun og‘ma asiptota yo‘q.

$f(x)$ funksiya grafigining asimptotalarini toping.

28.4. $f(x)=\dfrac{x^2+3}{x^2-9}.$

28.5. $f(x)=x\cdot e^x.$

28.6. $f(x)=\dfrac{3x}{x+2}.$

28.7. $f(x)=e^{-\frac{1}{x}}.$

28.8. $f(x)=\dfrac{1}{x^2+5x-6}.$

28.9. $f(x)=x-arctgx.$

28.10. $y=\dfrac{1-x^3}{(2-x)\left(1+3x^2\right)}$

28.11. $y=\dfrac{2+xe^x}{3+e^x}$

28.12. $y=\dfrac{\left(2x^2-1\right)}{x}e^{-x}$

28.13. $y=\dfrac{1}{x}\sin\dfrac{1}{x}$

28.14. $y=\dfrac{\arccos x}{x^2-\frac{\pi}{4}}$

28.15. $y=\dfrac{\ln^2 x}{x}$

28.16. $y=\dfrac{\sqrt{1-\cos x}}{x}$

28.17. $y=\dfrac{2x}{1-x^2}$ funksiyani tekshiring va grafigini yasang.

Yechish. 1. Funksiyaning aniqlanish sohasi $(-\infty;-1)\cup(-1;1)\cup(1;+\infty)$. $x=-1$ va $x=1$ funksiyaning uzilish nuqtalari.

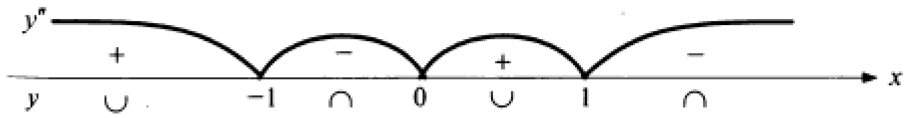
2. $f(-x)=-f(x)$ funksiya toq, funksiya grafigi koordinatalar boshiga nisbatan simmetrik va $[0;+\infty)$ intervalda funksiyani tekshirish yetarli.

3. $\lim\limits_{x\to 1-0}\dfrac{2x}{1-x^2}=+\infty;\qquad \lim\limits_{x\to 1+0}\dfrac{2x}{1-x^2}=-\infty.$ $x=1$ va $x=-1$ to‘g‘ri chiziqlar vertikal asimptotalar.

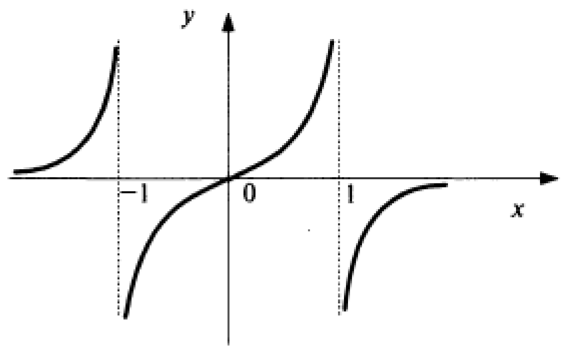
4. $\lim\limits_{x\to\pm\infty}\dfrac{2x}{1-x^2}=0.$ $y=0$ (absissa o‘qi) to‘g‘ri chiziq ikki tomonlama gorizontal asimptota.

5. x ning barcha mumkin bo‘lgan qiymatlarida $y'=\dfrac{2+2x^2}{\left(1-x^2\right)^2}>0.$ Ekstremum yo‘q, funksiya $(-\infty;-1)\cup(-1;1)\cup(1;+\infty)$ intervalda o‘suvchi.

6. $y''=\dfrac{4x\left(x^2+3\right)}{\left(1-x^2\right)^3},$ $x=0$ da $y''=0.$ Ikkinchi tartibli hosilaning ishorasi quyidagi rasmda keltirilgan.



Funksiya $(-\infty;-1)$ va $(0;1)$ intervalda pastga qavariq va $(-1;0)$ va $(1;+\infty)$ intervalda yuqoriga qavariq. y'' uchta nuqta orqali o‘tishda $x=-1,\ x=0,\ x=1$ o‘z ishorasini o‘zgartiradi, lekin funksiya grafigi faqat bitta $x=0$ burilish nuqtasiga ega. Boshqa ikkita $x=-1,\ x=1$ nuqtalarda funksiya aniqlanmagan.



7. Grafikning o‘qlar bilan kesishish nuqtasi yagona – koordinatalar boshi $(0;0)$ nuqta.

Quyidagi berilgan funksiyalarni tekshirib, grafiklarini yasang:

28.18. $y=\dfrac{2x}{1+x^2}.$

28.19. $y=x^2\left(x-4\right)^2.$

28.20. $y=\dfrac{2x}{2+x^3}.$

28.21. $y=\left(x+1\right)e^{-x}.$

28.22. $y=xe^{-\frac{x^2}{2}}.$

28.23. $y=\dfrac{\ln x}{x}.$

28.24. $y=\dfrac{e^x-e^{-x}}{e^x+e^{-x}}.$

28.25. $y=\sqrt[3]{1-\ln x}.$

28.26. $y=e^{\frac{1}{x}}.$

28.27. $y=xe^{\frac{1}{x}}.$

28.28. $y=\dfrac{1}{1-e^x}.$

28.29. $y=\sin x+\cos^2 x.$

28.30. $y=\dfrac{x}{\sqrt[3]{x^2+1}}+\dfrac{1}{\sqrt[3]{x-1}}.$

28.31. $y=\ln\left(x+\sqrt{x^2+1}\right).$

28.32. $y=\dfrac{1}{\sin x+\cos x}.$

28.33. $y=\sqrt[3]{x+1}-\sqrt[3]{x-1}.$

28.34. $y=\dfrac{1}{\sqrt[3]{x+1}}+\dfrac{1}{\sqrt[3]{x-1}}.$

28.35. $y=\sqrt[3]{1-x^3}$

28.36. $y=Sin^2x$

28.37. $y=\ln x-\ln(x-1)$

28.38. $y=\dfrac{x^3}{x^2-4}$

28.39. $y=16x(x-1)^3$

28.40. $y=2Sinx+Cos2x$ $\left([0,\pi)\right)$ oraliqda

28.41. $y=\dfrac{x^2}{x-2}$

28.42. $y=(x-1)\sqrt{x}$

28.43. $y=\ln\dfrac{x}{x-1}$

28.44. *y=Sin2x-x* $\left(-\dfrac{\pi}{2};\dfrac{\pi}{2}\right)$ oraliqda

28.45. *y=2x*$\pm$ *ctgx* (0, π) oraliqda

28.46. *y=x+* e^{-x}

28.47. *y=ln*(*x+* $\sqrt{x^2+1}$)

28.48. $y=\dfrac{x^3}{(x-2)^2}$

28.49. $y=2x^2+\dfrac{1}{x}.$

28.50. $y=\dfrac{\ln x}{\sqrt{x}}.$

28.51. $y=e^{\frac{1}{x+2}}.$

28.52. $y=\ln\left(1-x^2\right).$

28.53. $y=\dfrac{x^2}{1-x^2}.$

28.54. $y=x^3-4x^2+3x.$

28.55. $y=x+\dfrac{1}{x}.$

28.56. $y=x^2\cdot e^{-x}.$

28.57. $y=\dfrac{(x+1)^2}{x-2}.$

28.58. $y=\dfrac{3x-2}{5x^2}.$

28.59. $y=\dfrac{x^3}{3-x^2}.$

28.60. $y=x-\ln x.$

29-amaliy mashg‘ulot. Ko‘p o‘zgaruvchili funksiya differensiali. Xususiy hosila va yuqori tartibli differensiallar

29.1. Ushbu $z = xy^2 - \frac{x}{y}$ funksiyaning $M_0(3;-2)$ nuqtadagi xususiy va to‘la orttirmasini toping. $\Delta x = 0,1$ va $\Delta y = -0,05$.

Yechish. $x_0 = 3,\ y_0 = -2,\ x_0 + \Delta x = x = 3,1,\ y_0 + \Delta y = y = -2,05,\ M_1(3,1;-2,05)$.

Avval aniqlaymiz $z(M_0) = z(3;-2) = 3(-2)^2 + \frac{3}{2} = 13,50.$

$$z(x_0 + \Delta x; y_0) = z(3,1;-2) = 3,1\cdot(-2)^2 + \frac{3,1}{2} = 13,95;$$

$$z(x_0; y_0 + \Delta y) = z(3;-2,05) = 3\cdot(-2,05)^2 + \frac{3}{2,05} = 14,07;$$

$$z(M_1) = z(x_0 + \Delta x; y_0 + \Delta y) = z(3,1;-2,05) = 3,1\cdot(-2,05)^2 + \frac{3,1}{2,05} = 14,54.$$

Shunday qilib,

$$\Delta_x z = z(x_0 + \Delta x; y_0) - z(x_0; y_0) = 0,45;$$

$$\Delta_y z = z(x_0; y_0 + \Delta y) - z(x_0; y_0) = 0,57;$$

$$\Delta z = z(x_0 + \Delta x; y_0 + \Delta y) - z(x_0; y_0) = 14,54 - 13,50 = 1,04.$$

Ravshanki, $\Delta z = 1,04 \neq 0,45 + 0,57 = 1,02 = \Delta z_x + \Delta z_y$.

29.2. $z = x^2 y;\ M_0(1;2);\ \Delta y = -0,2.$

29.3. $z = \dfrac{x^2 y^2}{x^2 y^2 - (x-y)^2};\ M_0(2;2);\ \Delta_x = -0,2; \Delta y = 0,1.$

29.4. $z = \left(\dfrac{x^2 + y^2}{xy}\right)^2;\ M_0(1;1);\ \Delta x = 0,1;\ \Delta y = -0,1.$

Berilgan funksiyaning berilgan nuqtada to‘la orttirmasini toping.

29.5. $z = 3x^2 + xy - y^2 + 1;\ M_0(2;1);\ \Delta x = 0,1;\ \Delta y = 0,2.$

29.6. $z = 3x^2 + xy - y^2 + 1;\ M_0(2;1);\ \Delta x = 0,01;\ \Delta y = 0,01.$

29.7. $z = x^2 - xy + y^2;\ M_0(2;1);\ M_1(2,1;0,9).$

29.8. $z = \lg(x^2 + y^2);\ M_0(2;1);\ M_1(2,1;0,9).$

29.9. a) $z = x^2 + 2xy + 3y^2$ b) $u = \dfrac{x}{x^2 + y^2 + z^2}$ funksiyalarning xususiy hosilalarni toping.

Yechish. a) y ni o‘zgarmas deb, z'_x ni topamiz:

$$z'_x=(x^2+2xy+3y^2)'_x=(x^2)'_x+(2xy)'_x+(3y^2)'_x=2x+2y,$$

endi x ni o‘zgarmas deb, $\frac{\partial z}{\partial y}$ ni topamiz:

$$z'_y=(x^2+2xy+3y^2)'_y=(x^2)'_y+(2xy)'_y+(3y^2)'_y=2x+6y.$$

b) hosila olish qoidalari va formulalaridan foydalanib quyidagilarni topamiz:

$$u'_x=\left(\frac{x}{x^2+y^2+z^2}\right)'_x=\frac{x'_x(x^2+y^2+z^2)-x(x^2+y^2+z^2)'_x}{(x^2+y^2+z^2)^2}=$$

$$=\frac{x^2+y^2+z^2-2x^2}{(x^2+y^2+z^2)^2}=\frac{-x^2+y^2+z^2}{(x^2+y^2+z^2)^2}.$$

u'_y, u'_z larni mustaqil toping.

Quyidagi funksiyalarning xususiy hosilalarini toping:

29.10. $z=e^{x^2+y^2}$.

29.11. $u=t^5\sin^3 z$.

29.12. $v=x^4\cos^2 y-y^4\sin^3 x^5$.

29.13. $z=x^2\cos 2xy-y^2\sin(x+y)$.

29.14. $z(x,y)=\frac{x-y}{x+y}$

29.15. $u=e^{\frac{x}{y}}+e^{\frac{z}{y}}$

29.16. $z(x,y)=-\frac{Cosx}{Cosy}$

29.17. $z(x,y)=ln(x^2-y^2)$

29.18. $z(x,y)=x\ Sin(x+y)$

29.19. $z(x,y)=\frac{x}{\sqrt{x^2+y^2}}$

29.20. $z(x,y)=arcsin\frac{y}{x}$

29.21. $u=2y\sqrt{x}+3y^2\sqrt[3]{z^2}$

29.22. $z(x,y)=\sqrt{xy+\frac{x}{y}}$

29.23. $z(x,y)=e^{\sin\frac{y}{x}}$

29.24. $z=cos(ax-by)$

29.25. $z=\frac{x}{3y-2x}$

29.26. $z=\ln sin(x-2y)$

29.27. $z=2cos^2\left(x-\frac{y}{2}\right)$

29.28. $z=\ln tg\frac{y}{x}$

6.4.29. $z=arctg\sqrt{xy}$

29.30. $z=y\ e^{\frac{x}{y}}$

29.31. $u=(x-y)\ (x-z)\ (y-z)$.

29.32. $z=e^{x-y}(2x-1)$.

29.33. $z=\sin\left(x+\sqrt{y}\right)$.

29.34. $z=xe^y+x^y$.

29.35. $z=\ln\sqrt{x+y^2}$

29.36. $z=\ln\left(\sqrt{x}+\sqrt{y}\right)$.

29.37. $z=x^{\sqrt{y}}$.

29.38. $z = arctg\left(\frac{y}{x}+1\right).$

29.39. $z = xye^{xy}.$

29.40. $z = \frac{\cos y^2}{x}.$

29.41. $z = \arcsin\frac{y}{\sqrt{x^2+y^2}}.$

29.42. a) $z = \ln\left(x^2+y^2\right)$ b) $u = x^2yz^2$ funksiyalarning to‘la differensialini toping.

Yechish. a) xususiy hosilalarni topamiz

$$z'_x = \frac{\left(x^2+y^2\right)'_x}{x^2+y^2} = \frac{2x}{x^2+y^2}, \qquad z'_y = \frac{\left(x^2+y^2\right)'_y}{x^2+y^2} = \frac{2y}{x^2+y^2},$$

formulaga asosan $dz = \frac{2x}{x^2+y^2}dx + \frac{2y}{x^2+y^2}dy$ bo‘ladi.

b) $u = x^2yz^2$ funksiyaning xususiy hosilalarni topamiz

$$\frac{\partial u}{\partial x} = \left(x^2yz^2\right)'_x = yz^2(x^2)'_x = 2xyz^2,$$

$$\frac{\partial u}{\partial y} = \left(x^2yz^2\right)'_y = x^2z^2(y)'_y = x^2z^2,$$

$$\frac{\partial u}{\partial z} = \left(x^2yz^2\right)'_z = yx^2(z^2)'_z = 2x^2yz.$$

formulaga asosan $du = 2xyz^2dx + x^2z^2dy + 2x^2yzdz$ bo‘ladi.

Quyidagi funksiyalar to‘la differensiallarini toping:

29.43. $z=5x^3y^2$;

29.44. $z = \frac{y}{x} - \frac{x}{y}$;

29.45. $z = (\sin x)^{\cos y}$;

29.46. $z = e^{x^2+y^2}$;

29.47. $z = arctg\frac{x}{y}$;

29.48. $z = sin^2x + cos^2y$;

29.49. $z = x\, ln\frac{y}{x}$;

29.50. $z = x^y$;

29.51. $z = tg\frac{y}{x} + ctg\frac{x}{y}$;

29.52. $z = tg\left(2x+\sqrt{y}\right)$;

29.53. $z = e^{xy}$.

29.54. $z = x^my^n$;

29.55. $z = y\sqrt[3]{x}$;

29.56. $z = \sqrt{x^2+y^2}$;

29.57. $z = e^{\cos(xy)}$;

29.58. $z = \frac{x}{y}e^{xy}$;

29.59. $u = x^{y^2z}$.

29.60. $z = e^{xy}\left(x+y\right).$

29.61. $z = \ln\left(1+e^x+y^2\right).$

29.62. $z = \frac{\sqrt{x}-\sqrt{y}}{x+y}.$

29.63. $z = \sin\left(\frac{x}{y}\right).$

29.64. $z = \frac{x\arcsin y}{y}.$

29.65. $z=x^y+y^x$.

29.66. $z=\sqrt{x}\,sin\frac{y}{x}$; $\quad x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\frac{z}{2}$ ekani isbot qilinsin.

29.67. $z=ln(\sqrt{x}+\sqrt{y})$; $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\frac{1}{2}$ ekani isbot qilinsin.

29.68. a) $1{,}07^{3{,}97}$; b) $arcctg\left(\frac{1{,}97}{1{,}02}-1\right)$ larni taqribiy hisoblang.

Yechish. a) $1{,}07^{3{,}97}$ son $x=1{,}07$, $y=3{,}97$ da $f(x,y)=x^y$ funksiyaning xususiy qiymati. $f(1;4)=1$ ekani ma'lum. Shuning uchun $x_0=1,\ y_0=4$ u holda $\Delta x=x-x_0=0{,}07,\ \Delta y=y-y_0=-0{,}03.$ $f(x+\Delta x;y+\Delta y)$ ning qiymatini $f(x_0+\Delta x;y_0+\Delta y)\approx f(x_0;y_0)+f'_x(x_0;y_0)\Delta x+f'_y(x_0;y_0)\Delta y$ taqribiy hisoblash formulasi bo'yicha hisoblaymiz.

$$f'_x=yx^{y-1},\quad f'_y=x^y\ln x,\quad f'_x(1;4)=4,\quad f'_y(1;4)=0,$$

Shunday qilib, $1{,}07^{3{,}97}\approx 1+4\cdot 0{,}07+0\cdot(-0{,}03)=1{,}28.$

b) To'la differensial formulasidan taqribiy hisoblashda foydalanish uchun, oldin qiymati taqribiy hisoblanadigan funksiyaning analitik ifodasini tanlash zarur, keyin boshlang'ich nuqtani shunday tanlash kerakki funksiyaning va xususiy hosilalarning bu nuqtadagi qiymatlarini jadvalsiz hisoblash mumkin bo'lsin.

$arcctg\left(\frac{1{,}97}{1{,}02}-1\right)$ ifoda $f(x,y)=arcctg(\frac{x}{y}-1)$ funksiyaning $P_1(1{,}97;\ 1{,}02)$ nuqtadagi qiymati deyish mumkin. Boshlang'ich nuqta uchun $P_0=(2;1)$ ni olsak, $\Delta x=1{,}97-2=-0{,}03$, $\Delta y=1{,}02-1=0{,}02$ bo'ladi. Endi xususiy hosilalarni topib, ularning P_0 nuqtadagi qiymatlarini hisoblaymiz:

$$f'_x(x,y)=\left[arcctg\left(\frac{x}{y}-1\right)'_x\right]=-\frac{\left(\frac{x}{y}-1\right)'_x}{1+\left(\frac{x}{y}-1\right)^2}=-\frac{\frac{1}{y}}{1+\frac{(x-y)^2}{y^2}}=-\frac{y}{y^2+(x-y)^2};$$

$$f'_y(x,y)=\left[arcctg\left(\frac{x}{y}-1\right)'_y\right]=-\frac{\left(\frac{x}{y}-1\right)'_y}{1+\left(\frac{x}{y}-1\right)^2}=-\frac{-\frac{x}{y^2}}{\frac{\left(y^2+(x-y)^2\right)}{y^2}}=\frac{x}{y^2+(x-y)^2};$$

$$f'_x(2;1)=-\frac{1}{1+(2-1)^2}=-0.5;\qquad f'_y(2;1)=\frac{2}{1+(2-1)^2}=1.$$

$$arcctg\left(\frac{1.97}{1.02}-1\right)\approx arcctg\left(\frac{2}{1}-1\right)+(-0,5)(-0,03)+1\cdot 0,02=$$

$$=\frac{\pi}{4}+0,015+0,02=0,82.$$

29.69. $1{,}04^{2,03}$. **29.70.** $\sqrt{(1,04)^2+(3,01)^2}$. **29.71.** $\sin 28°\cdot\cos 61°$.

29.72. Berilgan $z=x^3-x^2y-y^3$ funksiya uchun birinchi, ikkinchi tartibli xususiy hosilalarni toping.

Yechish.

$$\frac{\partial z}{\partial x}=3x^2-2xy;\quad \frac{\partial z}{\partial y}=-x^2-3y^2.$$

$$\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}\left(3x^2-2xy\right)=6x-2y;$$

$$\frac{\partial^2 z}{\partial y\partial x}=\frac{\partial}{\partial y}\left(3x^2-2xy\right)=-2x;$$

$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial}{\partial x}\left(-x^2-3y^2\right)=-2x;\quad \left(\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x}\ \text{ekanini ko‘ramiz}\right)$$

$$\frac{\partial^2 z}{\partial y^2}=\frac{\partial}{\partial y}\left(-x^2-3y^2\right)=-6y.$$

29.73. Agar $z=arctg\frac{y}{x}$ bo‘lsa d^2zni toping.

Yechish. Birinchi differensialni topamiz:

$$dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy=-\frac{y}{x^2+y^2}dx+\frac{x}{x^2+y^2}dy.$$

Ikkinchi tartibli xususiy hosilalarni alohida hisoblaymiz:

$$\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}\left(-\frac{y}{x^2+y^2}\right)=\frac{2xy}{\left(x^2+y^2\right)^2};$$

$$\frac{\partial^2 z}{\partial y\partial x}=\frac{\partial}{\partial y}\left(-\frac{y}{x^2+y^2}\right)=\frac{y^2-x^2}{\left(x^2+y^2\right)^2};$$

$$\frac{\partial^2 z}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right)=-\frac{2xy}{\left(x^2+y^2\right)^2};$$

Endi ikkinchi tartibli differensialni tuzamiz:

$$d^2z=\frac{2\left[xydx^2+\left(y^2-x^2\right)dxdy-xydy^2\right]}{\left(x^2+y^2\right)^2}.$$

Quyidagi funksiyalar uchun talab qilingan xususiy hosila yoki differensiallarni toping:

29.74. $z = \sin x \sin y, d^2 z.$ **29.75.** $z = 4x^3 + 3x^2 y + 3xy^2 - y^3, \frac{\partial^2 z}{\partial x^2}.$

29.76. $z = xy + \sin(x+y), \frac{\partial^2 z}{\partial x^2}.$ **29.77.** $z = \ln tg(x+y), \frac{\partial^2 z}{\partial x \partial y}.$

29.78. $z = arctg\frac{x+y}{1-xy}, \frac{\partial^2 z}{\partial x \partial y}.$

Quyidagi funksiyalardan dz $d^2 z$ni toping:

29.79. $x = x^2 y - xy^2 + 7.$ **29.80.** $z = xy - \frac{y}{x}.$

29.81. $z = (x^2 + y^2)^3$ **29.82.** $z = x - 3\sin y$

29.83. a) $u=xy^2z^3$ funksiyaning $M_0(3;2;1)$ nuqtada gradiyenti va uning uzunligini toping.

Yechish. Buning uchun berilgan funksiya xususiy hosilalarini topamiz:

$$\frac{\partial u}{\partial x} = y^2 z^3; \frac{\partial u}{\partial y} = 2xyz^3; \frac{\partial u}{\partial z} = 3xy^2 z^2$$

$$\left.\frac{\partial u}{\partial x}\right|_{M_0} = 4; \left.\frac{\partial u}{\partial y}\right|_{M_0} = 12; \left.\frac{\partial u}{\partial z}\right|_{M_0} = 36$$

$$\left. gradu \right|_{M_0} = \{4;12;36\}$$

$$\left.|gradu|\right|_{M_0} = \sqrt{4^2 + 12^2 + 36^2} = \sqrt{1456}$$

b) $z = \ln(x^2 + y^2)$, $M(3;4)$ nuqtada.

$$\frac{\partial z}{\partial x} = \frac{2x}{x^2 + y^2}; \frac{\partial z}{\partial y} = \frac{2y}{x^2 + y^2}$$

$$\frac{\partial z(3,4)}{\partial x} = \frac{6}{25}; \frac{\partial z(3,4)}{\partial y} = \frac{8}{25};$$

$$|gradu| = \sqrt{\left(\frac{6}{25}\right)^2 + \left(\frac{8}{25}\right)^2} = \frac{2}{5}$$

Quyidagi funksiyalarning nuqtalardagi gradiyenti va uning uzunligini toping:

29.84. $z = \sqrt{4 + x^2 + y^2}$, (2; 1) da

29.85. $z = arctg\frac{x}{y}$, $(x_0; y_0)$ da

29.86. $u = xyz$, (1; 2; 1) da

29.87. $u = tgx - x + 3siny - sin^3 y + z + ctgz,$ $\left(\frac{\pi}{4}; \frac{\pi}{3}; \frac{\pi}{2}\right)$ da

Quyidagi funksiyalarning ekstremum nuqtalarini toping:

29.88. $z=6zy-x^2y-xy^2$ funksiyaning ekstremumini tekshiring.

Yechish. Birinchi tartibli xususiy hosilalarni topamiz: ular

$$\frac{\partial z}{\partial x}=(6xy-x^2y-xy^2)'_x=6y-2xy-y^2$$

$$\frac{\partial z}{\partial y}=6x-x^2-2xy$$

bo‘ladi. Ekstremumga ega bo‘lishining zaruriy sharti $\frac{\partial z}{\partial x}=0, \frac{\partial z}{\partial y}=0$ ga asosan $\begin{cases}6y-2xy-y^2=0,\\ 6x-x^2-2xy=0\end{cases}$ sistema hosil bo‘ladi. Bu tenglamalar sistemasidan $P_1(0;0), P_2(2;2)$ kritik nuqtalarni topamiz. Xususiy hosilalar mavjud bo‘lmagan nuqtalar yo‘q.

Ikkinchi tartibli xususiy hosilalar

$$\frac{\partial^2 z}{\partial x^2}=(6y-2xy-y^2)'_x=-2y,\quad \frac{\partial^2 z}{\partial y\partial x}=(6y-2xy-y^2)'_y=6-2x-2y$$

$$\frac{\partial^2 z}{\partial y^2}=(6x-x^2-2xy)'_y=-2x$$

bo‘ladi. Ikkinchi tartibli xususiy hosilalarning $P_1(0;0)$ nuqtadagi qiymatlarini topib ekstremumga ega bo‘lishining yetarli shartini tekshiramiz:

$$A=\frac{\partial^2 z(P_1)}{\partial x^2}=-2\cdot 0=0;\quad B=\frac{\partial^2 z(P_1)}{\partial x\partial y}6-2\cdot 0-2\cdot 0=6;$$

$$C=\frac{\partial^2 z(P_1)}{\partial y^2}=-2\cdot 0=0;$$

$$\Delta=A\cdot C-B^2=0\cdot 0-6^2=-36<0,$$

oxirgi tengsizlikdan ko‘rinadiki, ikki o‘zgaruvchili funksiya ekstremumining yetarli shartiga asosan: $P_1(0;0)$ nuqtada ekstemum yo‘q.

Endi $P_2(2;2)$ ikkinchi kritik nuqtani tekshiramiz:

$$A=\frac{\partial^2 z(P_2)}{\partial x^2}=-2\cdot 2=-4;\quad B=\frac{\partial^2 z(P_2)}{\partial x\partial y}=6-2\cdot 2-2\cdot 2=-2;$$

$$C=\frac{\partial^2 z(P_2)}{\partial y^2}=-2\cdot 2=-4.$$

$\Delta=(-4)(-4)-(-2)^2=16-4=12>0$ bo‘lib, $A=-4$ bo‘lganligi uchun $P_2(2;2)$ nuqtada berilgan funksiya ekstremumga ega bo‘lishining yetarli shartiga asosan, maksimumga ega bo‘ladi.

Shunday qilib, berilgan funksiya $P_1(0;0)$ nuqtada ekstremumga ega emas. $P_2(2;2)$ nuqatada funksiya maksimumga ega bo‘lib,

$$Z_{\max}=6\cdot2\cdot2-2^2\cdot2-2\cdot2^2=24-8-8=24-16=8$$

bo‘ladi.

29.89. $z=x^2-xy+y^2+9x-6y+20$

29.90. $z=y\sqrt{x}-y^2-x+6y$

29.91. $z=x^3+8y^3-6xy+1$

29.92. $z=2xy-4x-2y$

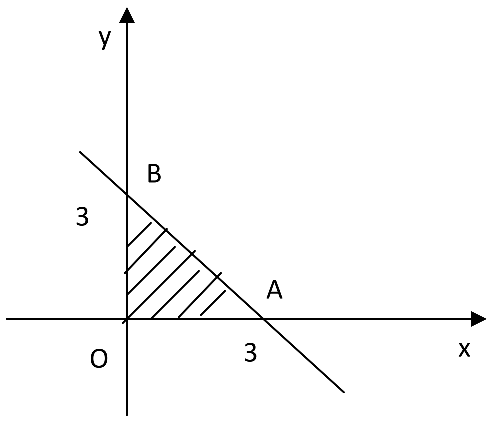
29.93. $f(x,y)=-x^2+xy-y^2-9y+6x-35.$

29.94. $f(x,y)=6x^2-7xy+2y^2+6x-3y.$

29.95. $f(x,y)=4x^2-5xy+3y^2-9x-8y.$

29.96. $z=x^2-2y^2+4xy-6x+5$ funksiyaning $x\geq0,\ y\geq0,\ x+y\leq3$ yopiq sohadagi eng katta va eng kichik qiymatlarini toping.

Yechish. 1) D soha AOB uchburchakdan iborat.



Berilgan funksiyaning 1-tartibli xususiy hosilalarini topib, ularni 0 ga tenglashtirib, hosil bo‘lgan tenglamalar sistemasidan soha ichidagi kritik nuqta koordinatlarini topamiz:

$$z'_x=\left(x^2-2y^2+4xy-6x+5\right)'_x=2x+4y-6,$$

$$z'_y=\left(x^2-2y^2+4xy-6x+5\right)'_y=-4y+4x,$$

$$\begin{cases}2x+4y-6=0,\\-4y+4x=0\end{cases}$$

Demak, bundan $x=y$ bo‘lib, $2x+4x-6=0,\ \ 6x=6,\ \ \ x=1,\ \ \ y=1$ bo‘ladi. $P_1(1;1)$ nuqta kritik nuqta bo‘ladi.

2) Berilgan funksiyani soha chegarasida tekshiramiz. AOB uchburchak OA, OB va AB tomonlarining tenglamalari mos ravishda, $y=0,\ \ x=0,\ \ x+y=3$ bo‘ladi. OA tomonda $y=0$ bo‘lganligi uchun, $z=x^2-2\cdot0+4x\cdot0-6x+5=x^2-6x+5$ bo‘ladi.

Bu tomonda $2x-6=0, \quad x=3$ bo‘lib, $P_2(3;0)$ kritik nuqta bo‘ladi. *OB* tomonda x=0 bo‘lib, $z=-2y^2+5$ bo‘ladi. Bundan, $-4y=0; \ y=0$ bo‘lib, O (0,0) kritik nuqta bo‘ladi. Uchburchakning AB tomoni tenglamasi $y=-x+3$ bo‘lib, bu tomonda

$$z=x^2-2(-x+3)^2+4x(-x+3)-6x+5=x^2-2x^2+12x-18-4x^2+12x-$$

$$-6x+5=-5x^2+18x-13$$

bo‘ladi, kritik nuqtaning absissasini $-10x+18=0$ tenglikdan topamiz: $10x=18, \ x=1{,}8$; x ning bu qiymatini AB tomon tenglamasiga qo‘ysak $y=-1{,}8+3=1{,}2, \quad y=1{,}2$ bo‘ladi.

Shunday qilib, AB tomondagi kritik nuqta $P_2(1{,}8;\,1{,}2)$ bo‘ladi.

3) Berilgan funksiyaning $P_1(1;1)$, $A(3;0)$, $O(0;0)$, $P_2(1{,}8;\,1{,}2)$ kritik nuqtalardagi hamda chegaralar tutashadigan *B*(0;3) nuqtadagi qiymatlarini hisoblaymiz:

$$z=f(P_1)=1^2-2\cdot 1^2+4\cdot 1\cdot 1-6\cdot 1+5=2;$$

$$z=f(A)=3^2-2\cdot 0^2+4\cdot 3\cdot 0-6\cdot 3+5=-4;$$

$$z=f(O)=0-2\cdot 0+4\cdot 0\cdot 0-6\cdot 0+5=5;$$

$$z=f(P_2)=1{,}8^2-2\cdot 1{,}2^2+4\cdot 1{,}8\cdot 1{,}2-6\cdot 1{,}8+5=3{,}2;$$

$$z=f(B)=0-2\cdot 3^2+4\cdot 0\cdot 3-6\cdot 0\cdot 3+5=-13$$

4) Berilgan funksiyaning barcha topilgan qiymatlarini taqqoslab, $z_{eng\,kat.}=f(O)=5, z_{eng\,kich.}=f(B)=-13$ xulosaga kelamiz.

Funksiyaning ko‘rsatilgan sohalardagi eng katta va eng kichik qiymatlarini toping:

29.97. $z=x^2+y^2-xy+x+y$. $x=0$, $y=0$, $x+y=-3$ to‘g‘ri chiziqlar bilan chegaralangan uchburchakda.

29.98. $f(x,y)=x-2y+5, x\geq 0, y\geq 0, x+y\leq 1.$

29.99. $f(x,y)=x^3+y^2, \ x^2+y^2\leq 1.$

29.100. $f(x,y)=\ln(x+y), (x-2)^2+(y-2)^2\leq 1.$

30-amaliy mashg'ulot. Shartsiz va shartli ekstremum masalasi. Lagranj metodi

x va y

$$\varphi(x;y)=0$$

tenglama (bog'lash tenglamasi) bilan bog'langanlik shartida $z=f(x;y)$ funksiyaning ekstremumini topish talab etilayotgan bo'lsin.

$z=f(x;y)$ funksiyaning $\varphi(x;y)=0$ bog'lanish tenglamasi o'rinli bo'lganda shartli ekstremumi bo'lishi mumkin bo'lgan nuqtalarini topish uchun quyidagi yordamchi funksiyani kiritish kerak

$$L(x;y;\lambda)=f(x;y)+\lambda\varphi(x;y)$$

(bu yerda λ birorta o'zgarmas) va uning x,y,λ bo'yicha xususiy hosilalarini nolga tenglab

$$\begin{cases} f'_x(x;y)+\lambda\varphi'_x(x;y)=0, \\ f'_y(x;y)+\lambda\varphi'_y(x;y)=0, \\ \varphi(x;y)=0. \end{cases}$$

sistemadan x,y va yordamchi ko'paytuvchi λ ni topish kerak.

Shartli ekstremumni topishning ushbu usuli Lagranj ko'paytuvchilari usuli $L(x;y;\lambda)$ funksiya esa Lagranj funksiyasi deyiladi.

Quyidagi funksiyalarning lokal ekstremumini toping.

30.1. $Z=x_1^2-x_2^2-3x_1x_2$

30.2. $Z=x_1^2+x_2^2-x_1x_2+x_1-x_2+1$

30.3. $Z=4x_1^2+2x_2^2-5x_1-3x_2$

30.4. $Z=5x_1+7x_2-3x_1^2-3x_2^2$

30.5. $Z=x_1^2+x_2^2+x_3^2$

30.6. $Z=3x_1-6x_1x_2+3x_2^2-x_3^2+4x_2x_3$

30.7. $Z=x_1x_2+x_2x_3+x_1x_3$

30.8. $Z=-x_1^2-x_2^2+6x_1+2x_2$

30.9. $Z=9x_1^2+4x_2^2-2x_1-4x_2$

30.10. $Z=4x_1^2+4x_2^2+8x_1-2x_2$

30.11. Berilgan $z=e^{x+2y}$ funksiyaning $x^2+y^2=1$ shartda ekstremumlarini toping.

Yechish. Lagranj funksiyasini tuzamiz

$$L(x;y;\lambda)=e^{x+2y}-\lambda\left(x^2+y^2-1\right).$$

Funksiyaning xususiy hosilalarini nolga tenglaymiz va quyidagi sistemani olamiz

$$\begin{cases} e^{x+2y}=2\lambda x, & (1) \\ 2e^{x+2y}=2\lambda y, & (2) \\ x^2+y^2=1. & (3) \end{cases}$$

(1) va (2) tenglamalardan $y = 2x$ ni olamiz va (3) tenglamaga qo‘yib $x = \frac{1}{\sqrt{5}}$, $y = \frac{2}{\sqrt{5}}$ va $x = -\frac{1}{\sqrt{5}}$, $y = -\frac{2}{\sqrt{5}}$ ikkita yechimni olamiz. Shunday qilib ikkita kritik nuqtani olamiz $\left(\frac{1}{\sqrt{5}}; \frac{2}{\sqrt{5}}\right) -$ maksimum nuqta, $\left(-\frac{1}{\sqrt{5}}; -\frac{2}{\sqrt{5}}\right) -$ minimum nuqta.

Quyidagi funksiyalarni shartli ekstremumga tekshiring:

30.11. $z = \frac{1}{x} + \frac{1}{y}$, $x + y = 2$. **30.12.** $z = x - y$, $x^2 + y^2 = 1$.

30.13 $z = xy^2$, $x + 2y = 4$. **30.14.** $z = \frac{x - y - 4}{\sqrt{2}}$, $x^2 + y^2 = 1$.

30.15. $z = \sqrt[4]{x}\sqrt[3]{y}$, $2x + 5y = 100$. $x^2 + y^2 = 1$.

30.16. $z = x^2 + 6x - 2y + 1$, $x^2 + y - 4 = 0$.

30.17. $z = 2x^2 + y^2$, $x + y - 2 = 0$. **30.18.** $z = e^{xy}$, $x + y = 1$.

30.19. $z = \frac{1}{x} + \frac{1}{y}$, $x + y = 2a$ $(a > 0)$. **30.20.** $z = xy$, $x^2 + y^2 = 1$.

30.21. $z = 6 - 4x - 3y$, $x^2 + y^2 = 1$. **30.22.** $z = \frac{1}{x} + \frac{1}{y}$, $\frac{1}{x^2} + \frac{1}{y^2} = 1$.

30.23. $z = x^2 + y^2$, $x + y = 1$. **30.24.** $z = e^{xy}$, $x + y = a$.

30.25. $z = \frac{1}{x} + \frac{1}{y}$, $x + y = 2$. **30.26.** $z = x + y$, $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{2}$.

30.27. $z = xy$, $x^2 + y^2 = 2$. **30.28.** $z = x^2 + y^2$, $\frac{x}{4} + \frac{y}{3} = 1$.

30.29. $u = x^2 + y^2 + z^2$, $\frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1$.

30.30. $u = xy^2z^3$, $x + 2y + 3z = 6$. $(x, y, z > 0)$

30.31. $u = xyz$, $x^2 + y^2 + z^2 = 1$ va $x + y + z = 0$.

30.32. $u = \frac{x^2}{9} + \frac{y^2}{4} + z^2$, $x^2 + y^2 + z^2 = 1$ va $\frac{x}{2} + \frac{y}{2} + \frac{z}{\sqrt{2}} = 0$.

30.33. Ikkita korxonada bir xil mahsulot ishlab chiqarish rejalashtirilgan bo‘lib, ushbu mahsulotga bo‘lgan talab 200 birlikni tashkil qiladi. I korxonada ishlab chiqiladigan x_1 miqdordagi mahsulotga $4x_1^2$ miqdorda, II korxonada x_2 miqdordagi mahsulot ishlab chiqarish uchun esa $20x_2 + 6x_2^2$ miqdorda xarajat sarf qilinadi. Har bir korxonada qanchadan miqdorda mahsulot ishlab chiqarilganda umumiy xarajatlar miqdori eng kam bo‘ladi?

30.34. Korxonada ikkita texnologiya asosida mahsulot ishlab chiqariladi. Birinchi texnologiya bo‘yicha ishlab chiqariladigan x_1 miqdordagi mahsulotga $a_0 + a_1x_1 + a_2x_1^2$ miqdorda, ikkinchi texnologiya asosida ishlab chiqariladigan x_2 miqdordagi mahsulotga $b_0 + b_1x_2 + b_2x_2^2$ miqdordagi xarajat qilinadi. Korxonada d birlik mahsulot ishlab chiqarishi kerakligini nazarda tutib, har bir texnologiya bo‘yicha qancha mahsulot ishlab chiqarilganda sarf qilingan umumiy xarajatlar miqdori minimal bo‘ladi? Masalaning matematik modelini tuzing.

30.35. Korxonada bir xil mahsulot 2 xil texnologiya asosida ishlab chiqariladi. Birinchi texnologiya bo‘yicha x_1 miqdorda mahsulot ishlab chiqarish uchun $3x_1 - 5x_1^2$ miqdorda, ikkinchi texnologiya bo‘yicha x_2 miqdorda mahsulot ishlab chiqarish uchun $-2x_2 + 4x_2^2$ miqdorda xarajat sarf qilinadi. Korxona bozor talabini inobatga olgan holda 60 birlik mahsulot ishlab chiqarishni rejalashtirgan. Umumiy xarajatlarni minimallashtirish uchun har bir texnologiya bo‘yicha qancha mahsulot ishlab chiqarish kerakligini aniqlang.

30.36. Korxonada ishlab chiqarilgan mahsulotni savdo do‘konlari va bozorda sotish rejalashtirilgan. x_1 miqdordagi mahsulotni savdo do‘konlarida sotish uchun $-2x_1 + x_1^2$ sharli birlikda, x_2 miqdordagi mahsulotni savdo agentlari yordamida sotish uchun $-6x_2 + 3x_2^2$ miqdorda xarajat sarf qilinadi. Firmada ishlab chiqarilgan 100 birlik mahsulotni eng kam harajat sarf qilib sotish yo‘lini aniqlang.

30.37. Korxonada ishlab chiqariladigan 120 tonna mahsulotning x_1 tonnasi avtotransport vositasida, x_2 tonnasi esa poezdda iste’molchilarga yetkazib beriladi. Poezdda tashilgan mahsulotga $(x_2 - 2)^2$ miqdorda, avtotransportda tashilgan mahsulotga $(x_1 - 3)^2$ miqdorda xarajat sarf qilinadi. Ishlab chiqariladigan mahsulotni iste’molchilarga qanday yo‘l bilan yetkazib berilganda sarf qilingan xarajatlar miqdori minimal (eng kam) bo‘ladi?

30.38. Korxona 2 ta ta’minotchidan xom ashyo sotib oladi. Birinchi ta’minotchidan keltirilgan x_1 miqdordagi xom ashyo uchun $-30x_1 + x_1^2$ miqdorda, ikkinchi ta’minotchidan keltirilgan x_2 miqdordagi xom ashyo uchun $-16x_2 + 2x_2^2$ miqdorda xarajat sarf qilinadi. Korxonaning xom ashyolarga bo‘lgan talabi 80 shartli birlikni tashkil qiladi. Korxona qaysi ta’minotchidan qancha xom ashyo sotib olsa, sarf qilingan umumiy xarajat minimal bo‘ladi?

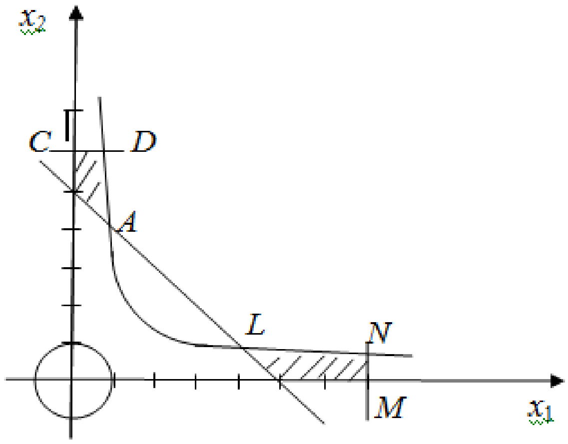
31-amaliy mashg‘ulot. Chiziqsiz programmalashtirish masalasi

31.1. Quyidagi masalalarni geometrik talqindan foydalanib yeching.

$$\begin{cases} x_1 x_2 \le 4 \\ x_1 + x_2 \ge 5 \\ x_1 \le 7 \\ x_2 \le 6 \end{cases}$$

$$x_1 \ge 0, \quad x_2 \ge 0$$

$$Z = x_1^2 + x_2^2 \to \max(\min)$$



1-chizma

Yechish. Bu masalaning mumkin bo‘lgan rejalar to‘plami qavariq to‘plam bo‘lmaydi, aksincha, ikkita ayrim K_1 va K_2 qismlardan iborat bo‘ladi (1-chizma). Maqsad funksiya o‘zining lokal minimum qiymatiga $A(1;\ 4)$ va $L(4;1)$ nuqtalarda erishadi $Z(A) = Z(L) = 17$. $D(2/3;\ 6)$ va $N(7;\ 4/7)$ nuqtalarda esa funksiya lokal maksimum qiymatlarga erishadi.

$$Z(D) = \frac{328}{9}, \qquad Z(N) = \frac{2417}{49}.$$

Lokal maksimum qiymatlarni taqqoslab Z funksiya N nuqtada global maksimumga erishishini ko‘rishimiz mumkin. D va N nuqtalarning koordinatalari va Z funksiyaning qiymati quyidagicha topiladi: $D(x_1^*; x_2^*)$ nuqta $x_2 = 6$ va $x_2 = \frac{4}{x_1}$ egri chiziqda yotgani uchun uning koordinatalari bu tenglamalarni qanoatlantirishi kerak, ya’ni:

$$\begin{cases} x_2^* = 6 \\ x_2^* = \dfrac{4}{x_1^*} \end{cases} \Rightarrow \begin{cases} x_1^* = \dfrac{2}{3} \\ x_2^* = 6 \end{cases}$$

$$Z^* = x_1^{*2} + x_2^{*2}; \qquad Z^* = 328/9 = Z(D).$$

Xuddi shuningdek, N nuqta $x_1 = 7$ to‘g‘ri chiziq va $x_2 = \dfrac{4}{x_1}$ egri chiziqning kesishgan nuqtasi bo‘lishi uchun uning x_1^0, x_2^0 koordinatalari bu tenglamalarni qanoatlantirishi kerak, ya’ni:

$$\begin{cases} x_1^0 = 7 \\ x_2^0 = \dfrac{4}{7} \\ Z^0 = (x_1^0)^2 + (x_2^0)^2 \end{cases} \Rightarrow \begin{cases} x_1^0 = 7 \\ x_2^0 = \dfrac{4}{7} \\ Z^0 = \dfrac{2417}{49} \end{cases}$$

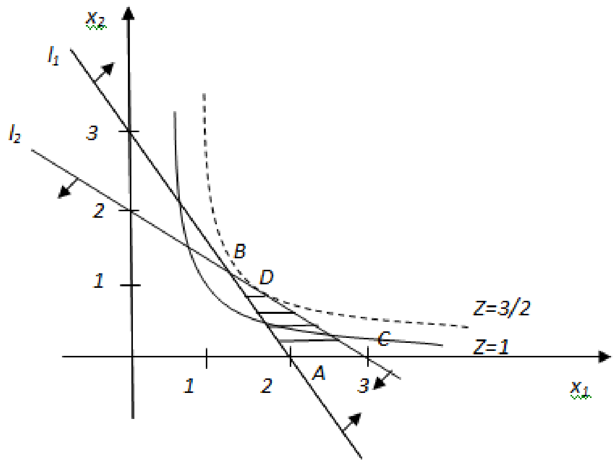
31.2. Quyidagi masalalarni geometrik talqindan foydalanib yeching.

$$\begin{cases} 3x_1 + 2x_2 \geq 6 \\ 2x_1 + 3x_2 \leq 6 \end{cases}$$

$$x_1 \geq 0, \quad x_2 \geq 0$$

$$Z = x_1 \cdot x_2 \rightarrow \max$$

Yechish. Masalaning rejalaridan tashkil topgan to‘plam ABC uchburchakdan iborat bo‘ladi (2-chizma).



2-chizma

Maqsad funksiya Z ga ixtiyoriy Q qiymat beramiz ($Q > 0$). Natijada hosil bo‘lgan

$$x_1 \cdot x_2 = Q$$

tenglama giperbolani ifodalaydi. Q ning qiymatini o'zgartirib borib, giperbolik egri chiziqni o'ziga parallel ravishda siljitib borish mumkin. Natijada 2-chizmadan ko'rish mumkinki, giperbolaning ABC uchburchakning BC tomoniga uringan nuqtasi $D(x_1^*;x_2^*)$ da Z funksiya maksimumga erishadi. Bu nuqtada Z funksiyaning qiymatini Z^* bilan belgilaymiz. Demak, x_1^*, x_2^*, Z^* noma'lumlarning qiymatlarini topishimiz kerak. Ushbu noma`lumlar quyidagi shartlarni qanoatlantirishi kerak:

$$2x_1^* + 3x_2^* = 6$$

$$Z^* = x_1^* \cdot x_2^*$$

Bundan tashqari $x_1^* \cdot x_2^* = Z^*$ giperbolaning $(x_1^*;x_2^*)$ nuqtadagi urinmasi og'ish burchagining tangensi –2/3 ga teng, chunki bu urunma $2x_1 + 3x_2 = 6,$ to'g'ri chiziq bilan ustma-ust tushadi. Bu to'g'ri chiziq og'ish burchagining tangensi esa –2/3 ga teng.

Ikkinchi tomondan

$$Z^* = x_1^* \cdot x_2^*$$

giperbolaga o'tkazilgan urunma og'ish burchagining tangensi

$$\frac{dx_2}{dx_1} = -\frac{x_2}{x_1}$$

formula yordamida topish mumkin. Demak,

$$-\frac{x_2}{x_1} = -\frac{2}{3}.$$

Bundan $3x_2^* = 2x_1^*$ yoki $2x_1^* - 3x_2^* = 0.$

Shunday qilib, masalaning opimal yechimi quyidagi sistemaning yechimidan iborat bo'ladi:

$$\begin{cases} 2x_1^* + 3x_2^* = 6 \\ 2x_1^* - 3x_2^* = 0 \\ Z^* = x_1^* \cdot x_2^* \end{cases} \Rightarrow \begin{cases} x_1^* = \dfrac{3}{2} \\ x_2^* = 1 \\ Z^* = \dfrac{3}{2} \end{cases}$$

Javob: $X^*\left(\frac{3}{2};1\right), \quad Z_{\max} = \frac{3}{2}.$

Grafik usulidan foydalanib, quyidagi chiziqsiz programmalashtirish masalalarini yeching:

31.3. $\begin{cases} x_1 + 2x_2 \ge 2 \\ x_1 + x_2 \le 6 \\ 2x_1 + x_2 \le 11 \end{cases}$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = 2(x_1 - 7)^2 + 4(x_2 - 3)^2 \to \min(\max)$

31.4. $\begin{cases} x_1 + 2x_2 \ge 2 \\ x_1 + x_2 \le 6 \\ 2x_1 + x_2 \le 10 \end{cases}$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = x_1 \cdot x_2 \to \max$

31.5. $\begin{cases} x_1 + x_2 \le 7 \\ 2x_1 - x_2 \le 8 \end{cases}$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = 4(x_1 - 2)^2 + 2(x_2 - 2)^2 \to \min(\max)$

31.6. $\begin{cases} x_1 + x_2 \ge 2 \\ x_1 - x_2 \ge -2 \\ x_1 + x_2 \le 6 \\ x_1 - 3x_2 \le 2 \end{cases}$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = 25(x_1 - 2)^2 + (x_2 - 2)^2 \to \max$

31.7. $\begin{cases} x_1 + 2x_2 \le 8 \\ 3x_1 + x_2 \le 15 \\ x_1 + x_2 \ge 1 \end{cases}$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = (x_1 - 6)^2 + (x_2 - 2)^2 \to \min(\max)$

31.8. $\begin{cases} 6x_1 + 4x_2 \ge 12 \\ 2x_1 + 3x_2 \le 24 \\ -3x_1 + 4x_2 \le 12 \end{cases}$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = x_1 \cdot x_2 \to \max$

31.9. $\begin{cases} 3x_1 + 2x_2 \ge 12 \\ x_1 - x_2 \le 6 \\ x_2 \le 4 \end{cases}$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = 9(x_1 - 5)^2 + 4(x_2 - 6)^2 \to \min$

31.10. $\begin{cases} 3x_1 + 2x_2 \ge 7 \\ 2x_1 - x_2 \le 8 \\ -3x_1 + 4x_2 \le 12 \end{cases}$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = (x_1 - 3)^2 + (x_2 - 4)^2 \to \min\ (\max)$

31.11. $\begin{cases} x_1 \cdot x_2 \le 8 \\ 0 \le x_1 \le 6 \end{cases}$

$0 \le x_2 \le 4$

$Z = x_1 + 3x_2 \to \min\ (\max)$

31.12. $\begin{cases} x_1 + x_2 \ge 1 \\ 2x_1 + 3x_2 \le 12 \end{cases}$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = (x_1 - 4)^2 + (x_2 - 6)^2 \to \min$

31.13. $x_1^2 + x_2^2 \le 16$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = 2x_1 + x_2 \to \min\ (\max)$

31.14. $x_1^2 + x_2^2 \le 16$

$x_1 \ge 0, \;\; x_2 \ge 0$

$Z = (x_1 - 2)^2 + (x_2 - 1)^2 \to \min\ (\max)$

31.15. $\begin{cases} 2x_1 + 3x_2 \ge 6 \\ 3x_1 - 2x_2 \le 18 \\ -x_1 + 2x_2 \le 8 \end{cases}$

31.16. $\begin{cases} x_1 \cdot x_2 \ge 2 \\ x_1^2 + x_2^2 \le 16 \end{cases}$

$x_1 \geq 0, \quad x_2 \geq 0$ $\qquad$ $x_1 \geq 0, \quad x_2 \geq 0$

$Z = (x_1 - 4)^2 + (x_2 - 3)^2 \to \min\ (\max)$ $\qquad$ $Z = 3x_1 + x_2 \to \min\ (\max)$

31.17. $\begin{cases} x_1 - x_2 \leq 4 \\ 2x_1 + x_2 \geq 8 \\ x_2 \leq 6 \end{cases}$ $\qquad$ **31.18.** $\begin{cases} 5x_1 + 4x_2 \leq -20 \\ 3x_1 + 2x_2 \leq 18 \end{cases}$

$x_1 \geq 0$ $\qquad$ $x_1 \geq 0, \quad x_2 \geq 0$

$Z = (x_1 - 4)^2 + (x_2 - 2)^2 \to \min\ (\max)$ $\qquad$ $Z = (x_1 - 5)^2 + (x_2 - 4)^2 \to \min\ (\max)$

31.19. Korxonaning ikki bo‘limida bir xil mahsulot ishlab chiqariladi. Kelgusi yilda ushbu mahsulotga bo‘lgan talab ko‘pi bilan 50 tonna bo‘lishi kutilmoqda. Birinchi bo‘limida ishlab chiqariladigan x_1 miqdordagi mahsulot uchun $9(x_1 - 30)^2$ miqdorda xarajat sarf qilinadi. Ikkinchi bo‘limida ishlab chiqariladigan x_2 tonna mahsulot uchun $4(x_2 - 50)^2$ miqdorda xarajat sarf qilinadi. Har bir bo‘limda qancha mahsulot ishlab chiqarilganda sarf qilingan xarajat miqdori eng kam bo‘ladi va mahsulotga bo‘lgan talab qondiriladi?

31.20. Ma’lum bir xududda A mahsulotga bo‘lgan maksimal talab 70 birlikni tashkil qiladi. Ushbu mahsulot 2 ta korxonada ishlab chiqariladi. Birinchi korxonada ishlab chiqarilgan x_1 mahsulotga $4(x_1 - 40)^2$ miqdorda xarajat sarf qilinadi. Ikkinchi korxonada ishlab chiqarilgan x_2 miqdordagi mahsulotga $25(x_2 - 60)^2$ miqdorda xarajat sarf qilinadi. Har bir korxonada qanchadan mahsulot ishlab chiqarilganda mahsulotga bo‘lgan talab qondiriladi va sarf qilingan jami xarajat miqdori minimal bo‘ladi?

31.21. Do‘konda ikki xil mahsulot sotiladi. Birinchi mahsulotning x_1 birligini $(x_1 - 40)^2$ so‘mdan, ikkinchi mahsulotning x_2 birligini $(x_2 - 15)^2$ so‘mdan sotiladi. Tovarlarning maksimal sotish hajmi 54 birlikni tashkil qiladi. Do‘konda qaysi mahsulotdan qanchadan sotilganda uning daromadi maksimal bo‘ladi?

31.22. Firma o‘zi ishlab chiqargan mahsulotlarni 2 xil yo‘l bilan, ya’ni do‘kon orqali yoki savdo agentlari orqali sotadi. Firma x_1 miqdordagi mahsulotni do‘konlarda sotish uchun $(x_1 - 60)^2$ sh.b. miqdorida, x_2 miqdordagi mahsulotni savdo agentlari orqali sotganda $(x_2 - 80)^2$ sh.b. miqdorida xarajat sarf qiladi. Firmada ko‘pi bilan 120 birlik mahsulot ishlab chiqariladi. Ushbu mahsulotlarni qanday yo‘l bilan sotganda firmaning umumiy xarajati minimal bo‘ladi?

32-amaliy mashg‘ulot. Qavariq programmalashtirish masalasi

32.1. Quyidagi funksiyaning qavariqligini tekshiring.

$$F(X)=5x_1^2+2x_2^2-3x_1x_2+3x_1-4x_2+6$$

Yechish. $F(X)$ funksiyadan x_1 va x_2 lar bo‘yicha birinchi va ikkinchi tartibli xususiy hosilalar olamiz:

$$\frac{\partial F(X)}{\partial x_1}=10x_1-3x_2+3; \qquad \frac{\partial F(X)}{\partial x_2}=-3x_1+4x_2-4;$$

$$\frac{\partial^2 F(X)}{\partial x_1^2}=10; \qquad \frac{\partial^2 F(X)}{\partial x_1 \partial x_2}=\frac{\partial^2 F(X)}{\partial x_2 \partial x_1}=-3; \qquad \frac{\partial^2 F(X)}{\partial x_2^2}=4.$$

Ikkinchi tartibli xususiy hosilalardan foydalanib Gesse matritsasini tuzamiz:

$$H=\begin{pmatrix} 10 & -3 \\ -3 & 4 \end{pmatrix}.$$

Ushbu matritsaning bosh minorlari

$$\Delta_1=10>0; \qquad \Delta_2=|H|=31>0.$$

Demak, $F(X)$ funksiya qat’iy botiq funksiya bo‘ladi.

32.2. Quyidagi masalani grafik usulda yeching va topilgan yechim uchun Kun-Takker shartlari o‘rinli ekanligini ko‘rsating.

$$\begin{cases} 2x_1+x_2\ge 2 \\ 2x_1+x_2\le 8 \\ x_1+x_2\le 6 \end{cases}$$

$$x_1\ge 0, \quad x_2\ge 0$$

$$Z=f(x_1;\, x_2)=x_1^2+x_2^2-4x_1-4x_2+8\to \min$$

Yechish. Masalani grafik usulda yechib, $X^0=(2;\,2)$ va $f(2;\,2)=0$ ekanligini aniqlaymiz.

Endi shunday Λ^0 mavjud bo‘lib, $(X^0;\,\Lambda^0)$ nuqtada Kun-Takker shartlarining bajarilishini ko‘rsatamiz.

Berilgan masala uchun Lagranj funksiyasini tuzamiz.

$$f(X;\,\Lambda)=\lambda_0(x_1^2+x_2^2-4x_1-4x_2+8)+$$
$$+\lambda_1(2x_1+x_2-2)+\lambda_2(8-2x_1-x_2)+\lambda_3(6-x_1-x_2)$$

X^0 nuqtada masalaning barcha shartlari qat’iy tengsizlikka aylanadi, ya’ni Sleyter sharti bajariladi, bu holda $\lambda_0=1$ deb qabul qilishimiz mumkin.

Lagranj funksiyasidan x_1, x_2, λ_1, λ_2, λ_3 lar bo‘yicha xususiy hosilalar olamiz:

$$\frac{\partial F}{\partial x_1}=2x_1-4+2\lambda_1-2\lambda_2-\lambda_3; \qquad \frac{\partial F}{\partial x_2}=2x_2-4+\lambda_1-\lambda_2-\lambda_3;$$

$$\frac{\partial F}{\partial \lambda_1}=2x_1+x_2-2; \qquad \frac{\partial F}{\partial \lambda_2}=-2x_1-x_2+8; \qquad \frac{\partial F}{\partial \lambda_3}=-2x_1-x_2+6;$$

$$\frac{\partial F(X^0;\Lambda^0)}{\partial \lambda_1}=2\cdot 2+2-2=4>0; \qquad \frac{\partial F(X^0;\Lambda^0)}{\partial \lambda_2}=8-2\cdot 2-2=4>0;$$

$$\frac{\partial F(X^0;\Lambda^0)}{\partial \lambda_3}=6-2-2=2>0; \qquad \lambda_i^0\frac{\partial F(X^0;\Lambda^0)}{\partial \lambda_i}=0$$

shartga ko‘ra, λ_1, λ_2 va λ_3 larning qiymatlari nolga teng.

Demak, $(X^0;\Lambda^0)=(2;2;0;0;0)$ nuqtada haqiqatdan ham, Kun-Takker shartlari bajarilayapti. Demak u egar nuqta bo‘ladi.

32.3. Kun-Takker shartlaridan foydalanib, $X^0=(1;0)$ nuqta quyidagi chiziqsiz programmalashtirish masalasining yechimi ekanligini ko‘rsating.

$$\begin{cases} 4x_1+5x_2\le 8 \\ 2x_1+x_2\le 4 \end{cases}$$

$$x_1\ge 0, \quad x_2\ge 0$$

$$Z=f(x_1;x_2)=x_1^2-2x_1+3x_2^2\to \min$$

Yechish. $X^0=(1;0)$ nuqtada chegaraviy shartlar qat’iy tengsizlikka aylanadi, demak Sleyter sharti bajariladi. Bu holda $\lambda_0=1$ deb qabul qilish mumkin, u holda Lagranj funksiyasi quyidagi ko‘rinishda bo‘ladi.

$$F(x_1;x_2;\lambda_1;\lambda_2)=x_1^2-2x_1+3x_2^2+\lambda_1(4x_1+5x_2-8)+\lambda_2(2x_1+x_2-4)$$

$$x_1\ge 0, \quad x_2\ge 0, \quad \lambda_1\ge 0, \quad \lambda_2\ge 0$$

Kun-Takker shartlarining bajarilishini tekshiramiz.

$$\frac{\partial F(X^0;\Lambda^0)}{\partial x_1}=(2x_1-2+4\lambda_1+2\lambda_2)_{X^0}\ge 0;$$

$$\frac{\partial F(X^0;\Lambda^0)}{\partial x_2}=(6x_2+5\lambda_1+\lambda_2)_{X^0}\ge 0;$$

$$\frac{\partial F(X^0;\Lambda^0)}{\partial \lambda_1}=(4x_1+5x_2-8)_{X^0}=-4<0;$$

$$\frac{\partial F(X^0;\Lambda^0)}{\partial \lambda_1}\cdot\lambda_1^0=0 \quad\Rightarrow\quad \lambda_1^0=0;$$

$$\frac{\partial F(X^0;\Lambda^0)}{\partial \lambda_2}=(2x_1+x_2-4)_{X^0}=-2<0;$$

$$\frac{\partial F(X^0;\ \Lambda^0)}{\partial \lambda_2}\cdot\lambda_2^0=0 \quad\Rightarrow\quad \lambda_2^0=0.$$

Demak, $(X^0;\ \Lambda^0)=(1;\ 0;\ 0;\ 0)$ nuqta Kun-Takker shartlarini qanoatlantiradi. Bundan u Lagranj funksiyasining egar nuqtasi bo‘lishi kelib chiqadi. Shuning uchun $X^0=(1;\ 0)$ nuqta berilgan masalaning yechimi bo‘ladi.

Endi Kun-Takker teoremasidan foydalanib qavariq programmalashtirish masalasini yechish jarayoni bilan tanishamiz.

Buning uchun quyidagi masalaga murojaat qilamiz:

$$\begin{cases} x_1+2x_2\le 8 \\ 2x_1-x_2\le 12 \end{cases}$$

$$x_1\ge 0,\quad x_2\ge 0$$

$$Z=f(x_1;\ x_2)=2x_1+4x_2-x_1^2-2x_2^2\to \max$$

Bu masaladagi maqsad funksiya chiziqli $f_1=2x_1+4x_2$ va $f_2=-x_1^2-x_2^2$ kvadratik funksiyalarning yig‘indisidan iborat. Bunda $f_2=-x_1^2-x_2^2$ funksiya manfiy aniqlangan kvadratik formadan iborat bo‘lgani uchun botiq funksiya bo‘ladi. Chiziqli $f_1=2x_1+4x_2$ funksiyani ham botiq funksiya, deb qarash mumkin. Shunday qilib, berilgan masalaning chegaraviy shartlari chiziqli tengsizliklardan, maqsad funksiyasi esa botiq funksiyadan iborat bo‘lgan qavariq programmalashtirish masalasidan iborat. Ushbu masalaga Kun-Takker teoremasini qo‘llash mumkin.

Lagranj funksiyasini tuzamiz:

$$F(x_1;\ x_2;\ \lambda_1;\ \lambda_2)=2x_1+4x_2-x_1^2-2x_2^2+$$
$$+\lambda_1(8-x_1-2x_2)+\lambda_2(12-2x_1+x_2).$$

Lagranj funksiyasining egar nuqtasining mavjudligini ifodalovchi Kun-Takker shartlarini yozamiz:

$$\begin{cases} \dfrac{\partial F}{\partial x_1}=2-2x_1-\lambda_1-2\lambda_2\le 0 \\ \dfrac{\partial F}{\partial x_2}=4-4x_1-2\lambda_1+\lambda_2\le 0 \\ \dfrac{\partial F}{\partial \lambda_1}=8-x_1-2x_2\ge 0 \\ \dfrac{\partial F}{\partial \lambda_1}=12-2x_1+x_2\ge 0 \end{cases} \qquad \text{(I)}$$

$$\begin{cases} x_1 \dfrac{\partial F}{\partial x_1} = x_1(2 - 2x_1 - \lambda_1 - 2\lambda_2) = 0 \\ x_2 \dfrac{\partial F}{\partial x_2} = x_2(4 - 4x_1 - 2\lambda_1 + \lambda_2) = 0 \\ \lambda_1 \dfrac{\partial F}{\partial \lambda_1} = \lambda_1(8 - x_1 - 2x_2) = 0 \\ \lambda_2 \dfrac{\partial F}{\partial \lambda_1} = \lambda_2(12 - 2x_1 + x_2) = 0 \end{cases} \qquad \text{(II)}$$

$$x_1 \geq 0,\ \ x_2 \geq 0,\ \ \lambda_1 \geq 0,\ \ \lambda_2 \geq 0 \qquad \text{(III)}$$

(I) sistemaga $\nu_1,\ \nu_2,\ w_1,\ w_2$ nomanfiy qo‘shimcha o‘zgaruvchilar kiritib, uni tenglamalar sistemasiga aylantiramiz:

$$\begin{cases} 2x_1 + \lambda_1 + 2\lambda_2 - \nu_1 = 2 \\ 4x_1 + 2\lambda_1 + \lambda_2 - \nu_2 = 4 \\ x_1 + 2x_2 + w_1 = 8 \\ 2x_1 - x_2 + w_2 = 12 \end{cases} \qquad \text{(IV)}$$

$$x_1 \geq 0,\ \ x_2 \geq 0,\ \ \lambda_1 \geq 0,\ \ \lambda_2 \geq 0$$

$$\nu_1 \geq 0,\ \ \nu_2 \geq 0,\ \ w_1 \geq 0,\ \ w_2 \geq 0$$

Ushbu sistemani yana quyidagicha yozish mumkin.

$$\begin{cases} \nu_1 = 2 - 2x_1 - \lambda_1 - 2\lambda_2 \\ \nu_2 = 4 - 4x_1 - 2\lambda_1 + \lambda_2 \\ w_1 = 8 - x_1 - 2x_2 \\ w_2 = 12 - 2x_1 + x_2 \end{cases} \qquad \text{(V)}$$

$$x_1\nu_1 = 0,\ \ x_2\nu_2 = 0,\ \ \lambda_1 w_1 = 0,\ \ \lambda_2 w_2 = 0 \qquad \text{(VI)}$$

Ushbu tengliklarni (II) ni nazarga olib quyidagicha yozish mumkin. Endi (IV) sistemaning (VI) shartni qanoatlantiruvchi bazis yechimini topamiz.

Demak, ushbu yechimni ifodalovchi nuqta Lagranj funksiyasining egar nuqtasi va berilgan masalaning optimal yechimini beradi.

(IV) sistemaning (1) va (2) tenglamasiga mos ravishda z_1 va z_2 sun’iy o‘zgaruvchilarni kiritib quyidagi chiziqli programmalashtirish masalasini tuzamiz:

$$\begin{cases} 2x_1 + \lambda_1 + 2\lambda_2 - \nu_1 + z_1 = 2 \\ 4x_1 + 2\lambda_1 + \lambda_2 - \nu_2 + z_2 = 4 \\ x_1 + 2x_2 + w_1 = 8 \\ 2x_1 - x_2 + w_2 = 12 \end{cases}$$

$$x_1 \geq 0,\ \ x_2 \geq 0,\ \ \lambda_1 \geq 0,\ \ \lambda_2 \geq 0$$

$$\nu_1 \geq 0,\ \ \nu_2 \geq 0,\ \ w_1 \geq 0,\ \ w_2 \geq 0$$

$$z_1 \geq 0, \;\; z_2 \geq 0$$

$$Z = Mz_1 + Mz_2 \to \min$$

Ushbu masalani sun’iy bazis vektor usuli bilan yechamiz:

Bazis	C_b	P_0	0	0	0	0	0	0	M	M	0	0
			X_1	X_2	Λ_1	Λ_2	V_1	V_2	Z_1	Z_2	W_1	W_2
Z_1	M	2	2	0	1	2	-1	0	1	0	0	0
Z_2	M	4	0	2	2	-1	0	-1	0	1	0	0
W_1	0	8	1	[4]	0	0	0	0	0	0	1	0
W_2	0	12	2	-1	0	0	0	0	0	0	0	1
$\Delta_j = Z_j - C_j$		$6M$	$2M$	$4M^*$	$3M$	M	$-M$	$-M$	0	0	0	0
Z_1	M	[2]	2	0	1	2	-1	0	1	0	0	0
X_2	0	1	0	1	1/2	-1/4	0	-1/4	0	1/4	0	0
W_1	0	6	1	0	-1	1/2	0	1/2	0	-1/2	1	0
W_2	0	13	2	0	1/2	-1/4	0	-1/4	0	1/4	0	1
$\Delta_j = Z_j - C_j$		$2M^*$	$2M$	0	M	$2M$	$-M$	0	0	0	0	0
X_1	0	1	1	0	1/2	1	-1/2	0	1/2	0	0	0
X_2	0	1	0	1	1/2	-1/4	0	-1/4	0	1/4	0	0
W_1	0	5	0	0	-3/2	-1/2	1/2	1/2	-1/2	-1/2	1	0
W_2	0	11	0	0	-1/2	-9/4	1	-1/4	-1	1/4	0	1
$\Delta_j = Z_j - C_j$		0	0	0	0	0	0	0	0	0	0	0

Tuzilgan chiziqli programmalashtirish masalasining optimal yechimi:

$$x_1^0 = 1; \quad x_2^0 = 1; \quad \lambda_1^0 = 0; \quad \lambda_2^0 = 0;$$

$$v_1^0 = 0; \quad v_2^0 = 0; \quad w_1^0 = 5; \quad w_2^0 = 11.$$

Ushbu yechim (IV) sistemaning (VI) shartni qanoatlantiruvchi bazis yechimi bo‘ladi.

Demak, $(X^0; \Lambda^0) = (1; 1; 0; 0)$ Lagranj funksiyasining egar nuqtasi bo‘ladi. Bunda $X^0 = (1; 1)$ berilgan masalaning optimal yechimi bo‘lib, unda $f(X^0) = 3$ bo‘ladi.

Quyidagi funksiyalarning qavariq yoki qavariq emasligini ko‘rsating.

32.4. $f(x_1; x_2) = 2x_1^2 + x_2^2 - x_1x_2 + 5x_1 - 6x_2 + 8$

32.5. $f(x_1; x_2) = x_1^2 + x_2^2 - 2x_1x_2 + 2x_1 + x_2$

32.6. $f(x_1;\,x_2)=x_1^2+x_2^2-4x_1+5x_2$

32.7. $f(x_1;\,x_2)=3x_1^2+5x_2^2-4x_1x_2-5x_1-12x_2$

32.8. $f(x_1;\,x_2)=5x_1^2-2x_2^2-x_1x_2-2x_1+3x_2$

32.9. $f(x_1;\,x_2)=2x_1^2+2x_2^2-3x_1x_2+3x_1+5x_2$

32.10. $f(x_1;\,x_2)=-2x_1^2-x_2^2+3x_1x_2+8x_1+6x_2$

32.11. $f(x_1;\,x_2)=3x_1^2+3x_2^2+5x_1x_2-6x_1-4x_2+20$

32.12. $f(x_1;\,x_2)=4x_1^2+2x_2^2-x_1x_2-4x_1-2x_2$

32.13. $f(x_1;\,x_2)=x_1^2-x_2^2+2x_1x_2+5x_1+7x_2+5$

Quyidagi masalalarda Kun-Takker shartlaridan foydalanib, berilgan X^0 nuqta qavariq programmalashtirish masalasining yechimi ekanligini aniqlang.

32.14. $\begin{cases} x_1+2x_2\le 8 \\ 3x_1+x_2\le 15 \\ x_1+x_2\ge 1 \end{cases}$ $\quad X^0=(0;\,4)$

$x_1\ge 0,\;\; x_2\ge 0$

$Z=f(x_1;\,x_2)=(x_1-6)^2+(x_2-2)^2\to\max$

32.15. $\begin{cases} 2x_1+3x_2\le 24 \\ x_1+2x_2\le 15 \\ 3x_1+2x_2\ge 24 \\ x_2\le 4 \end{cases}$ $\quad X^0=\left(\dfrac{32}{11};\,\dfrac{84}{11}\right)$

$x_1\ge 0,\;\; x_2\ge 0$

$Z=f(x_1;\,x_2)=6x_1^2+x_2^2-6x_1+6\to\max$

32.16. $\begin{cases} -2x_1+x_2\le 0 \\ x_1-2x_2\le 0 \\ x_1+x_2\ge 1 \end{cases}$ $\quad X^0=\left(\dfrac{4}{5};\,\dfrac{1}{4}\right)$

$x_1\ge 0,\;\; x_2\ge 0$

$Z=f(x_1;\,x_2)=x_1^2+4x_2^2\to\min$

32.17. $\begin{cases} 2x_1+3x_2\le 3 \\ 2x_1-x_2\le 8 \\ x_1+x_2\le 6 \end{cases}$ $\quad X^0=\left(\dfrac{6}{13};\,\dfrac{9}{13}\right)$

$x_1\ge 0,\;\; x_2\ge 0$

$Z=f(x_1;\,x_2)=x_1^2+x_2^2\to\min$

Quyidagi masalalarni grafik usulda yeching va topilgan yechim Kun-Takker shartlarini qanoatlantirishini ko‘rsating.

32.18. $\begin{cases} 3x_1 + 2x_2 \geq 6 \\ 8x_1 - x_2 \leq 8 \\ -6x_1 + 2x_2 \leq 6 \end{cases}$

$x_1 \geq 0, \ \ x_2 \geq 0$

$Z = f(x_1; x_2) = (x_1 - 3)^2 + (x_2 - 4)^2 \to \min$

32.19. $\begin{cases} x_1 + x_2 \geq 1 \\ 2x_1 + x_2 \leq 4 \\ x_1 + 2x_2 \leq 6 \end{cases}$

$x_1 \geq 0, \ \ x_2 \geq 0$

$Z = f(x_1; x_2) = 4x_1^2 + 6x_2^2 - 8x_1 - 12x_2 + 10 \to \max$

32.20. $\begin{cases} x_1 + 2x_2 \leq 12 \\ x_1 - x_2 \leq 8 \end{cases}$

$x_1 \geq 0, \ \ x_2 \geq 0$

$Z = f(x_1; x_2) = x_1^2 + x_2^2 - 2x_1 - 8x_2 + 17 \to \max$

32.21. $\begin{cases} x_1 + 2x_2 \geq 12 \\ 3x_1 + x_2 \leq 15 \end{cases}$

$x_1 \geq 0, \ \ x_2 \geq 0$

$Z = f(x_1; x_2) = 2x_1^2 + 4x_2^2 - 4x_1 - 8x_2 + 6 \to \max$

32.22. $\begin{cases} 3x_1 + 2x_2 \geq 12 \\ x_1 - x_2 \leq 6 \\ x_2 \leq 4 \end{cases}$

$x_1 \geq 0, \ \ x_2 \geq 0$

$Z = f(x_1; x_2) = 9x_1^2 + 4x_2^2 - 90x_1 - 48x_2 + 369 \to \max$

Quyidagi masalalarni Kun-Takker teoremasidan foydalanib yeching.

32.23. $\begin{cases} x_1 + 2x_2 \geq 2 \\ x_1 + x_2 \leq 10 \end{cases}$

$x_1 \geq 0, \ \ x_2 \geq 0$

$Z = f(x_1; x_2) = 9x_1^2 + 4x_2^2 + 2x_1 - 4x_2 \to \min$

32.24. $x_1 + 2x_2 \leq 2$

$x_1 \geq 0, \ \ x_2 \geq 0$

$$Z = f(x_1; x_2) = 4x_1^2 + 9x_2^2 + 8x_1 + 4 \to \max$$

32.25. $\begin{cases} x_1 + x_2 \le 7, \\ x_1 \le 5, \\ x_1 \ge 0, x_2 \ge 0, \end{cases}$

$$Z = f(x_1, x_2) = x_1^2 + x_2^2 - 4x_1 - 6x_2 \to \min.$$

32.26. $\begin{cases} x_1 + x_2 \ge 1 \\ 2x_1 + x_2 \le 4 \\ x_1 + 2x_2 \le 6 \end{cases}$

$x_1 \ge 0, \;\; x_2 \ge 0$

$$Z = f(x_1; x_2) = 4x_1^2 + 4x_2^2 + 8x_1 - 2x_2 \to \min$$

33-amaliy mashg'ulot. Aniqmas integral

33.1. $\int x^6 dx = \frac{x^7}{7} + C.$

33.2. $\int \sin\left(x + \frac{\pi}{3}\right) dx = \int \sin\left(x + \frac{\pi}{3}\right) d\left(x + \frac{\pi}{3}\right) = -\cos\left(x + \frac{\pi}{3}\right) + C.$

33.3. $\int \frac{dx}{4 + x^2} = \int \frac{dx}{2^2 + x^2} = \frac{1}{2} arctg \frac{x}{2} + C.$

Jadvaldan foydalanib quyidagi integrallarni toping:

33.4. $\int x^{10} dx.$

33.5. $\int \frac{dx}{x^7}.$

33.6. $\int \sqrt[4]{x} dx.$

33.7. $\int \frac{dx}{x^2 + 9}.$

33.8. $\int \frac{dx}{x^2 - \frac{1}{2}}.$

33.9. $\int \frac{dx}{\sqrt{x^2 + 3}}.$

33.10. Jadvaldan va aniqmas integralning asosiy xossalaridan foydalanib integrallarni toping.

Yechish.

$$\int \frac{2 - x^4}{1 + x^2} dx = \int \frac{1 - x^4 + 1}{1 + x^2} dx = \int \left(1 - x^2\right) dx + \int \frac{dx}{1 + x^2} = x - \frac{x^3}{3} + arctg\, x + C$$

Bu yerda

$$\int dx = x + C, \;\; \int x^n dx = \frac{x^{n+1}}{n+1} + C, \; \int \frac{dx}{1 + x^2} = arctg\, x + C$$

formulalardan foydalandik.

Jadvaldan va aniqmas integralning asosiy xossalaridan foydalanib quyidagi integrallarni toping:

33.11. $\int \frac{x^4+x^2-6x}{x^3}dx.$

33.12. $\int (\frac{5}{x}-\frac{10}{\sqrt[4]{x^3}}-\frac{3}{x^2+7})dx.$

33.13. $\int \sqrt{x}(x^2+1)dx.$

33.14. $\int \frac{3+\sqrt{4-x^2}}{\sqrt{4-x^2}}dx.$

33.15. $\int \frac{(x^3+2)^2}{\sqrt{x}}dx.$

33.16. $\int (4\sin x+8x^3-\frac{11}{\cos^2 x})dx$

33.17. $\int x\sqrt{x}dx.$

33.18. $\int \frac{dx}{\sqrt[5]{x}}.$

33.19. $\int \frac{2-\sqrt{1-x^2}}{\sqrt{1-x^2}}dx$

33.20. $\int \left(x^2+2x+\frac{1}{x}\right)dx$

33.21. $\int \frac{x-2}{x^3}dx$

33.22. $\int \left(\sqrt{x}+\sqrt[3]{x}\right)dx$

33.23. $\int \frac{\left(\sqrt{x}-1\right)^3}{x}dx$

33.24. $\int \frac{x-1}{\sqrt[3]{x^2}}dx.$

33.25. $\int \frac{\cos 2x}{cos^2 x\, sin^2 x}dx.$

33.26. $\int ctg^2 x\, dx.$

33.27. $\int \frac{\sqrt{x}dx}{1+2\sqrt{x}}$ integralni toping.

Yechish. ($t=1+2\sqrt{x}$ almashtirish kiritamiz; bu yerdan

$$x=\frac{(t-1)^2}{4};\quad dx=\frac{2(t-1)}{4}dt\,)$$

$$\int \frac{\sqrt{x}dx}{1+2\sqrt{x}}=\int \frac{\frac{t-1}{2}\cdot\frac{2(t-1)}{4}dt}{t}=\frac{1}{4}\int \frac{(t-1)^2}{t}dt=\frac{1}{4}\int \frac{t^2-2t+1}{t}dt=$$

$$=\frac{1}{4}\int \left(t-2+\frac{1}{t}\right)dt=\frac{1}{4}\int tdt-\frac{1}{2}\int dt+\frac{1}{4}\int \frac{dt}{t}=\frac{t^2}{8}-\frac{t}{2}+\frac{1}{4}\ln|t|+C=$$

$$=\frac{\left(1+2\sqrt{x}\right)^2}{8}-\frac{1+2\sqrt{x}}{2}+\frac{1}{4}\ln\left|1+2\sqrt{x}\right|+C.$$

33.28. $\int \frac{\sqrt{x+4}}{x}dx$ integralni toping.

Yechish. ($t^2=x+4$ almashtirish kiritamiz, bu yerdan $x=t^2-4;\ dx=2tdt,$ shuningdek, $t=\sqrt{x+4}$)

$$\int \frac{\sqrt{x+4}}{x}dx = \int \frac{t}{t^2-4}2tdt = 2\int \frac{t^2dt}{t^2-4} = 2\int \frac{t^2-4+4}{t^2-4}dt = 2\int \left(1+\frac{4}{t^2-4}\right)dt =$$

$$= 2\int dt + 8\int \frac{dt}{t^2-4} = 2t + 8\cdot\frac{1}{4}\ln\left|\frac{t-2}{t+2}\right| + C = 2\sqrt{x+4} + 2\ln\left|\frac{\sqrt{x+4}-2}{\sqrt{x+4}+2}\right| + C.$$

33.29. $\int \sqrt{25-x^2}dx$ integralni toping.

Yechish. ($x = 5\sin t$ almashtirish kiritamiz, $dx = 5\cos t dt$, $t = \arcsin\frac{x}{5}$, shuningdek $\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$ oraliqda $\sqrt{25-x^2} = \sqrt{25-25\sin^2 t} = 5\sqrt{1-\sin^2 t} = 5\cos t$)

$$\int \sqrt{25-x^2}dx = \int 5\cos t \cdot 5\cos t dt = 25\int \cos^2 t dt = 25\int \frac{1+\cos 2t}{2}dt =$$

$$= \frac{25}{2}\int dt + \frac{25}{2}\int \cos 2t dt = \frac{25}{2}t + \frac{25}{4}\sin 2t + C = 12{,}5\arcsin\frac{x}{5} + \frac{x\sqrt{25-x^2}}{2} + C.$$

O‘zgaruvchini almashtirish usulidan foydalanib integrallarni toping:

33.30. $\int x\cos\left(x^2\right)dx.$

33.31. $\int \frac{e^{\sqrt{2x-1}}}{\sqrt{2x-1}}dx.$

33.32. $\int x^3\left(1-2x^4\right)^3 dx.$

33.33. $\int x\sqrt{a-x}dx,\ a-x=t^2$

33.34. $\int \frac{dx}{x\ln x}$

33.35. $\int \frac{\cos x\, dx}{\sqrt{1+2\sin^2 x}}.$

33.36. $\int \frac{\sin 2x}{\sqrt{2+\cos^2 x}}dx.$

Bo‘laklab integrallash usulidan foydalanib integrallarni toping:

33.37. $\int arctgx dx.$

Yechish. $\int arctgxdx = xarctgx - \int \frac{x}{1+x^2}dx = xarctgx - \frac{1}{2}\int \frac{2xdx}{1+x^2} =$

$$\left[\begin{array}{ll} u = arctgx & \left| du = \frac{dx}{1+x^2} \right. \\ dv = dx & \left| v = \int dx = x \right. \end{array}\right]$$

$$= xarctgx - \frac{1}{2}\int \frac{d\left(1+x^2\right)}{1+x^2} = xarctgx - \frac{1}{2}\ln\left(1+x^2\right) + C.$$

33.38. $\int e^x \sin x dx.$

Yechish. $\int e^x \sin x dx = e^x \sin x - \int e^x \cos x dx = e^x \sin x - \left(e^x \cos x + \int e^x \sin x dx\right).$

$$\begin{bmatrix} u=\sin x & \bigg| & du=\cos x dx \\ dv=e^x dx & \bigg| & v=e^x \end{bmatrix} \quad \begin{bmatrix} u_1=\cos x & \bigg| & du_1=-\sin x dx \\ dv_1=e^x dx & \bigg| & v_1=e^x \end{bmatrix}$$

Shuning uchun $2\int e^x \sin x dx = e^x \sin x - e^x \cos x + C_1$

$$\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x) + C, \text{ bu yerda } C = \frac{C_1}{2}.$$

Bo‘laklab integrallash usulidan foydalanib integrallarni toping:

33.39. $\int (x+1)e^x dx$ **33.40.** $\int e^{2x} \cos x \, dx$

33.41. $\int \frac{\ln x}{x^3} dx$ **33.42.** $\int x^2 e^{3x} dx$

33.43. $\int x\, cosx dx$ **33.44.** $\int ln^2 x dx$

33.45. $\int \frac{x \cos x \, dx}{sin^3 x}$ **33.46.** $\int \frac{arcsin\frac{x}{2}}{\sqrt{2^2-x^2}} dx$

Quyidagi integrallarni toping:

33.47. $\int \frac{dx}{x^3}$ **33.48.** $\int \frac{dx}{\sqrt{2-x^2}}$

33.49. $\int \frac{dx}{2x^2-6}$ **33.50.** $\int (1+e^x)^2 dx$

33.51. $\int \frac{2x+3}{x^2-5} dx$ **33.52.** $\int tg^2 \varphi d\varphi$

33.53. $\int \frac{2x\, dx}{x^4+3}$ **33.54.** $\int \frac{\sin x \, dx}{\sqrt{1+2\cos x}}$

33.55. $\int \frac{dx}{\sqrt{e^x+1}}$; $(e^x+1=t^2)$ **33.56.** $\int \frac{e^x dx}{3+4e^x}$;

33.57. $\int e^{\sin x} \cos x \, dx$; **33.58.** $\int \frac{sin4x}{cos^4 2x+4} dx$;

33.59. $\int \frac{\sqrt{2-x^2}+\sqrt{2+x^2}}{\sqrt{4-x^4}} dx$; **33.60.** $\int x\, sinx \, dx$;

33.61. $\int x^3 e^x dx$; **33.62.** $\int \sqrt{a^2-x^2} dx,\ a>0,\ u=\sqrt{a^2-x^2}$;

33.63. $\int sin\sqrt{x} dx$; **33.64.** $\int (x^2+2x+3)\cos x \, dx$;

33.65. $\int \frac{3-2ctg^2 x}{cos^2 x} dx$.

O‘zgaruvchini almashtirishdan foydalanib integralni hisoblang:

33.66. $\int \frac{dx}{e^{2x-1}}$; **33.67.** $\int \sqrt[5]{3x+2} dx$

33.68. $\int \frac{dx}{(4x+3)^5}$ **33.69.** $\int \frac{dx}{3x+1}$

33.70. $\int \frac{dx}{\sqrt{2-x}}$ **33.71.** $\int \frac{xdx}{\sqrt{x^2+2}}$

33.72. $\int \frac{x^2 dx}{2x^3+5}$ **33.73.** $\int 9x+1/4)\sin(2x^2+x)dx$

33.74. $\int \sqrt[3]{2+\cos 3x} \sin x dx$ **33.75.** $\int e^{-\sqrt{2x}} dx$

33.76. $\int e^x\sqrt{2+5e^x}\,dx$ **33.77.** $\int \cos\frac{2x+1}{5}dx$

33.78. $\int \frac{\sin\ln x}{x}dx$ **33.79.** $\int \frac{\sqrt[3]{arctgx/3}}{9+x^2}dx$

33.80. $\int \frac{dx}{2x^2+1}$ **33.81.** $\int \frac{dx}{\sqrt{3+2x^2}}$

33.82. $\int \frac{e^{-x}}{1-e^{-2x}}dx$ **33.83.** $\int \frac{x^2+1}{x+1}dx$

33.84. $\int \frac{x^3dx}{1-x}$ **33.85.** $\int \frac{2x-1}{2x+1}dx$

33.86. $\int \frac{dx}{3x^2-5}$ **33.87.** $\int \frac{xdx}{\sqrt{16-x^4}}$

33.88. $\int \frac{x^2dx}{\sqrt{x^6+1}}$ **33.89.** $\int \frac{2x+1}{3x^2+2}dx$

33.90. $\int \frac{2x+1}{\sqrt{x^2+1}}dx$ **33.91.** $\int \frac{\sqrt{x}+\ln^2 x}{x}dx$

Bo‘laklab integrallash usulidan foydalanib integrallarni hisoblang:

33.92. $\int xe^{5x}dx$ **33.93.** $\int x^2e^{-x/2}dx$

33.94. $\int x^3e^{2x}dx.$ **33.95.** $\int \ln(1-x)dx$

33.96. $\int (x^2-3x)\ln xdx$ **33.97.** $\int x^2\ln^2 xdx$

33.98. $\int \frac{\ln(1-x)}{\sqrt{x}}dx$ **33.99.** $\int x\sin 3xdx$

33.100. $\int \frac{x}{\cos^2 x}dx$ **33.101.** $\int \sqrt{x^2-4}dx$

33.102. $\int \sqrt{2-x^2}dx$ **33.103.** $\int x\cos^2 xdx$

33.104. $\int arctg\sqrt{7x-1}dx$ **33.105.** $\int \frac{\arcsin\sqrt{x}}{\sqrt{x}}dx$

33.106. $\int x^2\cos dx$ **33.107.** $\int e^x\sin\frac{x}{2}dx$

33.108. $\int \cos(\ln x)dx$ **33.109.** $\int e^{\sqrt{x}}dx$

33.110. $\int \ln(\sqrt{1-x}+\sqrt{1+x})dx$ **33.111.** $\int \frac{x\cos x}{\sin^2 x}dx.$

33.112. $\int xtg^2 2xdx$ **33.113.** $\int x\ln\frac{1-x}{1+x}dx.$

33.114. $\int \cos^2(\ln x)dx$ **33.115.** $\int x^2 arctg3xdx.$

33.116. $\int \frac{\arcsin x}{x^2}dx$ **33.117.** $\int (\arcsin)^2dx.$

33.118. $\int 3^x\cos xdx.$ **33.119.** $\int e^{3x}\sin 2xdx.$

33.120. $\int \ln(1+x^2)dx.$ **33.121.** $\int \frac{\ln x}{(x+1)^2}dx$

33.122. Firmaning marjinal daromad funksiyasi $MR = 150 - 15Q$ ko‘rinishida berilgan. Agar $Q = 10$ birlik mahsulot ishlab chiqarilganda firmaning umumiy daromad $TR = 1000$ p.b. ni tashkil etsa, u holda yalpi daromad funksiyasi qanday ko‘rinishda bo‘ladi?

Yechish. Umumiy daromad funksiyasi $TR(Q)$ ni quyidagi integraldan foydalanib topamiz:

$$TR(Q) = \int (150 - 15Q)\,dQ = 150Q - \frac{15Q^2}{2} + C.$$

$Q = 10$ da $TR(Q) = 1000$ bo‘lganidan foydalanib, $C = 250$ ekanligini aniqlaymiz. Demak, firmaning yalpi daromad funksiyasi $TR = 150Q - \frac{15Q^2}{2} + 250$ ko‘rinishda bo‘ladi.

33.123. Firmaning marjinal xarajat funksiyasi $MC = 5Q^2 - 5Q - 8$ ko‘rinishga ega. Agar $Q = 6$ birlik mahsulot ishlab chiqarganda firmaning xarajati 500 p.b. ni tashkil etsa, u holda uning umumiy xarajat funksiyasini toping.

Yechish. Umumiy xarajat funksiyasi $TC(Q)$ ni quyidagi integraldan foydalanib topamiz:

$$TC(Q) = \int MC(Q)\,dQ = \int (5Q^2 - 5Q - 8)\,dQ = 5\frac{Q^3}{3} - 5\frac{Q^2}{2} - 8Q + C.$$

$Q = 6$ da $TC(Q) = 500$ ekanidan foydalanib, $C = 278$ ekanligini aniqlaymiz. Demak, umumiy xarajatlar funksiyasi quyidagi ko‘rinishda bo‘ladi:

$$TC(Q) = 5\frac{Q^3}{3} - 5\frac{Q^2}{2} - 8Q + 278.$$

33.124. Firmaning marjinal foyda funksiyasi $MF(Q) = -3Q^2 + 100Q - 200$ ko‘rinishiga ega. Agar $Q = 10$ birlik mahsulot ishlab chiqarganda firmaning foydasi $F = 1500$ p.b. ga teng bo‘lsa, u holda uning yalpi foyda funksiyasi qanday ko‘rinishga ega bo‘ladi?

Yechish. Marjinal foyda funksiyasi ma’lum bo‘lganda yalpi foyda funksiyasini quyidagi integral yordamida topamiz:

$$F(Q) = \int MF(Q)\,dQ = \int (-3Q^2 + 100Q - 200)\,dQ = -Q^3 + 50Q^2 - 200Q + C.$$

$Q = 10$ da $F(Q) = 1500$ ekanligini nazarga olib, $C = -500$ ekanligini topamiz. Demak, yalpi foyda funksiyasi $F(Q) = -Q^3 + 50Q^2 - 200Q - 500$ ko‘rinishga ega.

33.125. Marjinal mehnat unumdorlik funksiyasi $MQ(L)=\frac{5}{\sqrt{L}}-1$ ko‘rinishda berilgan. Agar ishlab chiqarishda $L=4$ ta kishi ishlaganda 10 ta mahsulot ishlab chiqarilsa, u holda ishlab chiqarish funksiyasi qanday ko‘rinishda bo‘ladi?

Yechish. Ishlab chiqarish funksiyasi $Q(L)$ ni quyidagi integraldan foydalanib topamiz:

$$Q(L)=\int MQ(L)\,dL=\int\left(\frac{5}{\sqrt{L}}-1\right)dL=10\sqrt{L}-L+C.$$

$L=4$ da $Q(L)=10$ ekanligini inobatga olib $C=-6$ ekanligini aniqlaymiz. Demak, ishlab chiqarish funksiyasi $Q(L)=10\sqrt{L}-L-6$ ko‘rinishga ega ekan.

33.126. Firmaning marjinal daromad funksiyasi $MR=15-4Q$ ko‘rinishga ega. Agar $Q=5$ birlik mahsulotga mos bo‘lgan umumiy xarajat 75 p.b. bo‘lsa, u holda firmaning umumiy harajat funksiyasi qanday ko‘rinishga ega?

33.127. Firmaning marjinal xarajatlar funksiyasi $MC(Q)=3Q^2-54Q+300$ ko‘rinishga ega. Agar $Q=10$ mahsulot ishlab chiqarganda firmaning xarajati 2500 p.b. ga teng bo‘lsa, u holda firmaning umumiy xarajat funksiyasi qanday ko‘rinishda bo‘ladi?

33.128. Velosiped ishlab chiqaruvchi firmaning marjinal xarajatlar funksiyasi $MC(Q)=6Q^2-6Q-12$ ko‘rinishga ega. Agar $Q=5$ birlik mahsulot ishlab chiqarganda firmaning umumiy xarajati 150 p.b. ga teng bo‘lsa, firmaning umumiy xarajatlar funksiyasini aniqlang.

33.129. Talabning baho bo‘yicha egiluvchanlik funksiyasi

$$E_D(P)=\frac{-P^2}{170-P^2},\quad 0<P<13$$

ko‘rinishga ega. Agar tovar narxi $P=7$ p.b. ga teng bo‘lganda unga bo‘lgan talab $Q_D=550$ bo‘lsa, u holda talab funksiyasi qanday ko‘rinishda bo‘ladi?

33.130. Taklifning baho bo‘yicha egiluvchanlik funksiyasi

$$E_S(P)=\frac{15P^2+35P}{(3+2P)(20+5P)},$$

ko‘rinishga ega. Agar tovar narxi $P=3$ p.b. ga teng bo‘lganda taklif 1080 birlik bo‘lsa, u holda taklif funksiyasi qanday ko‘rinishda bo‘ladi?

34-amaliy mashg‘ulot. Integrallash metodlari

34.1. Integralni toping: $\int \frac{x^2dx}{(x-1)^2(x+1)}$.

Yechish. Integral osti funksiyasini oddiy kasrlar yig‘indisi ko‘rinishida ifodalaymiz:

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{A_1}{x-1} + \frac{A_2}{(x-1)^2} + \frac{A_3}{(x+1)}.$$

o‘ng tomondagi ifodani umumiy maxrajga keltirgandan keyin:

$$\frac{x^2}{(x-1)^2(x+1)} = \frac{A_1(x^2-1) + A_2(x+1) + A_3(x-1)^2}{(x-1)^2(x+1)}.$$

hosil bo‘lgan tenglik uchun

$$x^2 = A_1(x^2-1) + A_2(x+1) + A_3(x-1)^2 \qquad (1)$$

o‘rinli bo‘ladi.

(1) ga $x=1$ ni qo‘yib, $1=2A_2$ tenglikka ega bo‘lamiz, bundan kelib chiqadiki, $A_2 = \frac{1}{2}$.

$x=-1$ da $1=4A_3$ shuning uchun $A_3 = \frac{1}{4}$.

$x=0$ ni qo‘ysak (1) da $0=-A_1+A_2+A_3$ tenglik hosil bo‘ladi. Oxirgi tenglikka A_2 va A_3 ning topilgan qiymatlarini qo‘yib $A_1 = \frac{3}{4}$ ni keltirib chiqaramiz.

Natijada

$$\int \frac{x^2dx}{(x-1)^2(x+1)} = \int \left(\frac{3}{4} \cdot \frac{1}{x-1} + \frac{1}{2} \cdot \frac{1}{(x-1)^2} + \frac{1}{4} \cdot \frac{1}{x+1} \right) dx =$$

$$\frac{3}{4}\ln|x-1| - \frac{1}{2(x-1)} + \frac{1}{4}\ln|x+1| + C.$$

34.2. Integralni hisoblang: $\int \frac{dx}{(x-1)(x^2-x+1)}$.

Yechish. Integral ostidagi ifodani oddiy kasrlar yig‘indisi ko‘rinishiga keltiramiz:

$$\frac{1}{(x-1)(x^2-x+1)} = \frac{A_1}{x-1} + \frac{M_1x+N_1}{x^2-x+1}.$$

Umumiy mahrajga keltirib, o‘ng va chap tomonlarning suratlarini tenglashtirishdan hosil bo‘ladigan tenglik quyidagicha bo‘ladi:

$$1 = A_1(x^2-x+1) + (M_1x+N_1)(x-1)$$

Agar $x=1$ bo‘lsa, $A_1=1$ agar $x=0$ bo‘lca, $1=A_1-N_1$ tenglikka o‘tamiz va natijada $N_1=0$ kelib chiqadi. $x=-1$ ni tenglikka qo‘yib, $1=3A_1+(-M_1+N_1)(-2)$ ni hosil qilamiz, bu yerdan $M_1=-1$. U holda

$$\int\frac{dx}{(x-1)(x^2-x+1)}=\int\frac{dx}{x-1}-\int\frac{xdx}{x^2-x+1}\ \text{bo‘ladi.}$$

Birinchi integral uchun differensial belgisi ostida funksiya shakllantiramiz: $dx=d(x-1)$ ikkinchisi uchun esa – maxrajida to‘la kvadrat ajratamiz: $x^2-x+1=(x-1/2)^2+3/4$ va $t=x-\frac{1}{2}$ almashtirishdan foydalanamiz. U holda $dt=dx,\ x=t+\frac{1}{2}$ bo‘ladi va

$$\int\frac{dx}{(x-1)(x^2-x+1)}=\int\frac{d(x-1)}{x-1}-\int\frac{t+1/2}{t^2+3/4}dt=$$

$$=\ln|x-1|-\int\frac{tdt}{t^2+3/4}-\frac{1}{2}\int\frac{dt}{t^2+3/4}=\ln|x-1|-\frac{1}{2}\int\frac{d(t^2+3/4)}{t^2+3/4}-\frac{1}{\sqrt{3}}arctg\frac{2t}{\sqrt{3}}=$$

$$=\ln|x-1|-\frac{1}{2}\ln|t^2+3/4|-\frac{\sqrt{3}}{3}arctg\frac{2x-1}{\sqrt{3}}+C=\ln|x-1|-\frac{1}{2}\ln|x^2-x+1|-\frac{\sqrt{3}}{3}arctg\frac{2x-1}{\sqrt{3}}+C.$$

Integrallarni toping:

34.3. $\int\frac{dx}{x^2-x-2}$

34.4. $\int\frac{x^2}{(1-x)^3}dx$

34.5. $\int\frac{dx}{x^3-x^2}$

34.6. $\int\frac{dx}{x^3+x}$

34.7. $\int\frac{dx}{x^3+1}$

34.8. $\int\frac{xdx}{x^3-1}$

34.9. $\int\frac{dx}{x(x+1)^2}$

34.10. $\int\frac{dx}{(x^2-1)(x+2)}$

34.11. $\int\frac{(x^2+2)dx}{(x+1)^2(x-1)}$

34.12. $\int\frac{xdx}{x^2+3x-4}$

34.13. $\int\frac{x^2-x}{x^2-6x+10}dx$

34.14. $\int\frac{x^3+1}{x^3-5x^2+6x}dx$

34.15. $\int\frac{3x^2+8}{x^3+4x^2+4x}dx$

34.16. $\int\frac{x^4+3x^3+2x^2+x+1}{x^2+x+1}dx$

34.17. $\int\frac{dx}{x^4+x^2}$

34.18. $\int\frac{dx}{x^4+1}$

34.19. $\int\frac{3x+5}{(x^2+2x+5)^2}dx$

34.20. $\int\frac{dx}{x^4+x^2+1}$

34.21. $\int\frac{dx}{(4+x^2)^2}$

34.22. $\int\frac{x^3-3}{x^4+10x^2+25}dx$

34.23. $\int \frac{dx}{x^4-1}$

34.24. $\int \frac{dx}{x^3-4x^2+5x-2}$

34.25. $\int \frac{x^2-x+14}{(x-4)^3(x-2)}dx$

34.26. $\int \frac{dx}{x^8+x^6}$

34.27. Ushbu $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$ integralni toping.

Yechish. $x=t^6 \quad dx=6t^5dt$ demak

$$\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}=\int \frac{6t^5dt}{t^3+t^2}=6\int \frac{t^3dt}{t+1}=6\int \frac{\left(t^3+1\right)-1}{t+1}dt=6\left[\int \frac{t^3+1}{t+1}dt-\int \frac{dt}{t+1}\right]=$$

$$=6\left[\int \frac{(t+1)\left(t^2-t+1\right)}{t+1}dt-\ln|t+1|\right]=6\left[\int\left(t^2-t+1\right)dt-\ln|t+1|\right]=$$

$$=6\left(\frac{t^3}{3}-\frac{t^2}{2}+t-\ln|t+1|\right)+C=2t^3-3t^2+6t-6\ln|t+1|+C=$$

$$=2\sqrt{x}-3\sqrt[3]{x}+6\sqrt[6]{x}-6\ln\left|\sqrt[6]{x}+1\right|+C.$$

Integrallarni toping:

34.28. $\int \frac{\sqrt[3]{x}dx}{\sqrt[3]{x^2}-\sqrt{x}}.$

34.29. $\int \frac{dx}{\sqrt{x}+\sqrt[4]{x}}.$

34.30. Ushbu $\int \frac{x+\sqrt{1+x}}{\sqrt[3]{1+x}}dx$ integralni toping.

Yechish. $1+x=t^6 \quad x=t^6-1, \quad dx=6t^5dt$

$$\int \frac{x+\sqrt{1+x}}{\sqrt[3]{1+x}}dx=\int \frac{\left(t^6-1\right)+t^3}{t^2}6t^5dt=6\int\left(t^9+t^6-t^3\right)dt=$$

$$=6\left(\frac{t^{10}}{10}+\frac{t^7}{7}-\frac{t^4}{4}\right)+C=6t^4\left(\frac{t^6}{10}+\frac{t^3}{7}-\frac{1}{4}\right)+C=$$

$$=6\sqrt[3]{(1+x)^2}\cdot\left(\frac{1+x}{10}+\frac{\sqrt{1+x}}{7}-\frac{1}{4}\right)+C.$$

Integrallarni toping:

34.31. $\int \frac{dx}{\sqrt[3]{(2x+1)^2}-\sqrt{2x+1}}.$

34.32. $\int \frac{dx}{1+\sqrt[3]{x+1}}.$

34.33. Ushbu $\int \sqrt{\frac{1-x}{1+x}}\cdot\frac{dx}{x}$ integralni toping.

Yechish. $\frac{1-x}{1+x}=t^2$ $1-x=(1+x)t^2,\ x=\frac{1-t^2}{1+t^2}$

$$dx=\frac{\left(1-t^2\right)'\left(1+t^2\right)-\left(1+t^2\right)'\left(1-t^2\right)}{\left(1+t^2\right)^2}dt=-\frac{4t}{\left(1+t^2\right)^2}dt.$$

$$\int\sqrt{\frac{1-x}{1+x}}\cdot\frac{dx}{x}=\int t\cdot\frac{1+t^2}{1-t^2}\frac{(-4t)dt}{\left(1+t^2\right)^2}=4\int\frac{t^2dt}{\left(t^2-1\right)\left(t^2+1\right)}=4\int\frac{\left(t^2-1\right)+1}{\left(t^2-1\right)\left(t^2+1\right)}dt=$$

$$=4\left[\int\frac{t^2-1}{\left(t^2-1\right)\left(t^2+1\right)}dt+\int\frac{dt}{\left(t^2-1\right)\left(t^2+1\right)}\right]=$$

$$=4\left[\int\frac{dt}{t^2+1}+\frac{1}{2}\int\left(\frac{1}{t^2-1}-\frac{1}{t^2+1}\right)dt\right]=4\int\frac{dt}{t^2+1}+2\int\frac{dt}{t^2-1}-2\int\frac{dt}{t^2+1}=$$

$$=2\left(\int\frac{dt}{t^2+1}+\int\frac{dt}{t^2-1}\right)=2\left(\operatorname{ar}ctgt+\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\right)+C=$$

$$=2arctg\sqrt{\frac{1-x}{1+x}}+\ln\left|\frac{\sqrt{\frac{1-x}{1+x}}-1}{\sqrt{\frac{1-x}{1+x}}+1}\right|+C=2arctg\sqrt{\frac{1-x}{1+x}}+\ln\left|\frac{\sqrt{1-x}-\sqrt{1+x}}{\sqrt{1-x}+\sqrt{1+x}}\right|+C.$$

34.34. Integralni toping: $\int\frac{\sqrt{x}dx}{x^2\cdot\sqrt{x-1}}.$

34.35. Integralni toping: $\int\sqrt[3]{x}\cdot\sqrt[3]{1+3\cdot\sqrt[3]{x^2}}\,dx.$

Yechish. $m=\frac{1}{3},\ n=\frac{2}{3},\ p=\frac{1}{3}.$ Ushbu holatda $\frac{m+1}{n}=\frac{\frac{1}{3}+1}{\frac{2}{3}}=2-$butun son

$1+3\cdot\sqrt[3]{x^2}=t^3.$ $x=\frac{1}{3\sqrt{3}}\left(t^3-1\right)^{\frac{3}{2}}$ demak $dx=\frac{\sqrt{3}}{2}\left(t^3-1\right)^{\frac{1}{2}}\cdot t^2dt.$

$$\int\sqrt[3]{x}\cdot\sqrt[3]{1+3\cdot\sqrt[3]{x^2}}\,dx=\int\frac{\sqrt{t^3-1}}{\sqrt{3}}\cdot t\cdot\frac{\sqrt{3}}{2}\cdot\sqrt{t^3-1}\cdot t^2dt=$$

$$=\frac{1}{2}\int\left(t^3-1\right)t^3dt=\frac{1}{2}\int\left(t^6-t^3\right)dt=\frac{1}{2}\left(\frac{t^7}{7}-\frac{t^4}{4}\right)+C=$$

$$=\frac{t^7}{14}-\frac{t^4}{8}+C=\frac{1}{14}\sqrt[3]{\left(1+3\sqrt[3]{x^2}\right)^7}-\frac{1}{8}\sqrt[3]{\left(1+3\sqrt[3]{x^2}\right)^4}+C.$$

Integrallarni toping:

34.36. $\int \sqrt{x}\left(1+\sqrt[3]{x}\right)^4 dx.$

34.37. $\int \frac{dx}{x^4\sqrt{x^2+1}}.$

34.38. $\int \frac{dx}{x+\sqrt[3]{x^2}}.$

34.39. $\int \frac{\sqrt{x}}{1+\sqrt[4]{x^3}}dx.$

34.40. $\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x\left(1-\sqrt[3]{x}\right)}dx.$

34.41. $\int \frac{\sqrt{x}dx}{x-\sqrt[3]{x^2}}.$

34.42. $\int \frac{\sqrt{x}dx}{1+\sqrt{x}}.$

34.43. $\int \frac{\sqrt{x}dx}{1-\sqrt[3]{x}}.$

34.44. $\int \frac{\sqrt{x+2}}{x}dx.$

34.45. $\int \frac{x\,dx}{\sqrt{x+1}+\sqrt[3]{x+1}}dx.$

34.46. $\int \frac{dx}{(x+1)^{\frac{3}{2}}+(x+1)^{\frac{1}{2}}}.$

34.47. $\int \frac{\sqrt{1+x}+1}{\sqrt{1+x}-1}dx.$

34.48. $\int \frac{x-1}{\sqrt{2x-1}}dx.$

34.49. $\int \frac{dx}{\sqrt{1-2x}-\sqrt[4]{1-2x}}.$

34.50. $\int \frac{1}{(2-x)^2}\cdot\sqrt{\frac{2-x}{2+x}}dx.$

34.51. $\int \frac{dx}{\sqrt{(x-1)^3(x-2)}}.$

34.52. $\int \frac{dx}{\sqrt[3]{(x-1)^2(x+1)}}.$

34.53. $\int \frac{dx}{(1-x)\sqrt{1-x^2}}.$

34.54. $\int \frac{dx}{x\cdot\left(1+\sqrt[3]{x}\right)^3}.$

34.55. $\int x^3\cdot\sqrt{1+x^2}dx.$

34.56. $\int \frac{dx}{x^{11}\cdot\sqrt{x^4+1}}.$

34.57. $\int \frac{dx}{\sqrt{x}\left(1-\sqrt{x}\right)^2}.$

34.58. $\int x^5\cdot\sqrt[3]{\left(1+x^3\right)^2}dx.$

34.59. $\int \frac{dx}{x^3\cdot\sqrt[3]{2-x^3}}.$

34.60. $\int \sqrt{x}\left(1+\sqrt{x}\right)^3 dx.$

34.61. $\int \sqrt[3]{x^3-4}\cdot x^2dx.$

34.62. $\int \frac{dx}{\sqrt{1-2x-x^2}}.$

34.63. $\int \frac{(x-2)dx}{\sqrt{x^2-10x+29}}.$

34.64. $\int \frac{3x-5}{\sqrt{x^2-4x+5}}dx.$

34.65. $\int \frac{x+1}{\sqrt{2x-x^2}}dx.$

34.66. $\int \frac{\sqrt{1-x^2}}{x}dx.$ **34.67.** $\int x\cdot\sqrt[5]{x-2}\,dx.$

34.68. $\int \frac{\sqrt{x^2-1}}{x}dx.$ **34.70.** $\int \frac{x^3dx}{\sqrt{x-1}}$

34.71. $\int \frac{\sqrt{x}dx}{2\sqrt{x}+3}$ **34.72.** $\int \frac{\sqrt{x}dx}{\sqrt[3]{x}+1}$

34.73. $\int \frac{xdx}{\sqrt[3]{2x+1}}$ **34.74.** $\int \frac{dx}{(1+\sqrt[3]{x})\sqrt{x}}$

34.75. $\int \frac{dx}{\sqrt{x+1}+\sqrt{x}}$ **34.76.** $\int \frac{dx}{x^5\sqrt{x^2-1}}$

34.77. $\int x\sqrt{\frac{x-1}{x+1}}dx$ **34.78.** $\int \frac{1}{x}\sqrt{\frac{x-2}{x}}dx$

34.79. $\int \frac{\sqrt{x+1}+2}{(x+1)^2-\sqrt{x+1}}dx$

34.80. Integralni toping: $\int \frac{dx}{3+5\cos x}$.

Yechish. $t=tg\frac{x}{2}$ universal o‘rniga qo‘yish usulini qo‘llab,

$$\int \frac{dx}{3+5\cos x}=\int \frac{2dt}{(1+t^2)(3+\frac{5(1-t^2)}{1+t^2})}=$$

$$=2\int \frac{dt}{8-2t^2}=\int \frac{dt}{4-t^2}=\frac{1}{4}\ln\left|\frac{t+2}{t-2}\right|+C=\frac{1}{4}\ln\left|\frac{tg\frac{x}{2}+2}{tg\frac{x}{2}-2}\right|+C$$

ni hosil qilamiz.

34.81. Integralni toping: a) $\int \frac{dx}{\cos^3 x}$ b) $\int \frac{dx}{1+3\cos^2 x}$

Yechish. a) $t=\sin x$ ni qo‘yamiz. Shunda $dt=\cos xdx$ va $\int \frac{dx}{\cos^3 x}=\int \frac{\cos xdx}{\cos^4 x}=\int \frac{dt}{(1-t^2)^2}$ hosil bo‘ladi. Aniqmas koeffisiyentlar usulini qo‘llab, integral ostidagi funksiyani oddiy kasr ko‘rinishiga keltiramiz:

$$\frac{1}{(1-t^2)^2}=\frac{1}{(1-t)^2(1+t)^2}=\frac{1}{4}\left(\frac{1}{1-t}+\frac{1}{(1-t)^2}+\frac{1}{1+t}+\frac{1}{(1+t)^2}\right).$$

U holda

$$\int \frac{dt}{(1-t^2)^2}=-\frac{1}{4}\int \frac{d(t-1)}{t-1}+\frac{1}{4}\int (t-1)^{-2}d(t-1)+\frac{1}{4}\int \frac{d(t+1)}{t+1}+\frac{1}{4}\int (t+1)^{-2}d(t+1)=$$

$$=-\frac{1}{4}\ln|t-1|-\frac{1}{4(t-1)}+\frac{1}{4}\ln|t+1|-\frac{1}{4(t+1)}+C=\frac{1}{4}\ln\left|\frac{\sin x+1}{\sin x-1}\right|-\frac{\sin x}{2(\sin^2 x-1)}+C=$$

$$=\frac{1}{4}\ln\left|\frac{\sin x+1}{\sin x-1}\right|+\frac{1}{2}\frac{tgx}{\cos x}+C.$$

b) $t=tgx$ ni qo‘yamiz $\left(-\frac{\pi}{2}<x<\frac{\pi}{2}\right)$. Xuddi shu kabi $1+t^2=\frac{1}{\cos^2 x}$, bundan $\cos^2 x=\frac{1}{1+t^2}$ va $x=arctgt$, $dx=\frac{dt}{1+t^2}$. Shundan kelib chiqqan holda

$$\int\frac{dx}{1+3\cos^2 x}=\int\frac{dt}{(1+t^2)(1+3/(1+t^2))}=\int\frac{dt}{t^2+4}=\frac{1}{2}arctg\frac{t}{2}+C=\frac{1}{2}arctg\frac{tgx}{2}+C.$$

34.82. Integralni toping: $\int\sin x\cos^2 2xdx.$

Yechish. $\int\sin x\cos^2 2xdx=\int\sin x\frac{1+\cos 4x}{2}dx=\frac{1}{2}\int\sin xdx+\frac{1}{2}\int\sin x\cos 4xdx$ ni hosil qilamiz. Ikkinchi integralni topish uchun ko‘paytmani yig‘indiga keltirish formulasidan foydalanamiz:

$$\frac{1}{2}\int\sin xdx+\frac{1}{2}\int\sin x\cos 4xdx=-\frac{1}{2}\cos x+\frac{1}{4}\int(\sin 5x-\sin 3x)dx=$$

$$-\frac{1}{2}\cos x+\frac{1}{20}\int\sin 5xd(5x)-\frac{1}{12}\int\sin 3xd(3x)=-\frac{1}{2}\cos x-\frac{1}{20}\cos 5x+\frac{1}{12}\cos 3x+C$$

Integrallarni toping:

34.83. $\int\sin^3 xdx$

34.84. $\int\cos^7 xdx$

34.85. $\int\sin^3 x\cos^2 xdx$

34.86. $\int\frac{dx}{1+\sin x}$

34.87. $\int\frac{dx}{\sin x-\cos x}$

34.88. $\int\frac{dx}{2\sin x+\sin 2x}$

34.89. $\int\frac{dx}{3\sin^2 x+5\cos^2 x}$

34.90. $\int\frac{\cos x}{1+\cos x}dx$

34.91. $\int\frac{\sin x}{(1-\cos x)^3}dx$

34.92. $\int\frac{dx}{8-4\sin x+7\cos x}$

34.93. $\int\frac{1+tgx}{1-tgx}dx$

34.94. $\int\frac{dx}{(2+\cos x)(3+\cos x)}$

34.95. $\int tg^3(\frac{x}{2}+\frac{\pi}{4})dx$

34.96. $\int\cos\frac{x}{2}\cos\frac{x}{3}dx$

34.97. $\int\sin 9x\sin xdx$

34.98. $\int\sin x\sin 2x\sin 3xdx$

34.99. $\int\frac{\sin(x+\frac{\pi}{4})}{\sin x\cos x}dx$

34.100. $\int\sin^5 x\sqrt[3]{\cos x}dx.$

35-amaliy mashg‘ulot. Aniq integral

35.1. Hisoblang:

a) $\int_1^2 \frac{3x^4-5x^2+7}{x}dx$ b) $\int_0^{\pi}|\cos x|dx$

v) $\int_0^{\ln 2} e^x\sqrt{e^x-1}\,dx$ g) $\int_{-0,5}^{0}\ln(1-x^2)dx$

d) $\int_0^{\pi/2} x^2\cos x\,dx$ y) $\int_0^1\sqrt{1-x^2}\,dx$

Yechish.

a) $$\int_1^2 \frac{3x^4-5x^2+7}{x}dx=\int_1^2\left(3x^3-5x+\frac{7}{x}\right)dx=3\int_1^2 x^3\,dx-5\int_1^2 x\,dx+7\int_1^2\frac{dx}{x}=$$

$$=3\frac{x^4}{4}\bigg|_1^2-5\frac{x^2}{2}\bigg|_1^2+7\ln|x|\Big|_1^2=\frac{3}{4}(16-1)-\frac{5}{2}(4-1)+7(\ln2-\ln1)=3{,}75+7\ln2$$

b) $\int_0^{\pi}|\cos x|dx$

$$|\cos x|=\begin{cases}\cos x & x\in\left[0,\frac{\pi}{2}\right]\\ -\cos x & x\in\left[\frac{\pi}{2},0\right]\end{cases}$$ ekanligidan

$$\int_0^{\pi}|\cos x|dx=\int_0^{\pi/2}\cos x\,dx+\int_{\pi/2}^{\pi}(-\cos x)dx=\sin x\Big|_0^{\pi/2}-\sin x\Big|_{\pi/2}^{\pi}=(1-0)-(0-1)=2.$$

v) $\int_0^{\ln 2} e^x\sqrt{e^x-1}\,dx$

O‘zgaruvchini almashtirish usulidan foydalanmiz. $t=\sqrt{e^x-1}$ bo‘lsin, unda $t^2=e^x-1,\ e^x=t^2+1,\ 2t\,dt=e^x\,dx$ va $dx=\frac{2t\,dt}{t^2+1}$ bo‘ladi. Endi t o‘zgaruvchi bo‘yicha integrallash chegaralarini topamiz. Agar $x=0$ bo‘lsa, $t=\sqrt{e^0-1}=0,$ agar $x=\ln 2$ bo‘lsa, $t=\sqrt{e^{\ln2}-1}=1$. Endi izlanayotgan integral quyidagi ko‘rinishga keladi:

$$\int_0^{\ln2}e^x\sqrt{e^x-1}\,dx=\int_0^1(t^2+1)t\frac{2t}{t^2+1}dt=2\int_0^1 t^2\,dt=2\frac{t^3}{3}\bigg|_0^1=\frac{2}{3}(1^3-0^3)=\frac{2}{3}$$

g) $\int_{-0,5}^{0}\ln(1-x^2)dx$

Bo‘laklab integrallash formulasidan foydalanib $u=\ln(1-x^2),\ \ dv=dx$ bo‘lsin. Unda

$$du=\left(\ln\left(1-x^2\right)\right)'dx=-\frac{2x}{1-x^2}dx\ ,\ v=\int dv=\int dx=x$$

bo‘ladi.

$$\int_{-0,5}^{0}\ln\left(1-x^2\right)dx = x\ln\left(1-x^2\right)\Big|_{-0,5}^{0}+\int_{-0,5}^{0}\frac{2x^2}{1-x^2}dx = 0,5\ln\left(4/3\right)-2\int_{-0,5}^{0}\frac{\left(x^2-1\right)+1}{x^2-1}dx =$$

$$=0,5\ln 4/3-2\int_{-0,5}^{0}dx-2\int_{-0,5}^{0}\frac{dx}{x^2-1}=0,5\ln\left(4/3\right)-2x\Big|_{-0,5}^{0}-2\cdot\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\Big|_{-0,5}^{0}=0,5\ln 12-1$$

d) $\int_{0}^{\pi/2} x^2\cos x\,dx$ bu integral ham bo‘laklab integrallash formulasini qo‘llab topiladi.

$u=x^2$, $dv=\cos x\,dx$ bo‘ladi. Unda

$$du=(x^2)'dx=2x\ dx\ ,\ v=\int\cos x\,dx=\sin x$$

bo‘lsin.

$$\int_{0}^{\pi/2}x^2\cos x\,dx=x^2\sin x\Big|_{0}^{\pi/2}-\int_{0}^{\pi/2}2x\sin x\,dx=\frac{\pi^2}{4}-2\int_{0}^{\pi/2}x\ \sin x\,dx.$$

Oxirgi integralni hisoblash uchun yana bo‘laklab integrallash formulasini qo‘llaymiz. $u=x$, $dv=\sin\ x\ dx$, demak $du=dx$, $v=\int\sin\ x\ dx=-\cos x$ va

$$\int_{0}^{\pi/2}x^2\cos x\,dx=\frac{\pi^2}{4}-2x\left(-\cos x\right)\Big|_{0}^{\pi/2}-2\int_{0}^{\pi/2}\cos x\ dx=\frac{\pi^2}{4}-2\sin x\Big|_{0}^{\pi/2}=\frac{\pi^2}{4}-2.$$

y) $\int_{0}^{1}\sqrt{1-x^2}\,dx$ integralda $x=\sin t$. $t\in\left[0;\frac{\pi}{2}\right]$ deb olamiz. Agar $t=0$ bo‘lsa, $x=0.$

Agar $t=\frac{\pi}{2}$ bo‘lsa $x=1$ bo‘ladi. $dx=\cos t\,dt$ va

$$\int_{0}^{1}\sqrt{1-x^2}\ dx=\int_{0}^{\pi/2}\sqrt{1-\sin^2 t}\ \cos t\ dt=\int_{0}^{\pi/2}\left|\cos t\right|\ \cos t\ dt$$

$t\in\left[0:\frac{\pi}{2}\right]$ bo‘lganda $\cos t\geq 0$ bo‘lganligidan, $\left|\cos t\right|=\cos t$ ga egamiz.

$$\int_{0}^{1}\sqrt{1-x^2}\ dx=\int_{0}^{\pi/2}\cos^2 t\ dt=\int_{0}^{\pi/2}\frac{1+\cos 2t}{2}dt=\frac{1}{2}t\Big|_{0}^{\pi/2}+\frac{1}{4}\int_{0}^{\pi/2}\cos 2t\ d2t=\frac{\pi}{4}+\frac{1}{4}\left(\sin 2t\Big|_{0}^{\pi/2}\right)=\frac{\pi}{4}$$

Quyidagi aniq integralni hisoblang:

35.2. $\int_{1}^{5}\sqrt{x-1}dx$

35.3. $\int_{1}^{2}\frac{dx}{x^2-4x+5}$

35.4. $\int_{-1}^{4}\frac{x}{\sqrt{x+5}}dx$

35.5. $\int_{1}^{e}\frac{\sqrt[3]{1+\ln x}}{x}dx$

35.6. $\int_{0}^{1}\frac{xdx}{\sqrt{1-x^2}}$

35.7. $\int_{0}^{1}\frac{dx}{\sqrt{4-x^2}}$

35.8. $\int_0^{\pi/4} \sin 4x dx$

35.9. $\int_1^e x^2 \ln x dx$

35.10. $\int_0^{\pi} e^x \sin x dx$

35.11. $\int_0^1 \arcsin x dx$

35.12. $\int_0^1 \ln(x+1) dx$

35.13. $\int_0^{\pi/2} \sin x \cos^2 x dx$

35.14. $\int_0^1 \frac{dx}{e^x+1}$

35.15. $\int_0^4 \frac{dx}{1+\sqrt{2x+1}}$

35.16. $\int_0^1 \frac{x^2 dx}{\sqrt{4-x^2}}$

35.17. $\int_0^1 \sqrt{1+x^2} dx$

35.18. $\int_1^3 \frac{dx}{x+x^2}$

35.19. $\int_1^2 \frac{dx}{2x-1}$

35.20. $\int_1^e \frac{dx}{x(1+\ln^2 x)}$

35.21. $\int_1^e x \arcsin x dx$

35.22. $\int_0^{\pi/2} x^2 \cos x dx$

35.23. $\int_2^3 \frac{dx}{x^2}$

35.24. $\int_0^{\sqrt{3}} \frac{x dx}{\sqrt{4-x^2}}$

35.25. $\int_{\frac{\Pi}{8}}^{\frac{\Pi}{6}} \frac{dx}{\cos^2 2x}$

35.26. $\int_1^4 \frac{dx}{(1+\sqrt{x})^2}$

35.27. $\int_0^1 \frac{e^x dx}{1+e^{2x}}$

35.28. $\int_1^{\sqrt{2}} \sqrt{2-x^2} dx$

35.29. $\int_0^{\frac{\Pi}{2}} x \cos x dx$

35.30. $\int_0^{\frac{\Pi}{4}} tg^3 x dx$

35.31. $\int_4^5 x\sqrt{x^2-16}\, dx$

35.32. $\int_1^5 \frac{x\, dx}{\sqrt{1+3x}}$

35.33. $\int_1^2 \frac{4x+2}{2x-1} dx$

35.34. $\int_4^9 \frac{\sqrt{x}\, dx}{\sqrt{x}-1}$

35.35. $\int_e^{e^2} \frac{2\ln x+1}{x} dx$

35.36. $\int_0^4 \frac{dx}{\sqrt{2x+1}}$

35.37. $\int_{-2}^1 x^2\sqrt{1-x^3}\, dx$

35.38. $\int_{\ln 3}^{\ln 8} \frac{e^x\, dx}{\sqrt{e^x+1}}$

35.39. $\int_0^{\ln 2} x e^x\, dx$

35.40. $\int_1^e x \ln x\, dx$

35.41. $\int_1^e \ln^2 x\, dx$

35.42. $\int_0^{\sqrt{3}} arctgx\ dx$

35.43. $\int_{-1}^{1} x^2 e^{-2}\, dx$

35.44. $\int_0^{\pi} e^x \sin x\, dx$

35.45. $\int_0^{\pi/2} (x+3)\sin x\, dx$

35.46. $\int_{-\pi}^{\pi} x \sin x \cos x\ dx$

35.47. $\int_0^{\pi/2} \cos^2 x \sin x\ dx$

35.48. $\int_{-7}^{7} \frac{x^4 \sin x}{x^6+2} dx$

35.49. $\int_0^{\pi/2} \sin^3 x\ dx$

35.50. $\int_0^{\ln 2} \sqrt{e^x - 1}\, dx$

35.51. $\int_0^{1} \frac{dx}{x^2+4x+5}$

35.52. $\int_0^{\pi} \frac{dx}{3+2\cos x}$

35.53. $\int_1^{3} \frac{dx}{x\sqrt{x^2+5x+1}}$

35.54. $\int_0^{1/2} \sqrt{\frac{1+x}{1-x}}\, dx$

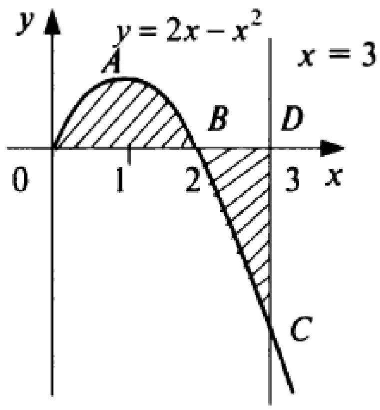
35.55. $\int_1^{2} \frac{dx}{x^2+2x}$

35.56. $\int_{-1}^{0} \frac{dx}{1+\sqrt[3]{x+1}}$

35.57. $\int_0^{\pi/4} tg^3 x\ dx$

35.58. $\int_{-3}^{3} x^2 \sqrt{9-x^2}\ dx$

35.59. $y = 2x - x^2$, $y = 0$, $x = 3$ chiziqlar bilan chegaralangan yassi shakl yuzini toping.



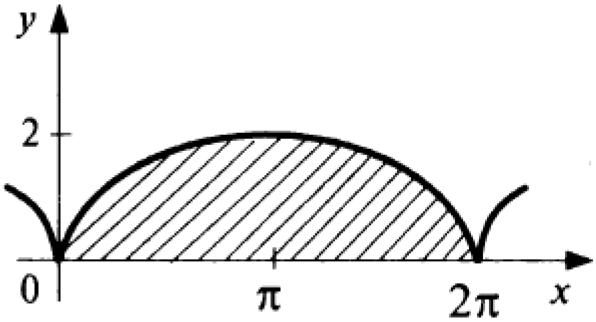
Yechish. Shaklning S yuzi ikkita egri chiziqli uchburchakdan iborat: OAB va BCD Ox o‘qining yuqori va quyi qismida joylashgan. Bu uchburchaklarning yuzalarini quyidagi formulalardan topamiz.

$$S_{OAB} = \int_0^2 (2x - x^2) dx = \left(x^2 - \frac{x^3}{3} \right)\Bigg|_0^2 = \frac{4}{3},$$

$$S_{BCD} = -\int_2^3 (2x - x^2) dx = \left(-x^2 + \frac{x^3}{3} \right)\Bigg|_2^3 = \frac{4}{3}.$$

U holda $S = S_{OAB} + S_{BCD} = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}\ \left(kv.bir.\right).$

35.60. $[0;2\pi]$ kesmada $x = t - \sin t$, $y = 1 - \cos t$ sikloida va Ox o'qi bilan chegaralangan figura yuzasini toping.

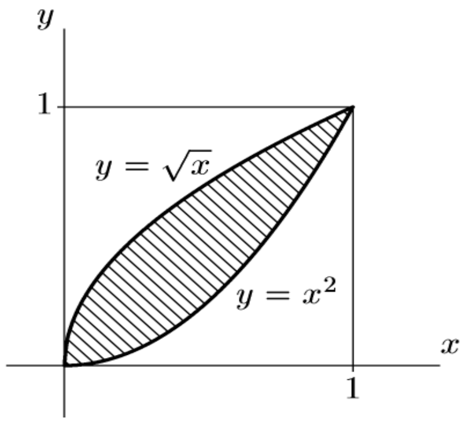


Yechish.

$$S = \int_0^{2\pi} \left(1 - \cos t\right)^2 dt = \int_0^{2\pi} \left(1 - 2\cos t + \cos^2 t\right) dt =$$

$$= \int_0^{2\pi} \left(\frac{3}{2} - 2\cos t + \frac{1}{2}\cos 2t\right) dt = \left(\frac{3}{2}t - 2\sin t + \frac{1}{4}\sin 2t\right)\Bigg|_0^{2\pi} = 3\pi \approx 9{,}4.$$

35.61. $y = x^2,\ \ y^2 = x$ chiziqlar bilan chegaralangan figuraning yuzini toping.



Yechish. Berilgan figura yuqoridan $y = \sqrt{x},\quad 0 \le x \le 1$ chiziq bilan, quyidan esa $y = x^2,\ \ 0 \le x \le 1$ chiziq bilan chegaralangan. Shuning uchun

$$S = \int_0^1 (\sqrt{x} - x^2)dx = \frac{2x^{\frac{3}{2}}}{3}\Bigg|_0^1 - \frac{1}{3}x^3\Bigg|_0^1 = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}.$$

Berilgan chiziqlar bilan chegaralangan figuralar yuzalarini hisoblang:

35.62. $y = 4 - x^2$ va Ox o'q bilan

35.63. $y = (x-1)^2$ va $x^2 - \frac{y^2}{2} = 1$

35.64. $y = x^2 + 1$ va $y = 3 - x$

35.65. $y = tg^2 x,\ x = \frac{\pi}{4},\ y = 0.$

35.66. $y = x^2,\ y = \frac{1}{x^2}, y = 0,\ x = 0,\ x = 3.$

35.67. $y^2 = 2x + 1,\ y = x - 1.$

36.68. $y = x^2 - 2x + 3, \quad y = 3x - 1.$ **35.69.** $y = \ln x, \quad y = e, y = 0.$

35.70. $y = tgx, \quad x = 0, x = \frac{\pi}{3}.$ **35.71.** $y = x^2 + 3, \quad xy = 4, \; y = 2, \; x = 0.$

35.72. $y = \ln \sin x$ egri chiziqning $x_1 = \frac{\pi}{3}$ dan $x_2 = \frac{2\pi}{3}$ gacha bo‘lgan yoyining uzunligini hisoblang.

Yechish. $y = \ln \sin x, \quad y' = \frac{\cos x}{\sin x}, \quad \sqrt{1 + (y')^2} = \sqrt{1 + \frac{\cos^2 x}{\sin^2 x}} = \frac{1}{\sin x}, \quad x \in \left[\frac{\pi}{3}; \frac{2}{3}\pi\right].$

AB yoyning l uzunligini hisoblaymiz:

$$l = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{dx}{\sin x} = \ln\left|tg\frac{x}{2}\right| \Bigg|_{\pi/3}^{2\pi/3} = \ln\sqrt{3} - \ln\frac{1}{\sqrt{3}} = 2\ln\sqrt{3} = \ln 3.$$

35.73. $y^2 = x^3$ yarim kubik parabolaning koordinatar boshidan $\left(\frac{4}{3}; \frac{8\sqrt{3}}{9}\right)$ koordinatali nuqtagacha bo‘lgan yoyining uzunligini toping.

Yechish. Egri chiziqning ko‘rsatilgan qismi birinchi chorakda joylashgan va $y = x^{3/2}$ tenglama bilan berilgan. Bu holatda $f'(x) = 1{,}5x^{1/2}$ formulani qo‘llab natijani hosil qilamiz.

$$S = \int_0^{4/3} \sqrt{1 + (f')^2}\, dx = \int_0^{4/3} \sqrt{1 + \frac{9}{4}x}\, dx = \frac{8}{27}\left(1 + \frac{9}{4}x\right)^{3/2} \Bigg|_0^{4/3} = \frac{8}{27}\left(4^{3/2} - 1\right) = \frac{56}{27}.$$

35.74. $y^2 = \frac{2}{3}(x-1)^3$ yarim kubik parabolaning $y^2 = \frac{x}{3}$ parabola ichki qismi bilan chegaralangan yoy uzunligini hisoblang:

Yechish. Egri chiziqlarning kesishish nuqtasini aniqlaymiz: $\frac{2}{3}(x-1)^3 = \frac{x}{3}$

$$x = 2, \text{ да } \; y = \sqrt{\frac{2}{3}}, \; chunki \;\; y = \sqrt{\frac{2}{3}(x-1)}(x-1), \quad u \;\; holda \;\; y' = \sqrt{\frac{3}{2}}\sqrt{x-1},$$

$$L = 2\int_1^2 \sqrt{1 + \frac{3}{2}(x-1)}dx = 2\frac{1}{\sqrt{2}}\int_1^2 \sqrt{3x-1}dx = \frac{2\sqrt{2}}{9}(5\sqrt{5} - 2\sqrt{2}).$$

Egri chiziqlar yoylari uzunliklari hisoblansin:

35.75. $y = 1 - \ln\cos x, \; x = 0$ dan $x = \frac{\pi}{6}$ gacha

35.76. $x = 8\sin t + 6\cos t, y = 6\sin t - 8\cos t, t = 0$ dan $t = \frac{\pi}{2}$ gacha

35.77. $x = \frac{1}{3}t^3 - t, y = t^2 + 2, t = 0$ dan $t = 3$ gacha

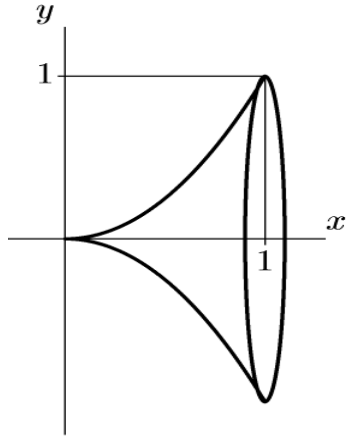
35.78. $\begin{cases} x=4(t-\sin t), \\ y=4(1-\cos t) \end{cases}$ $\frac{\pi}{2}\le t\le\frac{2\pi}{3}.$

35.79. $y=\sqrt{x-1}$ $A(1;0)$ nuqtadan $B(2;1)$gacha.

35.80. $y=\frac{1}{2}x^2-4x+\frac{15}{2},$ Ox o‘qi bilan kesishish nuqtalari orasidagi

35.81. $y=\frac{1}{2}x^2,$ $x=0$ dan $x=1$ gacha.

35.82. $y=x^2$ parabola va $y=0,$ $x=1$ to‘g‘ri chiziqlar bilan chegaralangan sohani Ox o‘qi atrofida aylantirishdan hosil bo‘lgan jism hajmini toping.



Yechish. $a=0, b=1, f(x)=x^2,$ bu yerdan

$$V_x=\pi\int_a^b f^2(x)dx=\pi\int_0^1\left(x^2\right)^2dx=\pi\cdot\frac{x^5}{5}\Bigg|_0^1=\frac{1}{5}\pi.$$

35.83. $y=x^2$ va $x=y^2$ parabolalar bilan chegaralangan figurani Ox o‘qi atrofida aylantirishdan hosil bo‘lgan jism hajmini hisoblang.

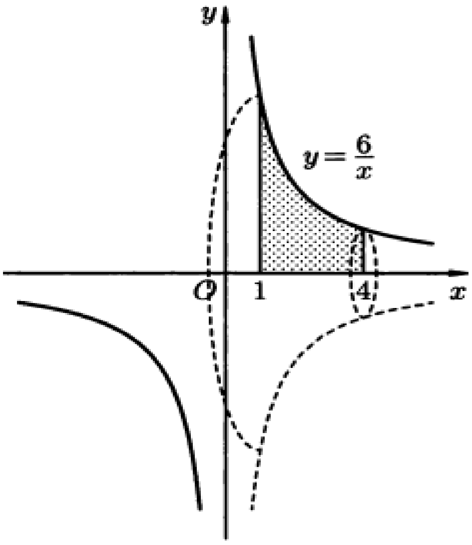
Yechish. $\begin{cases} y=x^2 \\ x=y^2 \end{cases}$ sistemasidan kesishish nuqtalarini topamiz:

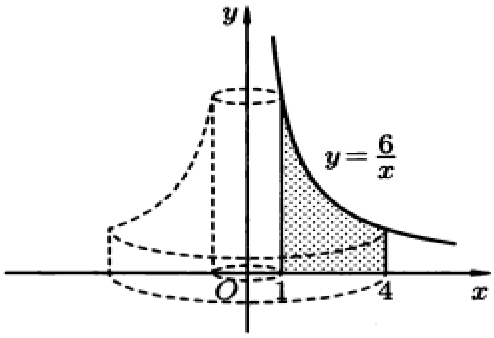
$$x_1=0,\ x_2=1,\ y_1=0,\ y_2=1$$

$$V=V_1+V_2=\pi\int_0^1 xdx-\pi\int_0^1 x^4dx=\pi\left(\frac{x^2}{2}-\frac{x^5}{5}\right)\Bigg|_0^1=\pi\left(\frac{1}{2}-\frac{1}{5}\right)=\frac{3}{10}\pi\ (kub.birlik)$$

35.84. $xy=6, x=1, x=4, y=0$ chiziqlar bilan chegaralangan shaklni Ox va Oy o‘qi atrofida aylantirishdan hosil bo‘lgan jism hajmini toping.

Yechish.





$$V_{Ox}=\pi\int_1^4\left(\frac{6}{x}\right)^2dx=36\pi\left(-\frac{1}{x}\right)\Bigg|_1^4=27\pi.$$

$$V_{Oy}=\pi\int_0^{1,5}(4-1)^2dy+\pi\int_{1,5}^{6}\left(\frac{6}{y}-1\right)^2dy=\pi\int_0^{1,5}9dy+\pi\int_{1,5}^{6}\left(\frac{36}{y^2}-\frac{12}{y}+1\right)dy=$$

$$=9\pi\, y\Big|_0^{1,5}+\pi\left(-\frac{36}{y}-12\ln y+y\right)\Bigg|_{1,5}^{6}=\left(36-12\ln 4\right)\pi.$$

Quyidagi chiziqlar bilan chegaralangan yassi shaklning Ox va Oy o‘qi atrofini aylantirishdan hosil bo‘lgan jismning hajmini toping:

35.85. $y=x^3,\quad y=4x.$

35.86. $y=\sin x,\quad y=0,\ 0\le x\le\pi.$

35.87. $y=\frac{4}{x},\quad x=1,\ x=4, y=0.$

35.88. $y=\ln x,\quad y=0,\ x=e.$

35.89. $y=-x^2+4,\quad y=x^2,\ x=0.$

35.90. $y=\sqrt{6x},\quad y=\sqrt{16-x^2},\ x=0.$

35.91. $y=x^2+1,\quad x=y^2+1,\ y=0,\ x=0,\ x=2.$

35.92. Agar kun davomida mehnat unumdorligi $f(t)=-0{,}1t^2+0{,}8t+10$ empirik formula bo‘yicha o‘zgarsa, kunlik ish vaqti 8 soat bo‘lganda Q bir kunlik ishlab chiqarilgan mahsulotni toping.

Yechish.

$$Q=\int_0^T f(t)dt=\int_0^8\left(-0{,}1t^2+0{,}8t+10\right)dt=\left(-0{,}1\frac{t^3}{3}+0{,}8\frac{t^2}{2}+10t\right)\Bigg|_0^8\approx 88{,}53$$

35.93. Quyida mahsulotga bo‘lgan talab va taklif funksiyalari berilgan. $Q=\frac{8000}{P^3}$ (talab funksiyasi), $Q=500P$ (taklif funksiyasi). Bu yerda $Q-$mahsulot miqdori, $P-$ bir birlik mahsulot narxi. Quyidagilarni aniqlang:

a) mahsulotning muvozanat narxi va miqdorini;

b) iste’molchilarning ortiqcha foydasini.

Yechish. Bozordagi mahsulotning muvozanat narxini va muvozanat miqdorini topamiz. Buning uchun quyidagi tenglamalar sistemasini yechamiz:

$$\begin{cases} Q=\frac{8000}{P^3}, \\ Q=500P \end{cases} \Leftrightarrow \begin{cases} \frac{8000}{P^3}=500P, \\ Q=500P \end{cases} \Leftrightarrow \begin{cases} P^4=16, \\ Q=500P \end{cases} \Leftrightarrow \begin{cases} P_0=2 \\ Q_0=1000. \end{cases}.$$

Demak, mahsulotning muvozanat bahosi $P_0=2$ birlik mahsulot narxi bo‘lib, muvozanat miqdori $Q_0=1000$ ga tengdir.

$$Q=\frac{8000}{P^3}, \qquad f(Q)=\sqrt[3]{\frac{8000}{Q}}=20Q^{-\frac{1}{3}}.$$

Bu yerda $f(Q)-$ $Q=\frac{8000}{P^3}$ funksiyaning teskari funksiyasi.

$$C.S.=\int_0^{1000}20Q^{-\frac{1}{3}}dQ-2\cdot1000=\frac{3\cdot20Q^{\frac{2}{3}}}{2}\Big|_0^{1000}-2000=30Q^{\frac{2}{3}}\Big|_0^{1000}-2000=$$

$$=30\cdot1000^{\frac{2}{3}}-2000=30\sqrt[3]{1000^2}-2000=1000.$$

Demak, iste’molchilarning ortiqcha foydasi 1000 p.b ni tashkil qiladi.

35.94. Korxonaning kunlik unumdorligi $f(t)=-0{,}0033t^2-0{,}089t+20{,}96$ funksiya bilan berilgan bo‘lsa, uning bir yilda (258 ish kuni) ishlab chiqargan mahsulot hajmini toping, bu yerda $1\le t\le 8$, $t-$ vaqt (soatlarda).

35.95. Agar mahsulotning kelishi $f(t)=2t+5$ funksiya bilan ifodalansa, do‘konda uch kunda hosil qilinadigan mahsulot zahirasini aniqlang.

35.96. Agar talab va taklif qonunlari $P=186-Q^2$, $P=20+\frac{11}{6}Q$ ko‘rinishga ega bo‘lsa, bozor muvozanatini o‘rnatish bo‘yicha taklifdan iste’molchi va taminotchining ortiqcha foydasini toping.

35.97. Biror mahsulotga bo‘lgan talab tenglamasi $P=134-Q^2$ ko‘rinishga ega. Agar muvozanat narx 70 ga teng bo‘lsa, iste’molchi ortiqcha foydasini aniqlang.

35.98. Biror mahsulotga bo‘lgan talab tenglamasi $P=\frac{100}{Q+15}$ ko‘rinishga ega. Agar tovarning muvozanat miqdori 10 ga teng bo‘lsa, iste’molchi ortiqcha foydasini aniqlang.

35.99. $y=x^2+1$ parabola *y=0, x=-1, x=4* to‘g‘ri chiziqlar bilan chegaralangan figuraning yuzini toping.

35.100. $y=\ln(\sin x)$ egri chiziq $x=\frac{\pi}{3}$ dan $x=\frac{\pi}{2}$ gacha bo‘lgan yoyining uzunligini toping.

35.101. $x^2-y^2=4, y=\pm 2$ chiziqlar bilan chegaralangan figurani *Oy* o‘qi atroqfida aylantirishidan hosil bo‘lgan jismning hajmini toping.

35.102. $y=\frac{1}{1+x^2}, x=\pm 1, y=0$ chiziqlar bilan chegaralangan figurani *Ox* o‘qi atrofida aylantirishidan hosil bo‘lgan jismning hajmini toping.

35.103. $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ *Ox* o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini hisoblang.

35.104. Mahsulotga bo‘lgan talab va taklif qonunlari $p=240-x^2$ va $p=x^2+2x+20$ ko‘rinishida bo‘lsa, iste’molchilarning va ta’minotchining ortiqcha foydasini aniqlang.

35.105. Agar mehnat unumdorligi $f(t)=11{,}3e^{-0{,}417t}$ formula bo‘yicha berilgan bo‘lsa, dastlabki besh soatda ishlab chiqarilgan mahsulot hajmini aniqlang, bu yerda $t-$vaqt (soatlarda).

Berilgan chiziqlar bilan chegaralangan figuralar yuzalarini hisoblang:

35.106. $y=e^x,\ y=e^{x/2},\ y=e^2.$

35.107. $y=x^4-2x^2,\ y=0.$

35.108. $y=3+2x-x^2, \quad y=x+1.$

35.109. $y=x^2+3,\ xy=4,\ y=2,\ x=0.$

35.110. $y=\sqrt{1-x},\ y=x+1.$

35.111. $y=\cos 2x,\ y=0,\ x=0,\ x=\pi/4.$

35.112. $x=0,\ x=2,\ y=2^x,\ y=2x-x^2.$

35.113. $y=\arcsin 2x,\ x=0,\ y=-\pi/2.$

35.114. $y=x^2+1,\ x=y^2,\ 3x+2y-16=0,\ x=0.$

Egri chiziqlar yoylari uzunliklari hisoblansin:

35.115. $y=2\sqrt{x}\ \ x=0\ \ dan\ \ x=1\ \ gacha.$

35.116. $y=\ln x\ \ \ x=\sqrt{3}\ \ dan\ \ x=\sqrt{8}\ \ gacha.$

35.117. $x=t-\sin t,\ y=1-\cos t\ \ \ t=0\ dan\ \ t=2\pi\ \ gacha.$

Ox o‘qi atrofida aylantirishdan hosil bo‘lgan aylanma jism sirtining yuzini toping:

35.118. $y=x^3$, $x\in\left[0\,;\sqrt[4]{1/3}\right]$.   **35.119.** $9y^2=x(3-x)^2$, $x\in[0\,;3]$.

35.120. $x^2+y^2=9$, $x\in[-2\,;1]$.   **35.121.** $x=cos^3t, y=sin^3t$, $t\in[0\,;\pi/2]$.

36-amaliy mashg'ulot. Xosmas integral. Aniq integralni taqribiy hisoblash

36.1. Xosmas integrallarni yaqinlashishga tekshiring:

$$a)\int_2^{+\infty}\frac{dx}{x^2-1};\qquad b)\int_{-\infty}^{0}x\cos xdx\qquad c)\int_{-\infty}^{+\infty}\frac{dx}{1+x^2};$$

Yechish.

a) ta'rif bo'yicha quyidagini olamiz:

$$\int_2^{+\infty}\frac{dx}{x^2-1}=\lim_{t\to+\infty}\int_2^{t}\frac{dx}{x^2-1}=\lim_{t\to+\infty}\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\Bigg|_2^t=\frac{1}{2}\lim_{t\to+\infty}\left(\ln\left|\frac{t-1}{t+1}\right|-\ln\frac{1}{3}\right)=$$

$$=\frac{1}{2}\left(\ln\lim_{t\to+\infty}\frac{t-1}{t+1}-\ln\frac{1}{3}\right)=\frac{1}{2}(\ln1+\ln3)=0,5\ln3$$

b) $\int\limits_{-\infty}^{0}x\cos xdx$

$$\int_{-\infty}^{0}x\cos xdx=\lim_{a\to-\infty}\int_a^0x\cos xdx=$$

$$\begin{array}{l|l} u=x & du=dx \\ dv=\cos xdx & v=\sin x \end{array}$$

$$=\lim_{a\to-\infty}\left(x\sin x\Big|_a^0+\cos x\Big|_a^0\right)=0-\lim_{a\to-\infty}a\sin a+1-\lim_{a\to-\infty}\cos a,$$

$\lim\limits_{a\to-\infty}a\sin a$; $\lim\limits_{a\to-\infty}\cos a$ mavjud emas. Demak, bu xosmas integral uzoqlashuvchi.

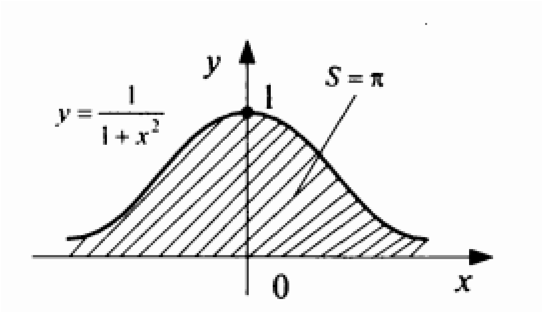
c) $c=0$, integral osti funksiyasi juft funksiya bo'lgani uchun quyidagiga ega bo'lamiz:

$$\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}=\int_{-\infty}^{0}\frac{dx}{1+x^2}+\int_0^{+\infty}\frac{dx}{1+x^2}=2\int_0^{+\infty}\frac{dx}{1+x^2}=2\lim_{t\to+\infty}\int_0^t\frac{dx}{1+x^2}=$$

$$=2\lim_{t\to+\infty}(arctgt-arctg0)=2\left(\frac{\pi}{2}-0\right)=\pi.$$

Geometrik nuqtai nazardan yaqinlashuvchi $\int\limits_a^{+\infty}f(x)dx$ xosmas integral $y=f(x)\geq0$ egri chiziq, $x=a$, $y=0$ to'g'ri chiziqlar bilan chegaralangan va Ox o'qi yo'nalishida cheksiz cho'zilgan figuraning chekli S yuzaga ega ekanligini anglatadi. Shunga o‘xshash, $\int_{-\infty}^{b} f(x)dx$ va $\int_{-\infty}^{+\infty} f(x)dx$ yaqinlashuvchi xosmas integrallarga ham geometrik talqin berish mumkin.

36.1-misolning c variantida natija $(-\infty;+\infty)$ intervalda $\frac{dx}{1+x^2}$ egri chiziq ostidagi yuza π (kv.bir.) ga tengligini bildiradi.



Integrallarni hisoblang:

36.2. $\int_{1}^{\infty}\frac{dx}{x}.$ **36.3.** $\int_{1}^{\infty}\frac{dx}{x^2}.$

36.4. $\int_{1}^{+\infty}\frac{dx}{(x+1)^4}.$ **36.5.** $\int_{-1}^{-\infty}\frac{dx}{\sqrt[3]{(2x+1)^2}}$

36.6. $\int_{1}^{+\infty}\frac{lnx}{x^3}dx.$ **36.7.** $\int_{2}^{+\infty}\frac{dx}{xln\,x}$

36.8. $\int_{-\infty}^{+\infty}\frac{dx}{x^2+4x+9}$ **36.9.** $\int_{0}^{+\infty}e^{-x}sinxdx.$

36.10. $\int_{-\infty}^{+\infty}arctgxdx.$ **36.11.** $\int_{-\infty}^{+\infty}xe^{2x}dx.$

36.12. $\int_{0}^{+\infty}xe^{-x+1}dx$ **36.13.** $\int_{-\infty}^{+\infty}xe^{-x^2}dx.$

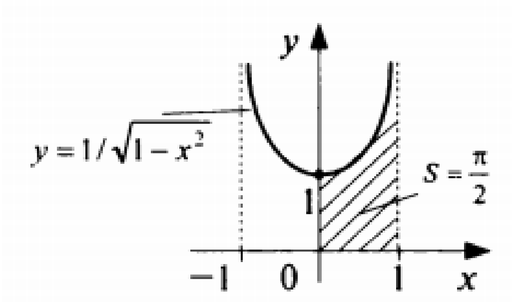
36.14. $\int_{1}^{\infty}\frac{dx}{x\sqrt{x^2-1}}.$ **36.15.** $\int_{-\infty}^{0}xe^{x}dx.$

36.16. $\int_{-\infty}^{0}\cos 3xdx$ **36.17.** $\int_{-\infty}^{0}\frac{1}{1+x^2}dx.$

36.18. Integrallarni hisoblang:

$a)\ \int_{0}^{1}\frac{dx}{\sqrt{1-x^2}};$ $b)\int_{-7}^{2}\frac{dx}{\sqrt[3]{(x-1)^2}}$

agar ular yaqinlashuvchi bo‘lsa.



Yechish: a) bunda $x=1$ nuqta integral ostidagi funksiyaning maxsus nuqtasidir. Bu holda

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to 1-0}\int_0^t \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to 1-0} \arcsin x\Big|_0^t = \lim_{t\to 1-0} \arcsin t = \frac{\pi}{2}.$$

Geometrik nuqtai nazardan chegaralanmagan funksiyaning xosmas integrali $y=f(x)$ egri chiziq, $x=a,\ x=b$ to‘g‘ri chiziqlar bilan chegaralangan va $x\to b-0$ da ($x\to a+0,\ x\to c\pm 0$) Oy o‘qi yo‘nalishida cheksiz cho‘zilgan figuraning chekli yuzga ega ekanligini anglatadi.

36.18. misolning a) variantida olingan natijaning geometrik ma’nosi $[0;1)$ yarim intervalda $y=\frac{dx}{\sqrt{1-x^2}}$ egri chiziq ostidagi yuza $\frac{\pi}{2}$ (kv.bir.) ga tengligini bildiradi.

b) integral osti $\frac{dx}{\sqrt[3]{(x-1)^2}}$ funksiya $[-7;2]$ kesmadagi $x=1$ ichki nuqtada aniqlanmagan va bu nuqta atrofida chegaralanmagan. Aniq integral xossalaridan foydalangan holda, bu integralni ikkita integralning yig‘indisi ko‘rinishda ifodalaymiz

$$\int_{-7}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}} = \int_{-7}^{1} \frac{dx}{\sqrt[3]{(x-1)^2}} + \int_{1}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}}$$

$$\int_{-7}^{2} \frac{dx}{\sqrt[3]{(x-1)^2}} = \lim_{t\to 1-0}\int_{-7}^{t} (x-1)^{-2/3}dx + \lim_{\tau\to 1+0}\int_{\tau}^{2} (x-1)^{-2/3}dx = \lim_{t\to 1-0} 3\,(x-1)^{1/3}\Big|_{-7}^{t} +$$

$$+\lim_{\tau\to 1+0} 3(x-1)^{1/3}\Big|_{\tau}^{2} = \lim_{t\to 1-0} 3(t-1)^{1/3} + 6 + 3 - \lim_{t\to 1+0} 3(\tau-1)^{1/3} = 9.$$

36.19. $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$.

36.20. $\int_1^5 \frac{dx}{\sqrt{5-x}}$.

36.21. $\int_{-1}^{0} \frac{dx}{(x+1)^2}$.

36.22. $\int_1^0 lnxdx$.

36.23. $\int_0^2 \frac{dx}{\sqrt[3]{1-x}}$. **36.24.** $\int_0^2 \frac{x^3dx}{\sqrt{4-x^2}}$.

36.25. $\int_0^1 \frac{dx}{x^2+x^4}$. **36.26.** $\int_{-1}^1 \frac{dx}{x^2}$

36.27. $\int_0^1 e^{-x^2}dx$ aniq integral trapetsiyalar va Simpson formulalaridan foydalanib taqribiy hisoblansin.

Yechish. $[0;1]$ kesmani $x_0=0;\ x_1=0,2;\ x_2=0,4;\ x_3=0,6;\ x_4=0,8;\ x_5=1$ nuqtalar yordamida 5 ta teng bo‘lakka bo‘lamiz. Keyin $f(x)=e^{-x^2}$ funksiyaning shu nuqtalardagi qiymatlarini hisoblaymiz.

$$f(x_0)=f(0)=e^0=1, \qquad f(x_1)=f(0,2)\approx 0,96079,$$

$$f(x_2)=f(0,4)\approx 0,85214, \qquad f(x_3)=f(0,6)\approx 0,69768,$$

$$f(x_4)=f(0,8)\approx 0,52729, \qquad f(x_5)=f(2)\approx 0,36788.$$

Trapetsiyalar formulasi bo‘yicha

$$\int_0^1 e^{-x^2}dx \approx \frac{1}{5}\left[\frac{1+0,36788}{2}+0,96079+0,85214+0,69769+0,52729\right]=0,74805.$$

Simpson formulasi bo‘yicha, hisoblash uchun $[0;1]$ kesmani $x_0=0,\ x_1=\frac{1}{4},\ x_2=\frac{1}{2},\ x_3=\frac{3}{4},\ x_4=1$ nuqtalar orqali 4 ta teng bo‘laklarga ajratamiz va bu nuqtalarda funksiyaning qiymatlari

$$y_0=1, \quad y_1=(0,25)\approx 0,9394, \quad y_2=(0,5)\approx 0,7788,$$

$$y_3=(0,75)\approx 0,5698, \quad y_4=(1)\approx 0,3679$$

bo‘ladi.

Simpson formulasiga asosan:

$$\int_0^1 e^{-x^2}dx \approx \frac{1}{12}[1+0,3679+4(0,9394+0,5698)+2\cdot 0,7788]\approx 0,7469.$$

Aniq integral to‘g‘ri to‘rtburchaklar, trapetsiyalar va Simpson formulalaridan foydalanib taqribiy hisoblang. Olingan natijani integralning aniq qiymati bilan taqqoslang.

36.28. $\int_1^{\pi} \sin x dx\ \ (n=6)$. **36.29.** $\int_0^1 \frac{dx}{1+x}\ \ (n=10)$.

36.30. $\int_{-4}^{6} \sqrt{x^2+9}dx\ \ (n=10)$. **36.31.** $\int_1^2 \frac{dx}{x}\ \ (n=10)$.

36.32. $\int_1^2 \frac{1+\sqrt{x}}{x^2}dx \ \ (n=6).$

36.33. $\int_1^3 \sqrt{x+1}dx \ \ (n=6).$

36.34. $\int_0^4 \frac{dx}{1+\sqrt{x}} \ \ (n=6).$

36.35. $\int_1^{\infty} \frac{arctgxdx}{1+x^2}.$

36.36. $\int_0^1 \ln xdx.$

36.37. $\int_{-1}^2 \frac{dx}{x}.$

36.38. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}.$

36.39. $\int_2^3 \frac{xdx}{\sqrt{x-2}}.$

36.40. $\int_0^2 \frac{dx}{\sqrt[3]{(x-1)^2}}.$

36.41. $\int_0^1 \frac{dx}{x\ln^2 x}.$

36.42. $\int_0^{e^4} \frac{dx}{x\sqrt{\ln x}}.$

36.43. $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}.$

Aniq integral trapetsiyalar formulasidan foydalanib taqribiy hisoblansin.

36.44. $\ln 2 = \int_1^2 \frac{dx}{x}.$

36.45. $\int_0^4 x^2 dx \ \ n=10.$

36.46. $\int_1^9 \sqrt{6x-5}dx \ \ n=8.$

Foydalanishga tavsiya etiladigan adabiyotlar ro‘yxati

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