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MATEMATIKADAN MASALALAR TO‘PLAMI

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MATEMATIKADAN MASALALAR TO’PLAMI.

Ushbu o’quv qo’llanma Oliy o’quv yurtlarining 5110700 – informatika o‘qitish metodikasi, 5321000 – oziq – ovqat texnologiyasi, 5112100 – texnologik ta‘lim yo‘nalishlari talabalariga mo’jallangan bo’lib, unda matematika tarkibiga kiruvchi chiziqli algebra, analitik geometriya va matematik analizdan qisqacha ma’lumotlar va ular yordamida yechiladigan misol va masalalar tizimli ravishda berilgan. Ushbu o‘quv qo‘llanma yuqoridagi yo‘nalishlar talabalariga mo‘ljallangan bo‘lishiga qaramasdan undan matematika o‘qitiladigan oliy o‘quv yurtlari ham foydalanishlari mumkin.

Taqrizchilar:

Arziqulov F. – ADU matematika kafedrasi dotsenti, f.-m.f.d.

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Ushbu o’quv qo’llanma Andijon davlat universiteti ilmiy kengashining 2020- yil 18 fevraldagi 7-sonli yig’ilishida muhokama qilinib chop etishga tavsiya qilingan.

So’z boshi

Hozirgi paytda, O’zbekiston Respublikasi Prezidentining 2017–yil 20-apreldagi Oliy ta’lim tizimini yanada rivojlantirish chora-tadbirlari to’g’risida”gi PQ–2909 va 2018–yil 5–iyundagi “Oliy ta’lim muassalarida ta’lim sifatini oshirish va ularning mamlakatda amalga oshirilayotgan keng qamrovli islohotlarda faol ishtirokini ta’minlash bo’yicha qo’shimcha chora–tadbirlar to’g’isida”gi PQ–3775 qarorida ta’kidlangan, oliy ta’lim muassasalarida ta’lim sifatini oshirish, ta’lim jarayoniga ilg’or pedagogic usullar, axborot kommunikatsiya texnologiyalari, electron ta’lim resurslari va multimediya taqdimotlarini keng tatbiq etish va buning uchun zarur shart–sharoitlarni yaratish to’g’risidagi vazifalar oliy o’quv yurtlarida ta’lim sifatini va samaradorligini tubdan yaxshilashga qaratilganligidan dalolatdir. Bu vazifalar oliy o’quv yurtlarida o’qitiladigan barcha fanlar, jumladan, matematika faniga ham taaluqlidir.

Oliy o’quv yurtlarida malakali, raqobatbardosh mutaxassislarni tayyorlashda matematika fanining o’rni salmoqlidir. Chunki bugungi kunda barcha sohalarda matematika va matematik usullardan samarali foydalanilmoqda. Shuning uchun ham bugungi kundagi asosiy vazifalardan biri brcha ta’lim muassasalarida matematikani o’qitishning zamon talablariga mos holda takomillashtirishdan iboratdir. Bu borada so’nggi yillarda respublikamiz prezidenti Sh.M. Mirziyoyev tashabbusi bilan bir qator muhim farmon va qarorlar qabul qilindi. Bularga misol tariqasida 2019-yil 9-iyuldagi PQ-4387 sonli qarori hamda O’zbekiston Respublikasi Prezidenti Sh.M. Mirziyoyevning Oliy majlisga murojaatnomasini keltirish mumkin.

Ma’lumki, bugungikunda oliy o’quv yurtlarining barcha mutaxassisliklarida oliy matematika (matematika) fani o’qitilmoqda. Talabalarni matematikadan chuqur bilim, ko’nikma va malakalarga ega bo’lishlarida o’quv adabiyotlarining, ayniqsa davlat tilida yozilgan qo’llanmalarning jumladan matematikadan yozilgan masalalarto’plamining o’rni beqiyosdir.

Hozirgi kunda oliy matematika va matematik analizdan masalalar to’plami va ularni yechish bo’yicha bir qancha adabiyotllar mavjud. Ular I.A.Maron mualliflligidagi “Differensialnoe i integralnoye ischislenie v primerax i zadachax” nomli, I.A. Kaplan mualliflligidagi “Prakticheskiye zanyatiya po visshey matematike” nomli, P.E. Danko, A.G. Popov, T.Ya. Kojevnikovalarning “Vishsaya matematika v uprajneniyax i zadachax” nomli, V.P. Minorskiy mualliflligidagi “Oliy matematikadan masalalar to’plami” nomli, G.M. Zaporojets mualliflligidagi “Rukovodstvo k resheniyu zadach po matematicheskomu analizu” nomli, E.F. Fayziboyev va N.M. Sirmirakislar mualliflligidagi “Integral xisob kursidan amaliy mashg’ulotlar” nomli, I.I. Lyashko, A.K. Boyarchuk, Ya.G.Gay, G.P.Golovachlar mualliflligidagi “Matematicheskiy analiz v primerax i zadach” nomli, A. Sa’dullayev va boshqalarning “Matematik analiz kursidan misol va masalalar to’plami” va xokazolardir.

Bunday qo’llanmalar ko’p bo’lishiga qaramasdan ularning aksariyati matematik analiz fanidan hamda davlat tilida emas. Bundan tashqari bu qo’llanmalarda masala va misollar tizimli berilmagan va ularni yechish uchun kerak bo’ladigan nazariy materiallar yetarli darajada berilmagan. Mualliflar tomonidan yozilgan ushbu qo’llanma yuqoridagi kamchiliklarni bartaraf qilishga qaratilgandir.

Ushbu o’quv qo’llanma oliy matematika (matematika) uchun ajratilgan soat eng ko’p bo’lgan yo’nalishlarga mo’ljallangan bo’lishiga qaramasdan, undan oliy matematika (matematika) uchun eng kam soat ajratilgan yo’nalishlarning talabalari ham foydalanishlari mumkin.

Ushbu o’quv qo’llanmaning qo’lyozmasini o’qib chiqib o’zlarining qimmatli maslahatlarini bergan fizika – matematika fanlari doktori F.Arziqulovga, f.m.f.nR.Azimovga va Andijon mashinasozlik institute oliy matematika kafedrasi dotsenti T.A.Djalilovaga mualliflar o’z minnatdorchiliklarini bildiradilar.

Mualliflar

I BOB. TO'PLAMLAR

1-§. To'plam tushunchasi. To'plamlar ustida amallar

To'plam matematikaning poydevorida yotgan boshlang'ich tushunchalardan biri bo'lgani uchun u ta'riflanmaydi. To'plam deyilganda biror bir xususiyati bo'yicha umumiylikka ega bo'lgan obyektlar majmuasi tushuniladi. Masalan 1-kurs talabalari to'plami, kesmadagi nuqtalar to'plami, matematikadagi raqamlar to'plami, bog'dagi mevali daraxtlar to'plami va xokazo. To'plamlar A,B,C,D,… kabi bosh harflar bilan belgilanadi. To'plamga kiruvchi obyektlar uning ***elementlari*** deyiladi va $a, b, c, d, \ldots$ kabi harflar bilan belgilanadi. Agar " a " element A to'plamga tegishli bo'lsa, uni $a \in A$ kabi, tegishli bo'lmasa $a \overline{\in} A$ yoki $a \notin A$ kabi yoziladi. Elementlari a, b, c, d bo'lgan A to'plam $A = \{a, b, c, d\}$ kabi yoziladi.

Birorta ham elementga ega bo'lmagan to'plam ***bo'shto'plam*** deb ataladi va $\emptyset$ kabi belgilanadi.

Masalan, $sinx = 5$ tenglamaning ildizlari to'plami, kvadrati manfiy son bo'ladigan haqiqiy sonlar to'plami, $x^2 + 9 = 0$ tenglamaning ildizlari to'plami bo'sh to'plamdan iborat.

Agar A to'plamga tegishli har bir element B to'plamga ham tegishli bo'lsa u holda A to'plam B to'plamning qismi deyiladi va $A \subset B$ kabi belgilanadi.

Masalan, bo'g'dagi mevali daraxtlar to'plamini A, barcha daraxtlar to'plamini B deb olsak, unda $A \subset B$ bo'ladi.

Ta'rifdan $A \subset A$ va $\emptyset \subset A$ tasdiqlar doim to'g'ri ekanligi kelib chiqadi.

Agar A va B to'plamlar uchun bir vaqtda $A \subset B$ va $B \subset A$ shartlar bajarilsa, bu to'plamlar teng to'plamlar deyiladi va $A = B$ kabi yoziladi.

Masalan, $A = \{-1, 1\}$ va $B = \{x^2 - 1 = 0$ tenglama ildizlari to'plami $\}$ bo'lsa, u holda $A = B$ bo'ladi.

A va B to'plamlar **birlashmasi (yig'indisi)** deb ularni barcha elementlaridan tuzilgan C to'plamga aytiladi va uni $C = A \cup B$ kabi yoziladi.

Masalan, $A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$ bo'lsa, $C = A \cup B =$

$$= \{1,2,3,4,5,6\}$$

To’plamlar birlashmasi uchun $A \cup B = B \cup A$ va $(A \cup B) \cup C = A \cup (B \cup C)$ $A \cup \emptyset = A,$ $A \cup A = A$ hamda $B \subset A$ bo’lganda $A \cup B = A$ munosabatlar o’rinlidir.

A va B to’plamlarning **kesishmasi (ko’paytmasi)** deb ularning har ikkalasiga tegishli bo’lgan elementlardan tuzilgan to’plamga aytiladi va uni $C = A \cap B$ kabi yoziladi.

Masalan, $A = \{1,2,3,4,5,6\}$ va $B = \{3,4,5,6\}$ bo’lsa, $A \cap B == \{3,4,5,6\}$ bo’ladi.

To’plamlar kesishmasi uchun quyidagilar o’rinli:

1. $A \cap B = B \cap A$;
2. $(A \cap B) \cap C = A \cap (B \cap C)$;
3. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$;
4. $A \cap A = A,$ $A \cap \emptyset = \emptyset$ va $B \subset A$ bo’lsa $A \cap B = B.$

A va B to’plamlarning ***ayirmasi*** deb A to’plamga tegishli, ammo B to’plamga tegishli bo’lmagan elementlardan tashkil topgan to’plamga aytiladi va uni $A \setminus B$ kabi yoziladi.

Masalan, $A = \{1,2,3,4,5,\}$ va $B = \{1,3,7,9\}$ bo’lsa, $A \setminus B = \{2,4,5\}$,
$B \setminus A = \{7,9\}$ bo’ladi.

To’plamlar ayirmasi uchun

$$A \setminus A = \emptyset, \qquad A \setminus \emptyset = A, \qquad \emptyset \setminus A = \emptyset$$

va $A \subset B$ bo’lsa, $A \setminus B = \emptyset$ munosabatlar o’rinlidir.

Agar qaralayotgan barcha to’plamlarni biror Ω to’plamning qism to’plamlari deb qarash mumkin bo’lsa, unda Ω ni ***universal to’plam*** deb ataladi.

Masalan, barcha sonli to’plamlaruchun $\Omega = (-\infty; +\infty)$ to’plam universal to’plamdir.

Agar A to’plam Ω universal to’plamning qismi bo’lsa, unda$\Omega \setminus$ A to’plam A ***to’plamning to’ldiruvchisi*** deyiladi va C(A) kabi belgilanadi.

Masalan, Ω={tumandagi barcha fermerlar}, A={ rejani bajargan fermerlar} bo’lsa, unda C(A)={rejani bajarmagan fermerlar} to’plami bo’ladi.

A va B to'plamalarning ***Dekart ko'paytmasi*** deb A x B kabi belgilanuvchi va $(x, y)(x \in A, y \in B)$ ko'rinishdagi juftliklardan tuzilgan yangi to'plamga aytiladi.

Masalan,$A = \{0, 2\}$va$B = \{0, 1\}$bo'lsa, u holda$A$ x $B =$
$= \{(0, 0), (0, 1), (2, 1), (2, 0)\}$bo'ladi.

To'plamlar chekli, cheksiz va bo'sh bo'lishi mumkin.

Agar A to'plamning elementlari bilan natural sonlar to'plami N ning dastlabki biror m ta elementlari orasida bir qiymatli moslik o'rnatib bo'lsa, unda A ***chekli to'plam*** deyiladi.

Masalan, A={O'zbekistondagi barcha odamlar}, B={Kitobdagi varaqlar}, C={Matematikadagi raqamlar}, D={Lotin alifbosidagi harflar} kabi to'plamlar chekli bo'ladi.

Chekli bo'lmagan A to'plam ***cheksiz to'plam*** bo'ladi.
Masalan, natural sonlar to'plami $N = \{1,2,3, \dots, n, \dots\}, A = \{[0; 1]$ kesmadaginuqtalar to'plami}, B={ $cosx = a$ $(|a| \leq 1)$ tenglama ildizlari}, D={tekislikdagi barcha to'g'ri chiziqlar} kabi to'plamlar cheksiz to'plamlardir.

Agar A va B to'plamlar orasida o'zaro bir qiymatli moslik o'rnatib bo'lsa, bu to'plamlar ***ekvivalent to'plamlar*** deyiladi va uni$A \sim B$ kabi yoziladi.

Masalan, A={toq sonlar}, B={juft sonlar} bo'lsa, unda $A \sim B$ bo'ladi.

1. $A = \{1,2,3\}$ va $B = \{2,4\}$ to'plamlar berilgan. Bu to'plamlar uchun $A \cup B$, $A \cap B$, $A \setminus B$, $A \triangle B$ va A x B lar topilsin.

2. $A = \{1,2,3,4,5\}$ va $B = \{2,4,6,8\}$ bo'lsa, $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$ lar topilsin.

3.$A = \{1,2,3,4,5\}$ va $B = \{1,3,7,9\}$ bo'lsa, $A \setminus B$, $B \setminus A$ lar topilsin.

4. $A = [0,2]$ va $B = [0,1]$ bo'lsa, A x B va B x A lar topilsin.

5. $A = \{n - 3, n - 2, n - 1, n, n + 1\}$ va $B = \{n - 1, n + 1, n + 2, n + 3, n + 4\}$ bo'lsa, $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$ lar topilsin.

6. $A = [n - 3, n + 1]$ va $B = (n - 1, n + 5)$ bo'lsa, $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$ lar topilsin.

7. $A = \{n-3, n-2, n-1\}$ va $B = \{n, n+1, n+2, n+3\}$ to'plamlar berilgan. Ular uchun $A \times B$ va $B \times A$ lar topilsin.

8. $E = \{1,2,3,4,5\}$ va $F = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$ to'plamlar berilgan. Bu to'plamlar ekvivalent bo'la oladimi? Nima uchun?

9. $E = \{2,4,6,8\}$ va $F = \{2,4,6,8,10\}$ to'plamlar berilgan. Bu to'plamlar ekvivalent bo'la oladimi?

10. $M = \{1,2,3\}$ to'plam berilgan. Uning barcha to'plam ostlari yozilsin.

11. $A = \{1,2\}$ va $B = \{1,2,3,4,5\}$ to'plamlar berilgan. A to'plamni B to'plamga to'ldiruvchi to'plam topilsin.

12. $A = \{-1,1\}$ to'plam bilan $(x-1)^2(x+1)^3 = 0$ tenglamaning barcha ildizlari to'plami o'zaro teng bo'la oladimi?

13. $x^2 - 1 = 0,\ x^2 - 2x + 1 = 0,\ x^2 + x + 1 = 0, |x| + x = 0$ tenglamalar yechimlari to'plamini yozing.

14. A to'plam $x^2 - 3x + 2 = 0$ tenglama ildizlaridan iborat va $B = \{0,2\}$ bo'lsa, $A \cup B,\ A \cap B,\ A \setminus B,\ B \setminus A$ lar topilsin.

15. $A = \{a, b, c\}$ to'plamning to'plam ostlari yozilsin.

2-§. Sonli to'plamlar. Haqiqiy sonning absolyut qiymati

Elementlari sonlardan iborat bo'lgan to'plam ***sonli to'plam*** deyiladi.

Quyidagilar eng asosiy sonli to'plamlardir:

- Natural sonlar to'plami: $N = \{1,2,3, \dots, n, \dots\}$;
- Butun sonlar to'plami: $Z = \{\dots, -n, \dots, -3, -2, -1, 0.1, 2, 3 \dots, n, \dots\}$;
- Ratsional sonlar to'plami:$Q = \left\{\frac{m}{n},\ m \in Z,\ n \in N\right\}$;
- Irratsional sonlar to'plami:$I = \{\text{cheksiz, davriy bo'lmagan, o'nli kasrlar}\}$
- Haqiqiy sonlar to'plami:$R = Q \cup J$

Haqiqiy sonlarni geometrik tasvirlash uchun ***haqiqiy sonlar o'qi*** yoki qisqacha ***sonlar o'qi***tushunchasi kiritiladi. Buning uchun biror l to'g'ri chiziz olinadi va quyidagi ishlar bajariladi:

- Bu to'g'ri chiziqda chapdan o'nga tomon yo'nalishni musbat yo'nalish deb olinadi.

- Bu to'g'ri chiziqda ixtiyoriy nuqta olinadi va uni ***sonlar o'qining boshi*** deyiladi.
- Bu to'g'ri chiziqda bir birlikni ifodalovchi masshtab kiritiladi.

Chekli a va $b(a < b)$ sonlari uchun $a < x < b$ qo'sh tengsizlikni qanoatlantiruvchi barcha x sonlar to'plami interval (oraliq) deb ataladi va uni(a, b) kabi yoziladi.

$a \leq x \leq b$ qo'sh tengsizlikni qanoatlantiruvchi barcha x sonlar to'plami segment yoki kesma deb ataladi va uni$[a, b]$ kabi yoziladi.

$a \leq x < b$ va $a < x \leq b$ lar yarim oraliqlar deyiladi va ularni$[a, b)$ va $(a, b]$ kabi yoziladi.

a va b chegaralardan birortasi∞ $(-\infty$ yoki$+\infty)$, ya'ni$(-\infty, b)$, $(a, +\infty)$, $(-\infty, b], [a, \infty)$ bo'lsa, u holda ularni ***yarim cheksiz oraliqlar*** deb ataladi.$(-\infty; +\infty)$ ni esa ***cheksiz oraliq*** deb ataladi.

Berilgan X sonli to'plam uchun shunday M(yoki m) soni mavjud bo'lsaki, ixtiyoriy $x \in X$ uchun $x \leq M$(yoki $x \geq m$) shart bajarilsa, X yuqoridan (quyidan) chegaralangan to'plam deyiladi. Agar X ham quyidan ham yuqoridan chegaralanganya'ni $m \leq x \leq M$ bo'lsa, u ***chegaralgan to'plam*** deyiladi.

Har qanday$x \in R$ sonning ***absolyut qiymati*** (moduli) deb shu sondan sonlar o'qining O boshigacha bo'lgan masofaga aytiladi.

Berilgan $x \in R$ sonning absolyut qiymati $|x|$ kabi belgilanadi va ta'rifga asosan,

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

formula bilan aniqlanadi. Masalan,$|4| = 4,\ |-3| = 3,\ |0| = 0.$

Sonning absolyut qiymati quyidagi xossalariga ega:

1. Har qanday $x \in R$ uchun $|x| \geq 0$.
2. Har qanday $x \in R$ uchun $|-x| = |x|$.
3. Har qanday $x \in R$ uchun $|x| \geq x$ va $x \geq -|x|$.
4. $|x| = 0$ tenglik $x = 0$ dagina o'rinli.
5. Har qanday $x \in R,\ \ y \in R$ uchun $|xy| = |x| \cdot |y|$.

6. Har qanday $x \in R$, $0 \neq y \in R$ uchun $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$.

7. Har qanday $x \in R$, $y \in R$ uchun $|x + y| \leq |x| + |y|$.

8. Har qanday $x \in R$, $y \in R$ uchun $|x - y| \geq |x| - |y|$.

1. Hisoblang:

1) $|-7| + |-3| - |-9|$;2) $|-9| \div |-3| \cdot |-7|$;

3) $\frac{|9-|-4|-|-31||}{|-6|+|-3|}$; 4) $\frac{|-4|-|3,5|+|-0,5|}{|-6,5|+|-3,5|}$.

2. O'zgaruvchining ko'rsatilgan qiymatlarida quyidagi ifodalarning son qiymatlari topilsin.

1) $\left|\frac{2x+5}{7-2x^2}\right|$ ni $x = 2$ da; 2) $\left|\frac{2x^2-4}{5-x}\right|$ ni $x = 0$ da;

3) $\left|\frac{x-8}{5-x^2}\right|$ ni $x = 4$ da;4) $\left|\frac{2x^2-3x+7}{3x^2+7}\right| + \left|\frac{6x}{5x^3+2}\right|$ ni $x = 1$ da.

3. x ning qanday qiymatlarida tengsizliklar o'rinli bo'ladi?

1) $|x - 3| < 2$; 2)$|x - 1| < 3$; 3) $|3x - 7| < 4$;

4) $|x + 1| > 3$; 5) $|3x - 6| > 9$; 6) $|0,5x - 3,5| > 2,5$.

4. x ning qanday qiymatlarida quyidagi ifodalar haqiqiy qiymatlarga ega bo'ladi?

1) $\sqrt{9 - x^2}$;2) $\sqrt{x^2 - 9}$; 3) $\frac{4}{\sqrt{x^2-36}}$; 4) $\sqrt{16 - x^2} + \sqrt{25 - x^2}$.

5. Quyidagi tenglamalar yechilsin.

1) $|3x - 6| = 4$; 2)$x^2 - 3|x| - 40 = 0$; 3) $|x^2 - 2x| = 2x - x^2$;

4) $|x^2 + 5x| = 6$; 5) $|2 - 3x| = |5 - 2x|$; 6)$\big|3 - |2 + x|\big| = 1$;

7)$|x^2 - 8x + 7| = -7 + 8x - x^2$; 8) $|x - 1|^2 - 8 = 2|x - 1|$;

9) $|x| = x^2 + x - 4$;10) $|x + 2| + |x| + |x - 2| = 4$.

6.Quyidagi tengsizliklar yechilsin.

1) $|2x - 3| < 6$; 2) $|x^2 - 5| < 4$;3) $x^2 - 2|x| < 3$;

4) $|x - 1| \geq 2$; 5) $|x - 3| \leq 6 - x$; 6) $x^2 - 3|x| \leq 4$;

7) $2|x - 1| \leq |x + 3|$; 8) $|x + 1| > 2|x + 2|$; 9) $1 < |x - 2| < 3$;

10) $\left|\frac{3x-11}{10(x+3)}\right| < \frac{1}{100}$.

7.Quyidagi tenglamalar yechimga ega bo'ladimi?

1) $|x| = x + 5$; 2) $|x| = x - 5$; 3) $|sinx| = sinx + 1$.

8. x ning qanday qiymatlarida quyidagi tengliklar o'rinli bo'ladi?

1) $\left|\frac{x-1}{x+1}\right| = \frac{x-1}{x+1}$; 2) $|x^2 - 5x + 6| = -(x^2 - 5x + 6)$;

3) $|(x^2 + 4x + 9) + (2x - 3)| = |x^2 + 4x + 9| + |2x - 3|$;

4) $|(x^2 - 4) - (x^2 + 2)| = |x^2 - 4| - |x^2 + 2|$.

3-§. Kompleks sonlar va ular ustida amallar

Kvadrati -1 ga teng bo'lgan sonni mavxum birlik deb ataladi va uni i bilan belgilanadi. Demak, $i^2 = -1$ bo'lib,undan $i = \sqrt{-1}$ kelib chiqadi. Mavxum birlikni kiritilishi bilan manfiy sonlardan kvadrat ildiz chiqarish mumkin bo'ladi. Masalan,$\sqrt{-36} = \sqrt{36 \cdot (-1)} = \sqrt{36} \cdot \sqrt{-1} = 6i$;

$\sqrt{-\frac{1}{9}} = \sqrt{\frac{1}{9} \cdot (-1)} = \sqrt{\frac{1}{9}} \cdot \sqrt{-1} = \frac{1}{3}i$.

Amalda mavhum birlikning darajalaridan foydalaniladi.

Quyida ularni keltiramiz.

$i^1 = i$;

$i^2 = -1$;

$i^3 = i^2 \cdot i = -1 \cdot i = -i$;

$i^4 = i^3 \cdot i = -i \cdot i = -i^2 = -(-1) = 1$;

$i^5 = i^4 \cdot i = 1 \cdot i = i$;

$i^6 = i^5 \cdot i = i \cdot i = i^2 = -1$;

$i^7 = i^6 \cdot i = -1 \cdot i = -i$;

$i^8 = i^7 \cdot i = -i \cdot i = -i^2 = -(-1) = 1$;

……………………………………………..

Bulardan $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$ekanligini aniqlaymiz.

a va b haqiqiy sonlar hamda i mavxum birlikdan hosil qilingan $a + bi$ ko'rinishdagi sonlarga ***kompleks sonlar*** deb ataladi. Bu yerda a kompleks sonining ***haqiqiy qismi***, bi kompleks sonning ***mavxum qismi*** va b mavxum qismining ***koeffisienti*** deyiladi. $a = 0$ bo'lsa bi kompleks soni sof mavxum son deyiladi.

Agar $b = 0$ bo'lsa, u holda kompleks son a ga teng bo'lib uni haqiqiy son deyiladi. Agar $a = b = 0$ bo'lsa, u holda kompleks son $0 + 0 \cdot i$ bo'lib nolga teng bo'ladi.

$a + bi$ko'rinishidagi yozuv kompleks sonning ***algebrak shakli*** deyiladi.

$a + bi$ va c+di kompleks sonlar teng bo'lishi uchun $a = c$, $b = d$ bo'lishi kerak.

$a + bi$ va $a - bi$ ko'rinishdagi kompleks sonlarga ***o'zaro qo'shma*** kompleks sonlar deyiladi.

Algebraik ko'rinishdagi kompleks sonlar ustida qo'shish, ayirish va ko'paytirish amallari ko'phadlar ustida bajariladigan amallar kabi bajariladi. Ular quyidagicha:

$$(a + bi) + (c + di) = (a + c) + (b + d)i;$$

$$(a + bi) - (c + di) = (a - c) + (b - d)i;$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i;$$

$$(a + bi) + (a - bi) = 2a;$$

$$(a + bi) - (a - bi) = 2bi;$$

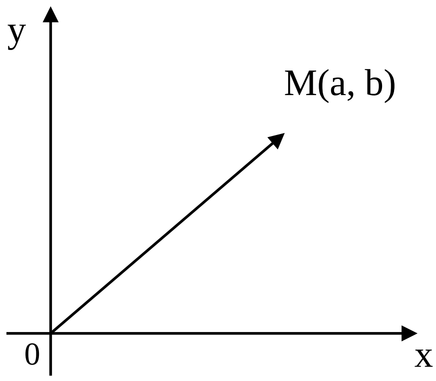
$$(a + bi)(a - bi) = a^2 + b^2;$$

Kompleks sonni Kompleks songa bo'lish uchun bo'luvchi va bo'linuvchi kompleks sonlarni bo'luvchi kompleks sonning qo'shmasiga ko'paytiriladi.

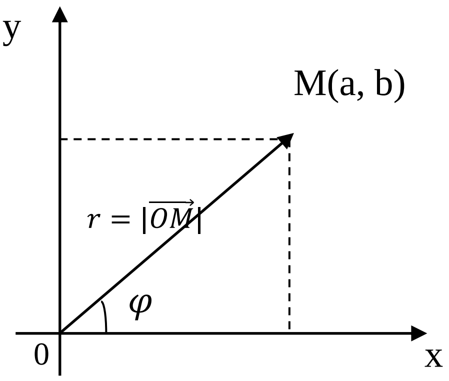
$$\frac{a + bi}{c + di} = \frac{(a + bi)(c - di)}{(c + di)(c - di)} = \frac{(ac + bd) + (bc - ad)i}{c^2 + d^2}$$

$a + ib$kompleks sonining geometrik tasviri boshi O(0,0) nuqtada va oxiri$M(a, b)$ nuqtada bo'lgan$\overrightarrow{OM}$ vektordan iborat (1-chizma).

Aytaylik Z= $a + ib$ kompleks son boshi O(0,0) nuqtada, oxiri $M(a, b)$ nuqtada bo'lgan $\vec{r}$ vektor bilan tasvirlangan bo'lsin(2-chizma). Z= $a + ib$ kompleks sonning ***moduli*** deb $\vec{r}$ vektorning uzunligiga aytiladi. Uni $|\vec{r}| = \sqrt{a^2 + b^2}$ (1) dan topiladi.



1- chizma



2-chizma

$\vec{r}$ vektorning Ox o'qini musbat yo'nalishi bilan hosil qilgan burchagiga kompleks sonning ***argumenti*** deyiladi. φ burchakni qiymatini $cos\varphi = \frac{a}{r}$ va $sin\varphi = \frac{b}{r}$(2) formulalardan topiladi.

$cos\varphi = \frac{a}{r}$va$sin\varphi = \frac{b}{r}$lardan $a = rcos$ц va $b = rsin\varphi$larni topamiz. a va b larning bu ifodalarini kompleksi son yozuviga qo'yib$Z = a + bi = rcos\varphi + +irsin\varphi = r(cos\varphi + isin\varphi)$ ni hosil qilamiz $Z = r(cos\varphi + isin\varphi)$ ga kompleks sonning trigonometrikshaklai deyiladi.

$cos\varphi + isin\varphi = e^{i\varphi}$ ga Eyler formulasi deyiladi. $Z = re^{i\varphi}$ ga kompleks sonning ko'rsatkichli shakli deyiladi.

Agar$Z_1 = r_1(cos\varphi_1 + isin\varphi_1)$ va $Z_2 = r_2(cos\varphi_2 + isin\varphi_2)$ bo'lsa, u holda quyidagilar o'rinlidir:

$$Z_1 \cdot Z_2 = r_1 \cdot r_2[\cos(\varphi_1 + \varphi_2) + isin(\varphi_1 + \varphi_2)];$$

$$\frac{Z_1}{Z_2} = \frac{r_1}{r_2}[\cos(\varphi_1 - \varphi_2) + isin(\varphi_1 - \varphi_2)];$$

$$Z_1{}^n = r_1{}^n(cosn\varphi_1 + isinn\varphi_1);$$

$$\sqrt[n]{Z_1} = \sqrt[n]{r_1}(cos\frac{\varphi_1 + 2k\pi}{n} + isin\frac{\varphi_1 + 2k\pi}{n}).$$

Agar$z = re^{i\varphi}$ bo'lsa, u holda$z^n = r^n e^{i\varphi n}$ va $\sqrt[n]{z} = \sqrt[n]{r}e^{\frac{\varphi + 2k\pi}{n}}$ bo'ladi.

1. Hisoblang:

1) i^{66} ; i^{143} ; i^{216} ; i^{137}; 2) $i^{43} + i^{48} + i^{64} + i^{45} - i^{101} - i^{71}$;

3) $(i^{36} + i^{25} + i^{17} + i^{14}) \cdot (i^{23} + i^{11})$;

4) $(i^{64} + i^{17} + i^{13} + i^{82})(i^{72} - i^{34})$.

2. Quyidagi tengliklardan x va y lar topilsin.

1) $3y+5xi=15-7i$; 2)$(2x+3y)+(x-y)i=7+6i$;

3) $(2x+y)-i=5+(y-x)i$;4) $(3i-1)x+(2-3i)y$=$2-3i$.

3. $z_1=2+3i$ va $z_2=5-7i$ kompleks sonlar berilgan:

a) z_1+z_2; b) z_1-z_2;

c) $z_1\cdot z_2$; d) $\frac{z_1}{z_2}$ lar topilsin.

4. $z_1=3+7i$ va $z_2=9-4i$ kompleks sonlar berilgan:

a) z_1+z_2; b) z_1-z_2;c) $z_1\cdot z_2$; d) $\frac{z_1}{z_2}$lar topilsin.

5. Quyidagi berilgan misollarda bo'lish amalini ikki hil usulda bajaring.

1) $\frac{3-i}{5-3i}$; 2) $\frac{3-7i}{3+2i}$; 3) $\frac{5-7i}{5+7i}$; 4) $\frac{9i}{6-3i}$.

6. Amallarni bajaring:

1) $\frac{3+2i}{3-2i}+\frac{5+2i}{3+2i}$; 2) $\frac{6+2i}{3-7i}-\frac{2+3i}{2+5i}$; 3) $\frac{6+2i}{1-i}-i^{27}$;

2) 4) $\left(\frac{1+i}{1-i}\right)^{12}+\left(\frac{1-i}{1+i}\right)^{12}$;5)$i^{123}+(1-i)^6+(1+i)^8$;

7. Quyidagi tenglamalar yechilsin:

1) $x^2-6x+13=0$; 2) $9x^2+12x+29=0$;3) $2{,}5x^2+x+1$=0.

8. Quyidagi berilgan kompleks sonlarni tekislikda tasvirlang:

1) $z_1=5$; 2) $z_2=-3i$; 3) $z_3=3+2i$;

4) $z_4=5-2i$; 5) $z_5=-3+2i$; 6) $z_6=-1-5i$.

9. Quyida berilgan kompleks sonlarni trigonometrik shakldа yozing:

1) $z=1+i$; 2) $z=-2+2\sqrt{3}\,i$;3) $z=-3i$; 4) $z=-3+3i$;

5) $z=2\sqrt{2}-2i\sqrt{6}$; 6) $z=-10$;7) $z=6i$; 8) $z=5$.

10. Quyida berilgan kompleks sonlarni ko'rsatkichli shaklda yozing.

1) $z=3\left(cos\frac{3\pi}{2}+isin\frac{3\pi}{2}\right)$; 2) $z=3\left(cos\frac{\pi}{4}+isin\frac{\pi}{4}\right)$;

3) $z=3-3i\sqrt{3}$; 4) $z=-3\sqrt{3}+3i$.

11. $z_1=3(cos330°+isin330°)$ va $z_2=2(cos60°+isin60°)$ kompleks sonlar berilgan. Quyidagilar topilsin:

a) $z_1\cdot z_2$; b) $\frac{z_1}{z_2}$; c) ${z_1}^4$;d) ${z_2}^5$; e) $\sqrt[3]{z_1}$; f) $\sqrt[4]{z_2}$.

12. $z_1 = 3\left(cos\frac{5\pi}{4} + isin\frac{5\pi}{4}\right)$ va $z_2 = 5\left(cos\frac{\pi}{2} + isin\frac{\pi}{2}\right)$ kompleks sonlar berilgan. Quyidagilar topilsin:

1) $z_1 \cdot z_2$; 2) $\frac{z_1}{z_2}$; 3) ${z_1}^5$; 4) $\sqrt[4]{z_2}$.

13. Quyidagilar hisoblansin.

a) $\sqrt[4]{-16}$; b) $\sqrt[6]{1}$; c) $\sqrt[6]{-1}$; d) $\sqrt[3]{-27}$.

II BOB. CHIZIQLI ALGEBRA

1-§. Matritsa va ular ustida amallar

$$A_{mxn} = \begin{pmatrix} a_{11}a_{12} & \dots & a_{1n} \\ a_{21}a_{22} & \dots & a_{2n} \\ \dots\dots\dots\dots\dots\dots\dots \\ a_{m1}a_{m2} & \dots & a_{mn} \end{pmatrix} \quad (1)$$

ga m ta satr va n ta ustundan iborat ***to'g'ri burchakli matritsa*** deyiladi. Bu matritsani ba'zan$A = (a_{ij})$ ko'rinishida ham yoziladi. Bu yerda$i = 1,2,\dots,m$; $j = 1,2,\dots,n$.

$$A = \begin{pmatrix} a_{11}a_{12} & \dots & a_{1n} \\ a_{21}a_{22} & \dots & a_{2n} \\ \dots\dots\dots\dots\dots\dots\dots \\ a_{n1}a_{n2} & \dots & a_{nn} \end{pmatrix} \quad (2)$$

ga n – tartibli ***kvadrat matritsa*** deyiladi.

Elementlari a_{11}, a_{22} , ... a_{nn} bo'lgan dioganal ***asosiy dioganal*** elementlari $a_{n1}, \dots, a_{1n}$ bo'lgan dioganal ***yordamchi dioganal elementlari*** deb ataladi.

$$A = \begin{pmatrix} a_{11} & 0 & \dots & 0 \\ 0 & a_{22} & \dots & 0 \\ \dots\dots\dots\dots\dots \\ 0 & 0 & \dots & a_{nn} \end{pmatrix} \quad (3)$$

matritsaga ***diagonal matritsa*** deyiladi.

Agar dioganal matritsada$a_{11} = a_{22} = \cdots = a_{nn}$ bo'lsa u holda uni ***skalyar matritsa*** deyiladi.

Agar skalyar matritsa bosh dioganalining barcha elementlari 1 ga teng bo'lsa, u holda uni ***birlik matritsa*** deyiladi va E bilan belgilanadi. U quyidagicha yoziladi:

$$E = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots\dots\dots\dots. \\ 0 & 0 & \dots & 1 \end{pmatrix} (4)$$

Xususiy holda 3-tartibli birlik matsitsa quyidagicha yoziladi:

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} (5)$$

Agar A_{mxn} to'g'ri burchakli matritsada $m = 1$ bo'lsa, u holda uni ***satr matritsa*** deyiladi va u quyidagicha yoziladi.

$$A = (a_{11} a_{12} \ \dots \ a_{1n}) \ (6)$$

Agar A_{mxn} to'g'ri burchakli matritsada $n = 1$ bo'lsa, u holda uni ***ustun matritsa*** deyiladi va u quyidagicha yoziladi:

$$B = \begin{pmatrix} a_{11} \\ a_{21} \\ \dots \\ a_{m1} \end{pmatrix} (7).$$

$$O = \begin{pmatrix} 0 \ 0 \dots 0 \\ 0 \ 0 \dots 0 \\ \dots \dots \dots \\ 0 \ 0 \dots 0 \end{pmatrix}$$ (8)ga ***nolmatritsa*** deyiladi.

A va B matritsa bir hil tartibli va ularning mos elementlari o'zaro teng bo'lsa, ya'ni $a_{ij} = b_{ij}$ bo'lsa, u holda ular **teng matritsalar** deyiladi. A va B matritsalar tengligi A=B yoki$(a_{ij}) = (b_{ij})$ko'rinishida yoziladi.

Agar

$$A = \begin{pmatrix} a_{11} a_{12} & \dots & a_{1n} \\ a_{21} a_{22} & \dots & a_{2n} \\ \dots \dots \dots & \dots & \dots \dots \\ a_{n1} a_{n2} & \dots & a_{nn} \end{pmatrix}, \quad B = \begin{pmatrix} b_{11} b_{12} & \dots & b_{1n} \\ b_{21} b_{22} & \dots & b_{2n} \\ \dots \dots \dots & \dots & \dots \dots \\ b_{n1} b_{n2} & \dots & b_{nn} \end{pmatrix}$$

bo'lsa, u holda quyidagilar o'rinlidir:

$$A \pm B = \begin{pmatrix} a_{11} \pm b_{11} a_{12} \pm b_{12} & \dots & a_{1n} \pm b_{1n} \\ a_{21} \pm b_{21} a_{22} \pm b_{22} & \dots & a_{2n} \pm b_{2n} \\ \dots \dots \dots \dots \dots \dots & \dots & \dots \dots \dots \\ a_{n1} \pm b_{n1} a_{n2} \pm b_{n2} & \dots & a_{nn} \pm b_{nn} \end{pmatrix};$$

$$kA = \begin{pmatrix} ka_{11} ka_{12} & \dots & ka_{1n} \\ ka_{21} \ \ ka_{22} & \dots & ka_{2n} \\ \dots \dots \dots & \dots & \dots \\ ka_{n1} \ \ ka_{n2} & \dots & ka_{n\square} \end{pmatrix};$$

$$AB = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} + \cdots + a_{1n}b_{n1} & \dots & a_{11}b_{1n} + a_{12}b_{2n} + \cdots + a_{1n}b_{nn} \\ a_{21}b_{11} + a_{22}b_{21} + \cdots + a_{2n}b_{n1} & \dots & a_{21}b_{1n} + a_{22}b_{2n} + \cdots + a_{2n}b_{nn} \\ \dots \dots \dots \dots \dots \dots & \dots & \dots \dots \dots \dots \dots \dots \\ a_{n1}b_{11} + a_{n2}b_{21} + \cdots + a_{nn}b_{n1} & \dots & a_{n1}b_{1n} + a_{n2}b_{2n} + \cdots + a_{nn}b_{nn} \end{pmatrix}$$

Matritsalar ustida amallar quyidagi hossalarga ega:

1) $A + B = B + A$; 2) $(A + B) + C = A + (B + C)$;

3) $A+0=A$;4) $AB\neq BA$;

5) $A(BC)=(AB)C$; 6) $(A+B)C=AC+BC$.

$$detA=|A|=\begin{vmatrix} a_{11}a_{12} & \dots & a_{1n} \\ a_{21}a_{22} & \dots & a_{2n} \\ \dots\dots\dots & \dots & \dots \\ a_{n1}a_{n2} & \dots & a_{nn} \end{vmatrix}$$

ga A matritsaning determinanti deyiladi.

$detA=0$bo'lsa, A matristsa **maxsus**,$detA\neq 0$ bo'lsa,**maxsusmas** matritsa deyiladi.

Agar A kvadratmatritsamaxsusmas bo'lsa, u holda$AA^{-1}=$ $=A^{-1}A=E$ tenglikni qanoatlantiradigan yagonaA^{-1} matritsa mavjud bo'ladi va u A matritsaga ***teskari matritsa*** deyiladi.

Teskari matritsa quyidagicha aniqlanadi:

$$A^{-1}=\frac{1}{detA}\begin{pmatrix} A_{11}A_{21} & \dots & A_{n1} \\ A_{12}A_{22} & \dots & A_{n2} \\ \dots\dots\dots & \dots & \dots \\ A_{1n}A_{2n} & \dots & A_{nn} \end{pmatrix}$$

A **matritsaning rangi** deb, uning noldan farqli minorlarining eng katta tartibiga aytiladi va rang (A) kabi belgilanadi.

Quyidagi almashtirishlar matritsalar ustida elementar almashtirishlar deyiladi:

a) faqat nollardan iborat satr (ustun)nio'chirish;

b) ikkita satr (ustun) ning o'rinlarini almashtirish ;

c) bir satr (ustun)ning barcha elementlarini biror songa ko'paytirib boshqa satr(ustun)ning mos elementlariga qo'shish;

d) satr(ustun)ning barcha elementlarini 0 dan farqli bir hil songa ko'paytirish.

$B=(b_{ij})$matritsa$A=(a_{ij})$ matritsaning **transponirlangani** deyiladi agar i va j indekslarning barcha mumkin bo'lgan qiymatlarida$a_{ij}=b_{ij}$ shart bajarilsa.

A matritsaning transponirlanganiniA^T kabi belgilanadi.

1. Quyida berilganA va B matritsalaryig'indisi va ayirmasi topilsin.

1) $A=\begin{pmatrix} 2 & 4 \\ -1 & 3 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 3 \\ 1 & -4 \end{pmatrix}$;

2) $A=\begin{pmatrix}1 & 2 & -3\\ 2 & -4 & 5\end{pmatrix}, \quad B=\begin{pmatrix}2 & -4 & 1\\ 3 & 0 & 2\end{pmatrix};$

3) $A=\begin{pmatrix}2 & 1 & 3\\ 0 & 0 & 0\\ -1 & 3 & 8\end{pmatrix}, \quad B=\begin{pmatrix}-5 & 3 & 16\\ 0 & 0 & 0\\ 7 & 10 & 0\end{pmatrix};$

4) $A=\begin{pmatrix}2 & -1\\ 3 & 5\\ 0 & -8\end{pmatrix}, \quad B=\begin{pmatrix}1 & 0 & -3\\ 2 & 4 & 8\end{pmatrix}.$

2. Berilgan A,B,C matritsalar uchun (A+B)+C=A+(B+C) tenglikni to'g'riligi isbotlansin.

$$A=\begin{pmatrix}-3 & 0 & 1\\ 2 & -1 & 5\\ 4 & 2 & 0\end{pmatrix}, \quad B=\begin{pmatrix}5 & 1 & 2\\ 3 & 0 & 2\\ 7 & -1 & 5\end{pmatrix}, \quad C=\begin{pmatrix}2 & 4 & -1\\ 0 & 5 & -3\\ 0 & 2 & 4\end{pmatrix}.$$

3. Quyida berilgan matritsalarni ko'rsatilgan sonlarga ko'paytmasi topilsin.

1) $A=\begin{pmatrix}2 & -1 & 4\\ 0 & 5 & -3\\ -2 & 1 & 0\end{pmatrix}$ni $k=3$ ga;

2) $B=\begin{pmatrix}4 & 3 & -2 & 1\\ 2 & 4 & -3 & 0\\ 3 & 2 & -1 & 4\\ 4 & 3 & -2 & 1\end{pmatrix}$ni $k=4$ga.

4. Berilgan A va B matritsalarga ko'ra 2A -3B va 3B+4A ni toping.

$$A=\begin{pmatrix}2 & -4 & 0\\ -1 & 5 & 1\\ 0 & 3 & -7\end{pmatrix}, \quad B=\begin{pmatrix}4 & -1 & -2\\ 0 & -3 & 5\\ 2 & 0 & -4\end{pmatrix}.$$

5. Berilgan A va B matritsalar ko'paytmasi topilsin.

1) $A=\begin{pmatrix}3 & -1\\ -1 & 2\end{pmatrix}, \quad B=\begin{pmatrix}1 & 1\\ 3 & 1\end{pmatrix};$

2) $A=\begin{pmatrix}5 & 1\\ 2 & -3\end{pmatrix}, \quad B=\begin{pmatrix}2 & 0\\ 0 & 1\end{pmatrix};$

3) $A=\begin{pmatrix}1 & 1 & 3\\ 0 & 2 & 1\\ -1 & 0 & 4\end{pmatrix}, \quad B=\begin{pmatrix}3 & -1 & 0\\ 0 & 1 & 1\\ 2 & 0 & 1\end{pmatrix};$

4) $A=\begin{pmatrix}0 & -1 & 2\\ 2 & 1 & 1\\ 3 & 0 & 1\\ 3 & 7 & 1\end{pmatrix}, \quad B=\begin{pmatrix}3 & 1\\ 2 & 1\\ 1 & 0\end{pmatrix};$

5) $A = \begin{pmatrix} 3 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$, $\qquad B = \begin{pmatrix} -1 & 2 \\ 2 & 0 \\ -3 & 1 \end{pmatrix}$.

6. Agar $A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ va $B = \begin{pmatrix} -7 & 4 \\ 5 & -3 \end{pmatrix}$ bo’lsa, $C = A^2 + 2B$ topilsin.

7. $AB - BA$ topilsin. Bu yerda

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 4 & 1 & 1 \\ 4 & 2 & 0 \\ 1 & 2 & 1 \end{pmatrix}.$$

8. Agar $A = \begin{pmatrix} 2 & 3 & 4 \\ 5 & -1 & 6 \end{pmatrix}$ va $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ bo’lsa, AE topilsin.

9. Agar $E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ va $A = \begin{pmatrix} 2 & 1 \\ -1 & 4 \\ 5 & -2 \end{pmatrix}$ bo’lsa, EAtopilsin.

10. Quyidagi matritsalarga teskari matritsalar topilsin.

1) $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$; $\qquad$ 2) $B = \begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$;

3) $C = \begin{pmatrix} 4 & -1 \\ 0 & 3 \end{pmatrix}$; $\qquad$ 4) $A = \begin{pmatrix} 2 & 5 & 7 \\ 6 & 3 & 4 \\ 5 & -2 & -3 \end{pmatrix}$;

5) $A = \begin{pmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & -1 \end{pmatrix}$ $\qquad$ 6) $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 1 & -1 \end{pmatrix}$.

2-§. Determinantlar

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ matritsaga mos kelgan ikkinchi tartibli determinant deb$a_{11}a_{22} - a_{21}a_{12}$ songa aytiladi.

Ikkinchi tartibli determinant quyidagicha yoziladi:

$$D = detA = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

matritsaga mos kelgan uchinchi tartibli determinant deb

$$a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{12}a_{23}a_{31} - a_{31}a_{22}a_{13} - a_{32}a_{23}a_{11} - a_{21}a_{12}a_{33}$$

ga aytiladi. U quyidagicha yoziladi:

$$detA = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{21}a_{32}a_{13} + a_{12}a_{23}a_{31} - a_{31}a_{22}a_{13} -$$

$$-a_{32}a_{23}a_{11} - a_{21}a_{12}a_{33} \ .$$

Determinantlar bir qator xossalarga ega:

1) Determinantning satrini unga mos ustuni bilan almashtirilsa u holda uning qiymati o'zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix}.$$

2) Determinantning ikkita satri (ustuni)ni o'zaro almashtirilsa uning qiymati qarama-qarshisiga o'zgaradi.

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = -\begin{vmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{vmatrix}.$$

3) Satr (ustun) elementlaridan umumiy ko'paytuvchini determinant oldiga chiqarish mumkin.

$$\begin{vmatrix} a_{11} & ka_{12} \\ a_{21} & k\square_{22} \end{vmatrix} = k\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}.$$

4) Ikkita satri (ustuni) elementlari bir hil bo'lgan determinantning qiymati 0 ga teng.

5) Ikkita satr(ustun) elementlari proporsional bo'lgan determinantning qiymati nolga teng.

6) Agar determinantning biror satr(ustun) elementlariga boshqa bir satr(ustun) elementlarini biror songa ko'paytirib qo'shilsa, u holda uning qiymati o'zgarmaydi.

$$\begin{vmatrix} a_{11} + ka_{12} & a_{12} \\ a_{21} + ka_{22} & a_{22} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}.$$

7) Determinantning bosh dioganali ostida yoki ustidagi barcha elementlari nolga teng bo'lsa, u holda uning qiymati dioganal elementlari ko'paytmasiga teng bo'ladi:

$$\begin{vmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33}.$$

$D = |a_{ij}|$ determinantning a_{ij} elementiga mos kelgan M_{ij} ***minori*** deb, shu element turgan satr va ustun elementlarini o'chirishdan qolgan elementlardan tuzilgan determinantga aytiladi.

Masalan ,

$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ bo'lsa, u holda $M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$.

$D = |a_{ij}|$ determinantning a_{ij} elementiga mos ***algebraik to'ldiruvchi*** deb $(-1)^{i+j}$ ishora bilan olingan M_{ij} minorga aytiladi. Uni $A_{ij} = (-1)^{i+j}M_{ij}$ ko'rinishda yoziladi.

D determinantning ixtiyoriy satr (ustun) elementlarini ularga mos algebraik to'ldiruvchilariga ko'paytmalarining yig'indisi shu determinantning qiymatiga teng.

$$D = a_{i1}A_{i1} + a_{i2}A_{i2} + \cdots + a_{in}A_{in} \text{yoki}$$

$$D = a_{1j}A_{1j} + a_{2j}A_{2j} + \cdots + +a_{nj}A_{nj}$$

1. Quyidagi ikkinchi tartibli determinantlar hisoblansin:

1) $\begin{vmatrix} 2 & 5 \\ -3 & -4 \end{vmatrix}$; 2) $\begin{vmatrix} -1 & 4 \\ 5 & 2 \end{vmatrix}$; 3) $\begin{vmatrix} 3 & -1 \\ 4 & 5 \end{vmatrix}$; 4) $\begin{vmatrix} 0 & 7 \\ -3 & 0 \end{vmatrix}$;

5) $\begin{vmatrix} a^2 & ab \\ ab & b^2 \end{vmatrix}$; 6) $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix}$; 7) $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$;

8) $\begin{vmatrix} a+b & b \\ 2a & a-b \end{vmatrix}$.

2. Quyidagi uchinchi tartibli determinantlar hisoblansin:

1) $\begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 3 \end{vmatrix}$;2) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$; 3) $\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & -1 & 8 \end{vmatrix}$; 4) $\begin{vmatrix} 2 & 3 & -4 \\ 5 & 6 & 7 \\ 8 & 0 & 3 \end{vmatrix}$;

5) $\begin{vmatrix} 5 & 0 & 0 \\ 3 & 2 & 0 \\ 0 & 7 & -1 \end{vmatrix}$; 6) $\begin{vmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -4 \end{vmatrix}$; 7) $\begin{vmatrix} a & -a & a \\ a & a & -a \\ a & -a & -a \end{vmatrix}$; 8) $\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix}$;

9) $\begin{vmatrix} 1+\cos a & 1+\sin a & 1 \\ 1-\sin a & 1+\cos a & 1 \\ 1 & 1 & 1 \end{vmatrix}$; 10) $\begin{vmatrix} \sin 3a & \cos 3a & 1 \\ \sin 2a & \cos 2a & 1 \\ \sin a & \cos a & 1 \end{vmatrix}$.

3. Quyidagi tenglamalardan x topilsin va ildizlarni determinantga qo'yib tekshirilsin:

1) $\begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0;$ 2) $\begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 0.$

4. Quyidagi berilgan determinantlarning barcha minorlari yozilsin.

1) $D = \begin{vmatrix} -1 & 2 & 0 \\ 3 & 7 & -1 \\ 5 & 4 & 2 \end{vmatrix}$; 2) $D = \begin{vmatrix} 3 & -2 & 4 \\ -2 & 0 & -3 \\ 5 & -3 & 4 \end{vmatrix}$;

3) $D = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & -1 & 5 & 2 \\ 3 & 2 & -1 & 4 \\ 1 & 4 & -3 & 2 \end{vmatrix}.$

5. Uchlari A(2;3),B(4;-1) va C(6;5) nuqtalarda bo'lgan uchburchakning yuzi topilsin:

6. Uchlari O(0;0), A(3;3) va B(5;0) nuqtalarda bo'lgan uchburchakning yuzi topilsin:

7.Uchlari $O(0;0)$, $A(0;4)$, $B(4;6)$ va $C(7;2)$ nuqtalarda bo'lgan to'rtburchakning yuzi topilsin:

8. A(1;3), B(2;4) va C(3;5) nuqtalar bir to'g'ri chiziqda yotadimi?

9. 1) $(x_1;y_1)$ va $(x_2;y_2)$; 2) (2;3) va (-1;5) nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi uchinchi tartibli determinant yordamida yozilsin.

10. Quyidagi determinantlar soddalashtirilsin va hisoblansin:

1) $\begin{vmatrix} 1 & 2 & 5 \\ 3 & -4 & 7 \\ -3 & 12 & -15 \end{vmatrix}$; 2) $\begin{vmatrix} 24 & 48 & 72 \\ -12 & 18 & 24 \\ 4 & -3 & -2 \end{vmatrix}$; 3) $\begin{vmatrix} 2cos^2\frac{\alpha}{2} & sin\alpha & 1 \\ 2cos^2\frac{\beta}{2} & sin\beta & 1 \\ 1 & 0 & 1 \end{vmatrix}.$

11. Quyidagi determinantlarni ikki xil usulda:1) uchburchak qoidasi bilan; 2) birinchi satr elementlari bo'yicha yoyish orqali hisoblansin:

1) $\begin{vmatrix} 6 & 4 & 3 \\ 2 & -3 & 2 \\ 3 & 0 & 4 \end{vmatrix}$; 2) $\begin{vmatrix} -3 & 2 & 4 \\ -2 & 4 & -3 \\ 3 & 2 & 0 \end{vmatrix}$;3) $\begin{vmatrix} -4 & -2 & -1 \\ 3 & 2 & 0 \\ 4 & -3 & 2 \end{vmatrix}.$

12. Quyida to'rtinchi tartibli determinantlar hisoblansin:

1) $\begin{vmatrix} 2 & -1 & 1 & 0 \\ 0 & -1 & 2 & 1 \\ 3 & -1 & 2 & 3 \\ 3 & -1 & 6 & 1 \end{vmatrix}$; 2) $\begin{vmatrix} 2 & 3 & -3 & 4 \\ 2 & 1 & -1 & 2 \\ 6 & 2 & 1 & 0 \\ 2 & 3 & 0 & 5 \end{vmatrix}$; 3) $\begin{vmatrix} 2 & 3 & -4 & 1 \\ 0 & 3 & 2 & 0 \\ -2 & 4 & -3 & 1 \\ 0 & 2 & 4 & -3 \end{vmatrix}$

13. Quyida berilgan determinantlarni uchburchak shakliga keltirib hisoblang.

1) $\begin{vmatrix} 2 & 3 & -3 & 4 \\ 2 & 1 & -1 & 2 \\ 6 & 2 & 1 & 0 \\ 2 & 3 & 0 & -5 \end{vmatrix}$; 2) $\begin{vmatrix} 1 & -2 & 5 & 9 \\ 1 & -1 & 7 & 4 \\ 1 & 3 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{vmatrix}$.

J: 1) 48 ; 2) 20.

14. Quyidagi determinantlarni oldin soddalashtirib, keyin hisoblang:

1) $\begin{vmatrix} x^2+a^2 & ax & 1 \\ y^2+a^2 & ay & 1 \\ z^2+a^2 & az & 1 \end{vmatrix}$; 2) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{vmatrix}$.

3-§. Ikki va uch noma'lumli chiziqli tenglamalar sistemasi. Kramer qoidasi. Gauss usuli

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

ga ikki noma'lumli ikkita chiziqli tenglamalar sistemasi deyiladi. a_{11}, a_{12}, a_{21} va a_{22} lar koeffisientlar, b_1 va b_2 lar ozod hadlar deyiladi. Bu sistemadan quyidagi determinantlarni tuzamiz.

$$D = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}; D_{x_1} = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}; \ D_{x_2} = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}.$$

D sistemaning asosiy determinanti, D_{x_1} va D_{x_2} lar sistemaning yordamchi determinantlari deyiladi.

Agar $D \neq 0$ bo'lsa, sistema yagona yechimga ega bo'ladi va u Kramer qoidasi bo'yicha quyidagi formuladan topiladi:

$$x_1 = \frac{D_{x_1}}{D}, \ x_2 = \frac{D_{x_2}}{D}.$$

Agar $D = 0$ va D_{x_1}, D_{x_2} lardan aqalli bittasi nolga teng bo'lmasa, sistema yechimga ega emas.

Agar $D = D_{x_1} = D_{x_2} = 0$ bo'lsa, u holda sistema cheksiz ko'p yechimga ega bo'ladi.

Uch noma'lumli uchta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1 \\ a_{21}x_1 + a_{32}x_2 + a_{23}x_3 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3 \end{cases}$$

uchun ham quyidagi determinantlarni tuzish mumkin:

$$D=\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}; \qquad D_{x_1}=\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix};$$

$$D_{x_2}=\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}; \qquad D_{x_3}=\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}.$$

Bu yerda ham D asosiydeterminant, $D_{x_1}, D_{x_2}, D_{x_3}$ lar yordamchi determinantlar deyiladi.

Agar $D \neq 0$ bo'lsa, sistema yagona yechimga ega bo'ladi va u Kramer qoidasi bo'yicha quyidagi formuladan topiladi:

$$x_1=\frac{D_{x_1}}{D},\ x_2=\frac{D_{x_2}}{D},\ x_3=\frac{D_{x_3}}{D}.$$

Agar $D=0$ va $D_{x_1}, D_{x_2}, D_{x_3}$ lardan aqalli bittasi nolga teng bo'lmasa, u holda berilgan sistema yechimga ega bo'lmaydi va uni ***birgalikda bo'lmagan sistema*** deyiladi.

n ta noma'lumli n ta chiziqli tenglamalar sistemasini n ning katta $(n \geq 4)$ qiymatlarida Kramer qoidasi bilan yechish qiyinchiliklarga olib keladi. Bunday hollarda sistemani Gauss usulidan foydalanib yechish qulay bo'ladi. Bunda noma'lumlar ketma-ket yo'qotilib, sistema uchburchaksimon shaklga keltiriladi.Agar sistema uchburchaksimon shaklga kelsa, u yagona yechimga ega bo'ladi va undagi noma'lumlar oxirgi tenglamadan boshlab topiladi.

1. Quyidagi chiziqli tenglamalar sistemasini Kramer usuli bilan yeching.

1) $\begin{cases} 5x+3y=12 \\ 2x-y=7 \end{cases}$; 2) $\begin{cases} 2x-3y=11 \\ 6x-9y=33 \end{cases}$;3) $\begin{cases} 2x+3y=7 \\ 4x-5y=2 \end{cases}$;

4)$\begin{cases} 3x+2y=7 \\ 4x-5y=40 \end{cases}$;5) $\begin{cases} 5x+2y=4 \\ 7x+4y=8 \end{cases}$;6) $\begin{cases} ax-3y=1 \\ ax-2y=2 \end{cases}$;

7)$\begin{cases} 3x+2y+z=3 \\ 5x-2y-2z=3 \\ x+y-z=-2 \end{cases}$; 8) $\begin{cases} 5x+8y+z=2 \\ 3x-2y+6z=-7 \\ 2x+y-z=-5 \end{cases}$;

9) $\begin{cases} 2x-3y+z=-7 \\ x+4y+2z=-1 \\ x-4y=-5 \end{cases}$;10) $\begin{cases} 2x-3y+z=2 \\ x+5y-4z=-5 \\ 4x+y-3z=-4 \end{cases}$;

11) $\begin{cases} x-2y+z=-4 \\ 3x+2y-z=8 \\ 2x-3y+2z=-6 \end{cases}$;12) $\begin{cases} 7x+4y-z=13 \\ 3x+2y+3z=3 \\ 2x-3y+z=-10 \end{cases}$.

Javoblar: 1) $(3,-1)$; 2) sistema cheksiz ko'p yechimga ega; 3) $\left(\frac{41}{22},\frac{12}{11}\right)$;4) $(5,-4)$; 5) $(0,2)$; 6) $\left(\frac{4}{a},1\right)$; 7) $(1,-1,2)$; 8) $(-3,2,1)$; 9) $\emptyset$; 11) $(1,2,-1)$;

2. Quyidagi chiziqli tenglamalar sistemasini Gauss usuli bilan yeching.

1) $\begin{cases} x_1+2x_2-x_3=5 \\ 2x_1+x_2-4x_3=9 \\ 5x_1-2x_2+4x_3=4 \end{cases}$;2) $\begin{cases} -2x_1+x_2+x_3=1 \\ 3x_1+5x_2-x_3=-1 \\ x_1+x_2+3x_3=3 \end{cases}$;

3) $\begin{cases} 3x_1+2x_2-x_3=4 \\ 2x_1-x_2+3x_3=9 \\ x_1-2x_2+2x_3=3 \end{cases}$; 4) $\begin{cases} x_1-2x_2+x_4=-3 \\ 3x_1-x_2-2x_3=1 \\ 2x_1+x_2-2x_3-x_4=4 \\ x_1+3x_2-2x_3-2x_4=7 \end{cases}$;

5) $\begin{cases} 2x_1-x_2+x_3-x_4=1 \\ 2x_1-x_2-3x_4=2 \\ 3x_1-x_3+x_4=-3 \\ 2x_1+2x_2-2x_3+5x_4=-6 \end{cases}$.

Javoblar: 1) (2; 1; -1), 2) (0; 0; 1).

4-§. Chiziqli tenglamalar sistemasini yechishning matritsalar usuli

n noma'lumli n ta chiziqli tenglamalar sistemasi

$$\begin{cases} a_{11}x_1+a_{12}x_2+\ldots+a_{1n}x_n=b_1 \\ a_{21}x_1+a_{22}x_2+\ldots+a_{2n}x_n=b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}x_1+a_{n2}x_2+\ldots+a_{nn}x_n=b_n \end{cases}$$

ni matritsa ko'rinishida $AX=B$ deb yozish mumkin. Bu yerda

$$A=\begin{pmatrix} a_{11}a_{12}\ldots a_{1n} \\ a_{21}a_{22}\ldots a_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1}a_{n2}\ldots a_{nn} \end{pmatrix}; \quad X=\begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}; \quad B=\begin{pmatrix} b_1 \\ b_2 \\ \ldots \\ b_n \end{pmatrix}.$$

Agar $\det A\neq 0$ bo'lsa, u holda bu sistemaning matritsa shaklidagi yechimi $X=A^{-1}B$ko'rinishda bo'ladi. Bu yerda A^{-1} matritsa A matritsaga teskari matritsa.

1. Quyidagi chiziqli tenglamalar sistemasini matritsalar usuli bilan yeching.

1) $\begin{cases} 3x-5y=13 \\ 2x+7y=81 \end{cases}$; 2) $\begin{cases} 3x-4y=-6 \\ 3x+4y=18 \end{cases}$; 3) $\begin{cases} 5x+3y=12 \\ 2x-y=7 \end{cases}$;

4) $\begin{cases} x_1 + 2x_2 = 10 \\ 3x_1 + 2x_2 + x_3 = 23 \\ x_2 + 2x_3 = 13 \end{cases}$;5) $\begin{cases} 2x_1 - 3x_2 + x_3 = -7 \\ x_1 + 4x_2 + 2x_3 = -1 \\ x_1 - 4x_2 = -5 \end{cases}$;

6) $\begin{cases} 5x_1 + 8x_2 + x_3 = 2 \\ 3x_1 - 2x_2 + 6x_3 = -7 \\ 2x_1 + x_2 - x_3 = -5 \end{cases}$;7) $\begin{cases} 3x_1 - x_2 + x_3 = 12 \\ x_1 + 2x_2 + 4x_3 = 6 \\ 5x_1 + x_2 + 2x_3 = 3 \end{cases}$;

8) $\begin{cases} 3x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 3x_2 + x_3 = 1 \\ 2x_1 + x_2 + 3x_3 = 11 \end{cases}$.

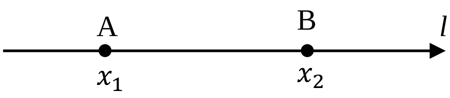
Javob: 1) $(16; 7)$; 2) $(2; 3)$; 3) $(3; -1)$; 4) $(4; 3; 5)$; 5) $(-1; 1; -2)$; 6)$(-3; 2; 1)$;7)$(0; -7; 5)$; 8) $(2; -2; 3)$.

III BOB. TEKISLIKDA VA FAZODA ANALITIK GEOMETRIYA

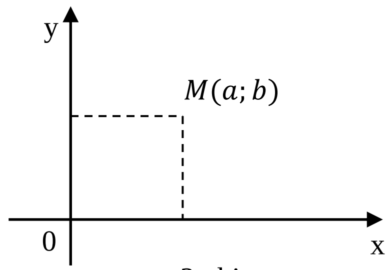
1-§. To’g’ri chiziqdagi va tekislikdagi nuqtaning koordinatalari.Ikki nuqta orasidagi masofa va kesmani berilgan nisbatda bo’lish

l o’qdagi $A(x_1)$ va $B(x_2)$ nuqtalar orasidagi masofa quyidagi formula orqali topiladi (1-chizma):

$$d = |x_2 - x_1| = \sqrt{(x_2 - x_1)^2} (1)$$

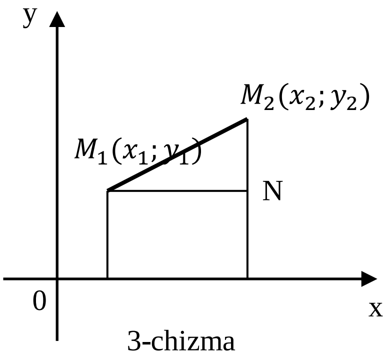


1-chizma

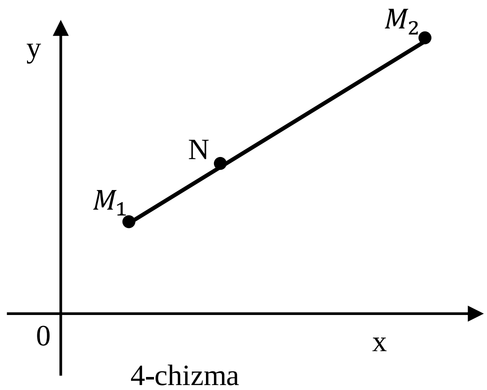


2-chizma

2-chizmada tekislikdagi to’g’ri burchakli Dekart koordinatalar sistemasi tasvirlangan. Bunda Ox- absissalar o’qi, Oy- ordinatalar o’qi, O nuqta koordinata boshi. a va b lar M nuqtaning koordinatalari ($a - M$ nuqtaning absissasi, $b - M$ nuqtaning ordinatasi).



3-chizma



4-chizma

$$d = M_1M_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} (2)$$

tekislikdagi ikkita nuqta orasidagi masofani topish formulasidir.

M_1M_2 kesmani $\frac{M_1N}{NM_2} = \lambda$ nisbatda bo’luvchi N nuqtaning koordinatalari

$$N_x = \frac{x_1 + \lambda x_2}{1 + \text{л}}, \quad N_y = \frac{y_1 + \lambda y_2}{1 + \lambda} \quad (3)$$

formuladan topiladi.

Agar N nuqta M_1M_2 kesmaning o'rtasi bo'lsa, u holda $\lambda = 1$ bo'lib, (3) formula

$$N_x = \frac{x_1 + x_2}{2}, \quad N_y = \frac{y_1 + y_2}{2} \quad (4)$$

ko'rinishga keladi va uni kesmani o'rtasini topish formulasi deyiladi.

1. Ordinatalar o'qida koordinatalar boshidan va $A(-2;5)$ nuqtadan baravar uzoqlikda turgan nuqta topilsin.Javob:(0;2,9)

2. Absissalar o'qida $A(-2;3)$ nuqtadan $3\sqrt{5}$ birlikka uzoqlashgan nuqta topilsin.Javob: $B(4;0)$.

3. Uchlari $A(-3;-1), B(5;3), C(6;-4)$ nuqtalarda bo'lgan uchburchakka tashqi chizilgan doiraning markazi va radiusi topilsin.Javob: $O(2;-1), R = 5$.

4. Koordinatalar to'g'ri chizig'ida $A(5)$ va $B(-10)$ nuqtalar orasidagi masofa topilsin.Javob: 15.

5. Agar A nuqtadan $B(-10)$ nuqtagacha masofa 6 ga teng bo'lsa, A nuqtaning koordinatasi qanday bo'ladi? Javob: $A_1(-4), A_2(-16)$.

6. Koordinatalar to'g'ri chizig'ida $A(-5), B(+4)$, va $C(-2)$ nuqtalar belgilansin va AB, BC va AC kesmalarning uzunliklari topilsin. $AB + BC = AC$ ekanligi tekshirilsin.

7. Uchlari $A(2)$ va $B(-5)$ nuqtalarda bo'lgan AB kesmani 3:1 nisbatda bo'luvchi N nuqtaning koordinatasi topilsin.Javob: $N\left(-\frac{13}{4}\right)$.

8. Nuqtaning koordinatalari $x + y = 0$ shartni qanoatlantirsa, $M(x, y)$ nuqta qaysi koordinatalar choragida bo'ladi? Javob: II yoki IV.

9. $A(-1;2)$nuqta bilan koordinata boshi orasidagi masofa topilsin. Javob:$\sqrt{5}$.

10. Ox o'qida shunday M nuqta topilsinki, u nuqtadan $A(2;6)$ nuqtagacha masofa 10 ga teng bo'lsin:Javob: $M_1(10;0),\ M_2(-6;0)$.

11.$A(5;1)$ va $B(-1;7)$ nuqtalar berilgan. AB kesmani 1:2:3 nisbatda bo'ling.

12. Parallelogrammning uchlaridan uchtasi $A(3;-3),\ B(-1;1),\ C(1;6)$nuqtalarda joylashgan. Uning to'rtinchi D uchi topilsin.Javob: D(5;2).

13. Uchburchakning uchlari $A(3;6)$, $B(-1;3)$ va $C(2;-1)$ nuqtalarda joylashgan. C uchidan tushirilgan balandlik uzunligi topilsin.

14. $A(-2;-1), B(-1;1)$ va $C(1;5)$ nuqtalarni bir to'g'ri chiziqda yotishi isbotlansin.

15. Uchlari $A(-4;2), B(0;-1)$ va $C(3;3)$ nuqtalarda bo'lgan uchburchak yasalsin va uning perimetri va burchaklari aniqlansin.J: $5\left(2+\sqrt{2}\right), 90°, 45°$.

16. Uchlari $A(-3;-2), B\ (0;-1)$ va $C(-2;5)$ nuqtalarda bo'lgan uchburchakning to'g'ri burchakli ekanligi isbot qilinsin.

17. $A(-4;0), B(-1;4)$ nuqtalar hamda Oy o'qqa nisbatan ularga mos ravishda simmetrik bo'lgan A_1, B_1 nuqtalar yasalsin. ABB_1A_1 trapetsiyaning peremetri aniqlansin.

18. B nuqta birinchi koordinatalar burchagining bissektiritsasiga nisbatan $A(4;-1)$ nuqtaga simmetrik. AB kesmaning uzunligi topilsin. J: $5\sqrt{2}$.

19. Uchlari $A(4;3), B(-3;2)$ va $C(1;-6)$ nuqtalarda bo'lgan uchburchakka tashqi chizilgan doiraning markazi va radiusi topilsin. J: $C(1;\ -1),\ R=15$.

20. Tekislikda $A(-7;0)$ va $B(0;1)$ nuqtalar hamda birinchi koordinatalar burchagining bissektirissasiga nisbatan ularga simmetrik bo'lgan A_1 va B_1 nuqtalar yasalsin. ABB_1A_1 trapetsiyaning peremetri aniqlansin. J: $18\sqrt{2}$.

21. $A(2;3)$ va $B(10;11)$ nuqtalarni birlashtiruvchi kesmani $AC:CB=3:5$ nisbatda bo'luvchi C nuqtaning koordinatalari topilsin.J: $C(5;6)$.

22. $A(-5;-2)$ va $B(4;2{,}5)$ nuqtalarni birlashtiruvchi kesmani $AM:MN:NB=3:4:2$ nisbatda bo'luvchi nuqtalarning koordinatalari topilsin. J: $M(-2;\ -0{,}5)$, $N(2;1{,}5)$.

23. $A(6;-2)$ va $B(12;-6)$ nuqtalarni birlashtiruvchi kesma 5ta teng bo'laklarga bo'lindi. Bo'linish nuqtalari koordinatalari topilsin.J: $C(7{,}2;\ -2{,}8)$, $D(8{,}4;-3{,}6),\ E(9{,}6;\ -4{,}4);\ F(10{,}8;\ -5{,}2)$.

24. $A(-2;1)$ va $B(3;6)$ nuqtalarni birlashtiruvchi AB kesmani $AN:NB=3:2$ nisbatda bo'luvchi $N(x;y)$ nuqta topilsin. J: $N(1,4)$.

25. $A(-2;1)$ va $B(3;6)$ nuqtalar berilgan. AB kesma nuqta bilan $AN:NB = 3:2$ nisbatda bo'lingan bo'lsa, N nuqtaning koordinatalari topilsin. J: $N(13;16)$

26. Uchlari $A(2;-1), B(4;3)$ va $C(-2;1)$ nuqtalarda bo'lgan uchburchak tomonlarining o'rtalari aniqlansin.

27. Uchlari $0(0;0), A(8;0)$ va $B(0;6)$ nuqtalarda bo'lgan uchburchakning OC medianasi va OD bissektirisasi uzunligi topilsin.J: $OC = 5, OD = \frac{24\sqrt{2}}{7}$.

28. Uchlari $A(2;0), B(5;3)$ va $C(2;6)$ nuqtalarda bo'lgan uchburchakning yuzi hisoblansin. J: 9 kv.b.

29. Uchlari $A(3;1)$, $B(4;6)$, $C(6;3)$ va $D(5;-2)$ nuqtalarda bo'lgan to'rtburchakning yuzi hisoblansin. J: 13 kv.b

30. Uchlari $A(3;2)$, $B(-1;-1)$ va $C(1;-6)$ nuqtalarda bo'lgan uchburchakning peremetri hisoblansin.

31. Uchlari $P(0;0)$, $Q(3;1)$ va $C(1;7)$ nuqtalarda bo'lgan uchburchakning to'g'ri burchakli ekanligini isbotlang.

32. y ning qanday qiymatlarida uchlari $A(1;3), B(2;-1), C(4;y)$ nuqtalarda bo'lgan uchburchak teng yonli bo'ladi.

33. Muntazam oltiburchakning ikkita $A_1(2;0)$ va $A_2(5;3)$qo'shni uchlarini bilgan holda, uning markazini toping.

34. Berilgan uchta $A(2;2), B(-5;1)$ va $C(3;-5)$ nuqtalardan barobar uzoqlikda bo'lgannuqtani toping.

35. Uchlari $A(4;2)$, $B(5;7)$ va $C(-3;4)$ nuqtalarda bo'lgan uchburchak har bir medianasining uzunligini toping.

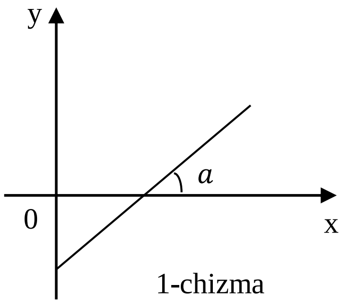
36. Agar $A(-2;2)$ va $B(1;-1)$ nuqtalar kvadratning ikkita qo'shni uchi bo'lsa, qolgan uchlari koordinatalarini toping. J: $C_1(5;1), D_1(2;5)$ va $C_2(-3;-5), D_2(-6;-1)$.

37. Agar $A(3;2)$ va $C(-2;5)$ nuqtalar kvadratning qarama-qarshi uchlari bo'lsa, uning qolgan uchlarikoordinatalari topilsin. J: $B(2;6)$, $D(-1;1)$.

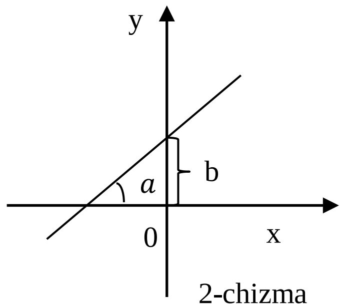
2-§. Chiziq tenglamasi. To'g'ri chiziq va uning turli xil tenglamalari

Chiziq tenglamasi deb shunday $F(x, y) = 0$ (1) tenglamaga aytiladiki, uni shu chiziqda yotgan har qanday nuqtaning koordinatalari qanoatlantiradi, unda yotmagan nuqtaning koordinatalari esa qanoatlantirmaydi.

$y = kx + b$ (2) tenglamaga to'g'ri chiziqning ***burchak koeffitsientli tenglamasi*** deyiladi. Bu yerdagi k parametr to'g'ri chiziqning Ox o'qi bilan hosil qilgan burchagining tangensiga teng bo'lib, uni to'g'ri chiziqning ***burchak koeffitsientli*** deyiladi(1-chizma). Demak, $k = tg\alpha$ (3), b esa to'g'ri chiziqning boshlang'ich ordinatasi deyiladi (2-chizma).



1-chizma



2-chizma

$Ax + By + C = 0$ (4) tenglamaga to'g'ri chiziqning ***umumiy tenglamasi*** deyiladi. Bu yerda A va B koeffitsientlar, C ozod had deyiladi. Bunda quyidagi hollar bo'lishi mumkin.

1) $C = 0$ bo'lsa, $Ax + By = 0$ bo'lib, bu holda to'g'ri chiziq koordinata boshidan o'tadi.

2) $A = 0$ bo'lsa, $By + C = 0$ bo'lib, bu holda to'g'ri chiziq Ox o'qiga parallel bo'ladi.

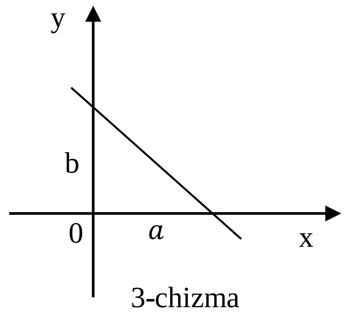
3) $B = 0$ bo'lsa, $Ax + C = 0$ bo'lib, bu holda to'g'ri chiziq Oy o'qiga parallel bo'ladi.

4) $C = 0, B = 0$ bo'lsa, $Ax = 0$ bo'lib, $(x = 0)$ bu holda to'g'ri chiziq Oy o'qi bilan ustma-ust tushadi.

5) $A = 0, C = 0$ bo'lsa, $By = 0$ bo'lib, $(y = 0)$ bu holda to'g'ri chiziq Ox o'qi bilan ustma-ust tushadi.

$Ax + By + C = 0 (B \neq 0)$ tenglamadan burchak koeffisentli tenglamaga quyidagicha o'tiladi: $Ax + By + C = 0,\ By = -Ax - C,\ y = -\frac{A}{B}x - \frac{C}{B},\ k = -\frac{A}{B},\ b = \frac{C}{B},\ y = kx + b.$

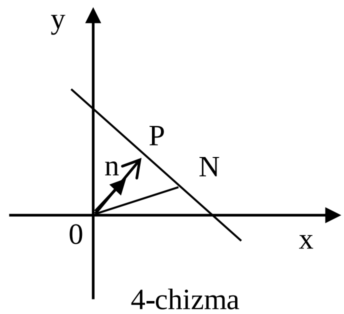
$Ax + By + C = 0$ tenglamada $A \neq 0, B \neq 0, C \neq 0$ bo'lsa, u holda undan $Ax + By = -C,\ \frac{Ax}{-C} + \frac{By}{-C} = 1,\ \frac{x}{-\frac{C}{A}} + \frac{y}{-\frac{C}{B}} = 1,\ -\frac{C}{A} = a,\ -\frac{C}{B} = b$ yoki $\frac{x}{a} + \frac{y}{b} = 1$ (5) tenglamani hosil qilamiz. Bu tenglamaga to'g'ri chiziqning ***kesmalar bo'yicha*** tenglamasi deyiladi. Bu yerda a va $b$lar to'g'ri chiziqning Ox va Oy o'qlardan ajratgan kesmalari (3-chizma).



3-chizma

$x cos\alpha + y sin\alpha - p = 0$ (6) tenglamaga to'g'ri chiziqning ***normal tenglamasi*** deyiladi.

Bu yerda n birlik vektor va uning koordinatalari $cos\alpha$ va $sin\alpha$ bo'ladi, ya'ni $n(cos\alpha; sin\alpha)$. Bundan tashqari $|OP| = p$ (4-chizma).



4-chizma

Agar tenglama umumiy tenglama ko'rinishida berilgan bo'lsa, u holda uni

$$\mu = \pm \frac{1}{\sqrt{A^2 + B^2}} \quad (7)$$

normallovchi ko'paytuvchiga ko'paytirib normal tenglamaga keltiriladi.

$M_0(x_0, y_0)$ nuqtadan o'tuvchi va yo'naltiruvchi vektori

$a = mi + nj\ \ (m, n) \neq 0$ bo'lgan to'g'ri chiziqning tenglamasi

$$\frac{x-x_0}{m}=\frac{y-y_0}{n} \quad (8)$$

bo'lib, unga to'g'ri chiziqning ***kanonik tenglamasi*** deyiladi.

Bu tenglamadan

$\frac{x-x_0}{m}=\frac{y-y_0}{n}=t,\ \begin{cases} y-y_0=nt \\ x-x_0=mt \end{cases};\ \begin{cases} y=y_0+nt \\ x=x_0+mt \end{cases};\ t\in(-\infty;+\infty)$(9)

ni hosil qilamiz. Bu tenglamaga to'g'ri chiziqning ***parametrik tenglamasi*** deyiladi.

$y-y_0=k(x-x_0)$ (10) tenglamaga $M_0(x_0,y_0)$ nuqtadan o'tuvchi ***to'g'ri chiziqlar dastasining tenglamasi*** deyiladi.

Berilgan ikkita $M_1(x_1,y_1)$ va $M_2(x_2,y_2)$nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi $\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}$(11)formuladan tuziladi.

Agar ikkita to'g'ri chiziq $y_1=k_1x+b_1$ va $y_2=k_2x+b_2$tenglamalar bilan berilgan bo'lsa, u holda ular orasidagi burchak

$$tg\varphi=\frac{k_2-k_1}{1+k_1k_2} \quad (12)$$

formuladan topiladi.

$k_2=k_1$ (13) bo'lsa, to'g'ri chiziqlar parallel, $k_1=-\frac{1}{k_2}$ (14) bo'lsa, to'g'ri chiziqlar perpendikulyar bo'ladi.

Agar to'g'ri chiziqlar $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ umumiy tenglamalar bilan berilgan bo'lsa, u holda ular orasidagi burchak

$$cos\varphi=\frac{A_1A_2+B_1B_2}{\sqrt{A_1^{\ 2}+B_1^{\ 2}}\cdot\sqrt{A_2^{\ 2}+B_2^{\ 2}}} \quad (15)$$

formuladan topiladi.

$\frac{A_1}{A_2}=\frac{B_1}{B_2}$(16) bo'lsa, to'g'ri chiziqlar parallel, $A_1A_2+B_1B_2=0$(17) bo'lsa, to'g'ri chiziqlar perpendikulyar bo'ladi.

$M_0(x_0,y_0)$nuqtadan$Ax+By+C=0$ to'g'ri chiziqqacha bo'lgan masofa

$$d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}(18)(d=|x_0cos\alpha+y_0sin\alpha-p|) \quad (19)$$

formuladan topiladi.

1.$A(1;3),\ B(2;2),\ C(2;-2),\ D(3;-3),\ E(0;-5)$ nuqtalardan qaysilari$x-y=0$ tenglama bilan berilgan chiziqda yotadi.

2. $2x^2-y^2+3x-4=0$ tenglama bilan aniqlanuvchi figuraga tegishli bir nechta nuqtani toping.

3. Quyidagi tenglamalar bilan qanday nuqtalar to'plami aniqlanadi? Ularni chizmada ko'rsating:

1) $x+7=0;$ 2) $x-4=0;$ 3) $y+3=0;$ 4) $x=0;$ 5) $y=0;$

6) $x^2-xy=0;$ 7) $x^2-y^2=0;$ 8) $xy=0;$ 9) $y^2-9=0;$

10) $x^2-8x+15=0;$11) $y^2+5y+4=0;$12) $y=|x|$.

4. Ushbu tenglamalarga mos chiziqlarni yasang.

1) $y=x^2;$2) $x^2+y^2=1;$ 3) $y=x^3;$ 4) $y=(x-1)^2+2$.

5. Quyidagi tenglamalar bilan berilgan figuralarning kesishish nuqtalari topilsin.

1) $x^2+y^2=32$va$x-y=0;$

2)$x^2-2xy+4x-3=0$va$5x-4y-1=0;$

3) $x^2+y^2-12x+16y=0$va$x=0;$

4) $x^2+y^2-2x+8y+7=0$va$y=0;$

5) $x^2+y^2-8x+10y+40=0$va$x^2+y^2=4;$

6) $x^2+y^2=36$ va $y-x=0;$

7) $x^2+y^2=64$ va $y+x=0$.

6.Oy o'qdan $b=3$ kesma ajratib, Ox o'q bilan 1)$45°$; 2) $135°$burchak hosil qiluvchi to'g'ri chiziqlarning tenglamalari tuzilsin va ular yasalsin.

7. Oy o'qdan $b=-3$ kesma ajratib, Ox o'q bilan 1) $60°$; 2) $120°$ burchak hosil qiluvchi to'g'ri chiziqlarning tenglamalari tuzilsin va ular yasalsin.

8. Koordinatalar boshidan va $A(-2;3)$ nuqtadan o'tuvchi to'g'ri chiziq yasalsin va tenglamasi tuzilsin. J: $y=-1{,}5x$.

9. 1) $2x-3y=6;$ 2) $2x+3y=0;$ 3) $y=-3;$ 4) $\frac{x}{4}+\frac{\square}{3}=1;$

5) $3x+2y-7=0;$ 6) $2x-3y+6=0$ to'g'ri chiziqlarning har biri uchun k va b parametrlar aniqlansin.

10. 1) $3x+4y=12$; 2) $3x-4y=0$; 3) $2x-5=0$;4) $2y+5=0$ to'g'ri chiziqlar yasalsin.

11. 1) $2x-3y=6$; 2) $3x-2y+4=0$; 3) $6x-3y+11=0$ to'g'ri chiziq tenglamalari kesmalar bo'yicha tenglama ko'rinishida yozilsin.

12. $A(3;5)$, $B(2;7)$, $C(-1;-3)$ va $D(-2;-6)$ nuqtalar $y=2x-1$ to'g'ri chiziqda yotadimi, yo o'sha to'g'ri chiziqdan "yuqoriroqda" yoki "quyiroqda" joylashganmi? J: A va C to'g'ri chiziqda yotadi, B undan "yuqorida" D esa "quyida" yotadi.

13. 1) $y>3x+1$; 2) $y<3x+1$; 3) $2x+y-4\geq 0$
4) $2x+y-4<0$ tengsizliklar qanday ma'noga ega?

14. Nuqtalarining koordinatalari ushbu:1) $y<2-x,\ x>-2,\ y>-2$; 2) $y>2-x,\ x<4,\ y<0$; 3) $\frac{x}{4}+\frac{y}{2}<1,\ y\geq x+2,$
$x\geq -4$ tengsizliklarni qanoatlantiruvchi sohalar yasalsin.

15. $3x-3y+1=0$ to'g'ri chiziqni Ox o'qi bilan hosil qilgan burchagi topilsin. J: $\frac{\pi}{4}$.

16. $2x-y+4=0$ to'g'ri chiziqning koordinata o'qlari bilan kesishish natijasida hosil bo'lgan uchburchakning yuzi topilsin. J: 4.

17. Quyidagi to'g'ri chiziqlar orasidagi burchaklar topilsin.

1) $\begin{cases} y=2x-3 \\ y=\frac{1}{2}x+1 \end{cases}$; 2) $\begin{cases} 5x-y+7=0 \\ 2x-3y+1=0 \end{cases}$; 3) $\begin{cases} 2x+y=0 \\ y=3x-4 \end{cases}$;

4) $\begin{cases} 3x+2y=0 \\ 6x+4y+9=0 \end{cases}$; 5) $\begin{cases} 3x-4y-6=0 \\ 8x+6y-11=0 \end{cases}$; 6) $\begin{cases} \frac{x}{a}+\frac{y}{b}=1 \\ \frac{x}{b}+\frac{y}{a}=1 \end{cases}$;

7) $\begin{cases} x+2y=0 \\ x+4y-6=0 \end{cases}$; 8) $\begin{cases} y=-\sqrt{3}x+1 \\ y=\sqrt{3}x-5 \end{cases}$.

18.$3x-2y+7=0,\ \ 6x-4y-9=0,\ \ 6x+4y-5=0,$
$2x+3y-6=0$ to'g'ri chizqlardan parallel va perpendikulyar bo'ganlari ko'rsatilsin.

19. $\frac{x}{3}+\frac{y}{2}=1$ va $y=\frac{3}{2}x-1$ to'g'ri chiziqlar o'zaro perpendikulyarmi yo yo'qmi?

20. Koordinatalar boshidan o'tib, $y=4-2x$ to'g'ri chiziq bilan $45°$ burchak tashkil qiluvchi to'g'ri chiziq tenglamasi yozilsin.

J:$y=3x$ va $y=-\frac{1}{3}x$.

21. $A(-1;1)$ nuqtadan o'tib, $2x+3y=6$ to'g'ri chiziq bilan $45°$ burchak hosil qiluvchi to'g'ri chiziq tenglamasi tuzilsin.

J: $x-5y+6=0; \quad 5x+y=-4$.

22.Uchburchak tomonlari $x+2y$=0, $x+4y-6$=0, $x-4y$=6 tenglamalar bilan berilgan. Uning ichki burchaklari topilsin. J:28°, 12°30' va 139°30'.

23. Uchlari $A(-2;0), B(2;6)$ va $C(4;2)$nuqtalarda bo'lgan uchburchakning BD balandligi va BE medianasi o'tkazilgan. AC tomon, BE mediana va BD balandlikning tenglamasi tuzilsin. J: $x-3y+2$=0; $5x-y=0$; $3x+y=12$.

24. Quyidagi nuqtalardan o'tuvchi to'g'ri chiziq tenglamalari tuzilsin.

1) $A(-1;3)$va $B(4;-2)$; 2) $A(2;3)$va $B(3;5)$;

3) $A(-3;2)$va $B(4;-7)$; 4) $A(-2;-2)$va $B(4;4)$.

25. $A(2;3)$ nuqtadan o'tuvchi to'g'ri chiziqlar dastasining tenglamasi yozilsin. Shu dastadan Ox o'q bilan: 1) $45°$; 2) $60°$; 3) $135°$; 4) $0°$ burchak tashkil etuvchi to'g'ri chiziqlar yasalsin va ularning tenglamalari tuzilsin.

26. $2x-5y-10=0$ to'g'ri chiziqning koordinata o'qlari bilan kesishgan nuqtalaridan bu to'g'ri chiziqqa perpendikulyarlar o'tkazilgan. Ularning tenglamalari tuzilsin. J: $5x+2y+4=0$; $5x+2y=25$.

27. Koordinatalar boshidan o'tib, $y=4-2x$ to'g'ri chiziq bilan $45°$ burchak tashkil etuvchi to'g'ri chiziq tenglamasi yozilsin. J:$y=3x$ va $y=-\frac{1}{3}x$.

28. Uchburchak tomonlari $y=-\frac{x}{3}$, $x=3$,$x-2y+3$=0 tenglamalar bilan berilgan. Uning uchlari va burchaklari topilsin.

J: (3; -1), (3; 3),$\left(-\frac{9}{5};\ \frac{3}{5}\right)$, 45°, 71°34', 63°26'.

29. Tomonlari $x + y = 4$, $3x - y = 0$, $x - 3y - 8 = 0$ tenglamalar bilan berilgan uchburchak yasalsin, uning burchaklari va yuzi topilsin.

30. Uchlari $A(-4; 2)$, $B(2; -5)$ va $C(5; 0)$ nuqtalarda bo'lgan uchburchak medianalarining va balandliklarining kesishgan nuqtasi topilsin. J: (1; -1), $\left(\frac{8}{3}; -2\right)$.

31. $(-4; 6)$ nuqtadan o'tuvchi to'g'ri chiziq koordinatalar burchagidan yuzi 6 kv birlikka teng uchburchak ajratadi. Bu to'g'ri chiziq tenglamasi yozilsin.

32. 1) $3x - 4y - 20 = 0$; 2) $x + y + 3 = 0$; 3) $y = kx + b$; 4) $2x - 5y + 7 = 0$ to'g'ri chiziqlarning tenglamalari normal ko'rinishga keltirilsin.

33. Normal uzunligi $p = 2$ va uning Ox o'qqa og'ish burchagi β: 1) $45°$; 2) $135°$; 3) $225°$; 4) $315°$ bo'lgan to'g'ri chiziqlar yasalsin. Bu to'g'ri chiziqlarning tenglamalari yozilsin.

34. $\square(4; 3), B(2; 1), C(1; 0)$ nuqtalardan $3x + 4y - 10 = 0$ to'g'ri chiziqqacha bo'lgan masofalar topilsin. Nuqtalar va to'g'ri chiziqlar yasalsin.

35. Koordinatalar boshidan $12x - 5y + 39 = 0$ to'g'ri chiziqqacha bo'lgan masofa topilsin.

36.$2x - 3y - 6 = 0$ va $4x - 6y - 25 = 0$ to'g'ri chiziqlar parallel ekanligi ko'rsatilsin va ular orasidagi masofa topilsin.

37. $y = kx + 5$ to'g'ri chiziq koordinatalar boshidan $d = \sqrt{5}$ masofada bo'lsa, k topilsin.

38. Uchlari $A(-3; 0), B(2; 5)$ va $C(3; 2)$ nuqtalarda bo'lgan uchburchak BD balandligining uzunligi topilsin.

39. $A(-4; -3)$, $B(-5; 0)$, $C(5; 6)$ va $D(1; 0)$ nuqtalar trapetsiyaning uchlari bo'lishi tekshirilsin va uning balandligi topilsin.

40. $x + 2y - 5 = 0$ to'g'ri chiziqdan $\sqrt{5}$ ga teng uzoqlikda bo'lgan nuqtalar geometrik o'rnining tenglamasi tuzilsin.

41. $2x - 3y + 5 = 0$ va $3x + y - 7 = 0$ to'g'ri chiziqlarning kesishish nuqtasi $M(x, y)$ nuqtadan o'tuvchi va $y = 2x$ to'g'ri chiziqqa perpendikulyar to'g'ri chiziq tenglamasi yozilsin. J: $11x + 22y - 74$=0.

42. Uchburchak AB tomonining tenglamasi $x - 3y + 3 = 0$ va AC tomonining tenglamasi $x + 3y + 3 = 0$ hamda AD balandligining asosi $D(-1; 3)$ bo'lsa, uchburchak ichki burchaklarini toping. J: 36°52', B=127°52'.

43. Nuqtalarining koordinatalari:

1) $x - 2 < y < 0$ va $x > 0$;2) $-2 \leq y \leq x \leq 2$;

3) $2 < 2x + y < 8$, $x > 0$ va $y > 0$ tengsizliklarni qanoatlantiruvchi sohalar yasalsin.

44. $A(5; 7)$ nuqta va $x + 2y - 4 = 0$ to'g'ri chiziq berilgan. A nuqtaning berilgan to'g'ri chiziqdagi proyeksiyasi B topilsin.

45. Chiziq $x = Rcost$, $y = Rsint$ parametrik tenglama bilan berilgan. Chiziq tenglamasini ikki o'zgaruvchili tenglama ko'rinishiga keltiring. J: $x^2 + y^2 = R^2$.

46. Chiziq $x = 2cost$, $y = 3sint$ $(0 \leq t \leq 2\pi)$ parametrik tenglama bilan berilgan. Chiziq tenglamasini ikki o'zgaruvchili tenglama ko'rinishiga keltiring. J: $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

47. Uchlari $A(-5;\ 3)$, $B(3;\ 7)$, $C(4;\ -1)$ nuqtalarda bo'lgan ABC uchburchak AD balandligining tenglamasi tuzilsin. J: $x - 8y + 29 = 0$.

48. Uchlari $A(2; 2)$, $B(3; 5)$, $C(4; 2)$, $D(3; -1)$ nuqtalarda bo'lgan romb dioganallarining tenglamalari tuzilsin. J: $x - 3 = 0$, $\quad y - 2 = 0$.

49. Uchlari $A(1; 1)$, $B(4; 2)$,$C(5; -1)$, $D(2; -2)$ nuqtalarda bo'lgan kvadrat tomonlarining tenglamalari tuzilsin.J:x-3y+2=0(AB), $x - 3y - 8$=0 (CD), $3x + y - 14 = 0$ (BC), $3x + y - 4 = 0$ (AD).

50. $A(2; 4)$ va $B(5; -7)$ nuqtalarni birlashtiruvchi kesmaning o'rta perpendikulyari tenglamasi tuzilsin. J: $7x - 11y - 27 = 0$.

51. Uchlari $A(5; 2)$, $B(-1; -4)$ va $C(-5; -3)$ nuqtalarda bo'lgan ABC uchburchakning B uchidan AC tomoniga parallel qilib o'tkazilgan to'g'ri chiziq tenglamasi tuzilsin. J: $x - 2y - 7 = 0$.

52. Uchlari $A(2;1)$, $B(1;4)$, $C(3;6)$ va $D(6;5)$ nuqtalarda bo’lgan trapetsiya o’rta chizig’i tenglamasi tuzilsin. J: $x-y+1=0$.

53. Uchlari $A(-1;2), B(5;3)$ va $C(4;-2)$ nuqralarda bo’lgan uchburchak o’rta chizig’ining tenglamasi tuzilsin. J: $10x-2y-15$=0.

54. $A(2;3)$ nuqtadan o’tuvchi va $4x+3y-12=0$ to’g’ri chiziqqa perpendikulyar bo’lgan to’g’ri chiziq tenglamasi tuzilsin.J: $3x-4y+6=0$.

55. $A(-1;3)$ nuqtadan o’tuvchi va $5x-3y+7=0$ to’g’ri chiziqqa perpendikulyar bo’lgan to’g’ri chiziq tenglamasi tuzilsin.J: $3x+5y-12=0$.

56. $M_1(4;2)$ va $N_1(1;-7)$ nuqtalardan o’tuvchi to’g’ri chiziq bilan $M_2(-1;3)$ va $N_2(8;6)$ nuqtalardan o’tuvchi to’g’ri chiziqlar orasidagi burchak topilsin. J: $\varphi=\arccos 0{,}6$.

57. Tomonlarining tenglamalari $18x+6y-17=0$, $14x-7y+15=0$ va $5x+10y-9=0$ bo’lgan uchburchakning burchaklari topilsin. J: 45°, 90°, 45°.

58. Uchlari $A(-6;-3)$, $B(6;7)$ va $C(2;-1)$ nuqtalarda bo’lgan uchburchakning burchaklari topilsin.J:$\arccos 0{,}006$; $\arccos 0{,}9162$; $\arccos(-0{,}6508)$.

3-§. Ikkinchi tartibli egri chiziqlar. Aylana

$$Ax^2+2Bxy+Cy^2+2Dx+2Ey+F=0 \quad (1)$$

tenglama bilan berilgan tekislikdagi chiziqlar **ikkinchi tartibli egri chiziqlar** deyiladi.

Bu yerda A, B, C koeffisientlardan kamida bittasi noldan farqli, ya’ni $A^2+B^2+C^2 \neq 0$ shart bajarilishi kerak.

Berilgan $C(a,b)$ nuqtadan bir hil R masofada joylashgan tekislikdagi nuqtalar toplami (geometrik o’rni) ***aylana*** deyiladi. Bunda $C(a,b)$ nuqta aylananing ***markazi***, R soni aylananing ***radiusi*** deyiladi.

Markazi $C(a,b)$ nuqtada radiusi R bo’lgan aylananing tenglamasi

$$(x-a)^2+(y-b)^2=R^2 \quad (2)$$

ko’rinishida bo’ladi.

Agar aylananing markazi $O(0;0)$ nuqtada bo’lsa, u holda uning tenglamasi

$$x^2 + y^2 = R^2 \quad (3)$$

ko'rinishida bo'ladi.

Agar (1) tenglamada $A = C = 1,\ B = 0,\ \ D = -2A,\ \ E = -2B$ va $F = A^2 + B^2 - R^2$ bo'lsa, u holda (1)tenglama aylanani ifodalaydi.

1.Markazi$C(a; b)$ nuqtada va radiusi$R$ bo'lgan aylana tenglamasi tuzilsin:

1) $C(3; -2)$va $R = 5$; 2) $C(-2; -5)$va $R = \sqrt{3}$;

3) $C(-5; 0)$va $R = 3$; 4) $C(0; -7)$va $R = 2$.

2.Quyidagi tenglamalar bilan berilgan aylanalarning mazkazlari va radiuslari topilsin.

1) $x^2 + y^2 + 6x - 4y - 3 = 0$;2)$x^2 + y^2 - 10x - 6y - 2 = 0$;

3) $x^2 + y^2 - 10x + 9 = 0$;4) $x^2 + y^2 + 8x + 7 = 0$.

3.$A(-4; 6)$ nuqta berilgan. Diametri OA kesmadan iborat aylana tenglamasi tuzilsin. J: $x^2+y^2 + 4x - 6y = 0$.

4.$x^2 + y^2 + 5x = 0$aylana va$x + y = 0$ to'g'ri chiziq yasalsin va ularning kesishish nuqtalari topilsin. J: $O(0; 0), M(-25; 25)$.

5. $A(1; 2)$ nuqtadan o'tuvchi va koordinata o'qlariga urunuvchi aylana tenglamasi tuzilsin.

J: $(x - 1)^2+(y - 1)^2 = 1$ yoki $(x - 5)^2+(y - 5)^2 = 25$

6. Diametrining uchlari$A(3; 2)$ va $B(-1; 6)$ nuqtalarda joylashgan aylananing tenglamasi tuzilsin. J: $(x - 1)^2+(y - 4)^2 = 8$

7. Markazi$C(1; 2)$ nuqtada bo'lgan va , $6x + 8y - 15 = 0$ to'g'ri chiziqqa urinadigan aylananing tenglamasi yasalsin.

J: $(x - 1)^2+(y - 2)^2 = \frac{49}{100}$.

8. Koordinatalari quyidagi shartlarni qanoatlantiruvchi nuqtalar to'plami tekislikda qanday figurani aniqlaydi?

1) $x^2 + y^2 \geq 4$; 2) $1 \leq x^2 + y^2 \leq 9$; 3) $(x - 1)^2 + (y + 2)^2 \leq 1$;

4) $\begin{cases} x^2 + y^2 - 4y \leq 0 \\ |x| \geq 1 \end{cases}$; 5) $\begin{cases} (x - 3)^2 + (y - 3)^2 < 4 \\ x > y \end{cases}$.

9. $x^2+y^2+4x-6y=0$ aylanning Oy o’q bilan kesishgan tuqtalariga o’tkazilgan radiuslari orasidagi burchak topilsin.

J: $tg\,\alpha=-24, \alpha=112°37'$.

10. $A(-1;3)$, $B(0;2)$ va $C(1;-1)$ nuqtalardan o’tuvchi aylana tenglamasi tuzilsin.

<u>Ko’rsatma.</u> Izlanayotgan aylana tenglamasini $x^2+y^2+mx+ny+p=0$ ko’rinishda yozib, undagi x va y lar o’rniga qiymatlarini qo’yib, so’ngra m,n,p lar topiladi.

11. $A(4;4)$ nuqtadan va $x^2+y^2+4x-4y=0$ aylana bilan $y=-x$ to’gri chiziqkesishgan nuqtadan o’tuvchi aylana tenglamasi tuzilsin. J: x^2+y^2-8y=0.

12. $y=\sqrt{-x^2-4x^2}$ egri chiziqning joylashish sohasi aniqlanib, shakli chizilsin.

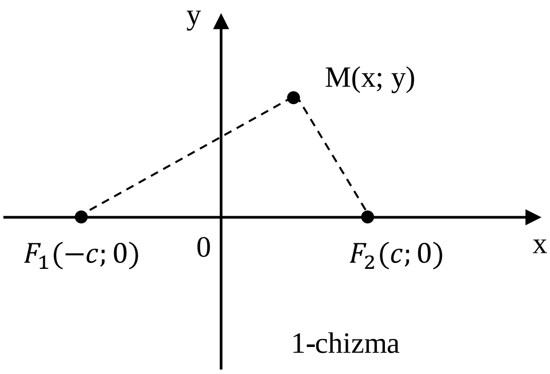
13. $A(1;-2)$, $B(0;-1)$ va $C(-3;0)$ nuqtalardan o’tuvchi aylanaga koordinatalar boshidan o’tkazilgan urunmalar tenglamalari tuzilsin. J: $y=0$, $15x+8y=0$.

14. $A(3;0)$ nuqta $x^2+y^2-4x+2y+1=0$ aylana ichida yotishi ko’rsatilsin va A nuqtada teng ikkiga bo’linuvchi vatar tenglamasi yozilsin.

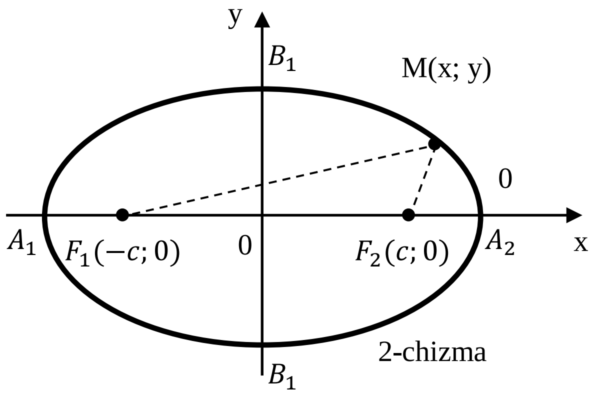
Ko’rsatma. Izlanuvchi vatar CA ga perpendikulyardir, bunda C -aylana markazi. J: $x+y=3$.

4-§. Ellips

Fokuslar deb ataluvchi berilgan ikkita F_1 va F_2 nuqtalargacha masofalarning yig’indisi o’zgarmas 2a songa teng bo’lgan tekislikdagi nuqtalarning geometrik o’rniga ***<u>ellips</u>*** deb ataladi(1-chizma).



1-chizma



2-chizma

$$|F_1M| + |F_2M| = \sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a \quad (1)$$

(1) tenglikni soddalashtirib va$a^2 - c^2 = b^2$ (2) (2)belgilash qilib

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (3)$$

ni hosil qilamiz. Bunga ellipsning ***kanonik tenglamasi*** deyiladi. Koordinatalari (3)tenglamani qanoatlantiruvchi nuqtalarning geometrik o'rni 2-chizmada berilgan.

$AA_1 = 2a$ ga ellipsning katta o'qi, $BB_1 = 2b$ ga ellipsning kichik o'qi deyiladi. a-ellipsning katta yarim o'qi,b-ellipsning kichik yarim o'qi deyiladi.c-ga ellipsning fokusi deyiladi.

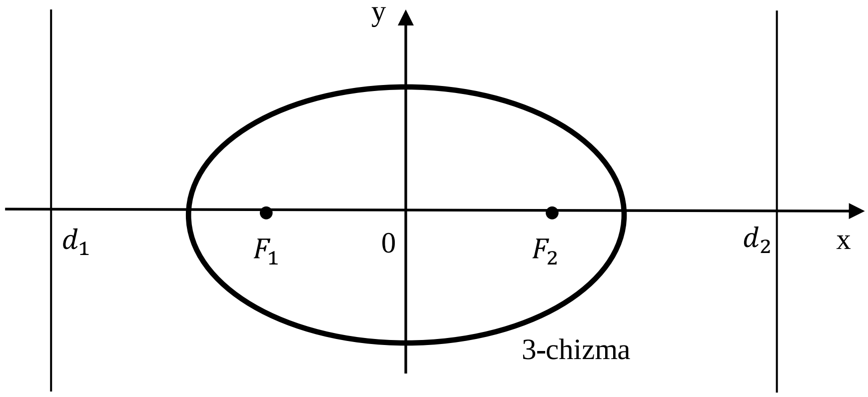
Ellipsning fokuslari orasidagi 2c masofani uning katta o'qi uzunligi $2a$ ga nisbati ***ellipsning eksentrisiteti*** deyiladi va uniε bilan belgilanadi. Demak,

$$\varepsilon = \frac{2c}{2a} = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a} = \sqrt{1 - \left(\frac{b}{a}\right)^2} \qquad (4)$$

Ellipsning ixtiyoriy $M(x, y)$ nuqtasidan uning F_1 va F_2 fokuslarigacha bo'lgan$|F_1M| = r_1,\ |F_2M| = r_2$ masofalar ellipsning ***fokal radiuslari*** deyiladi.

Fokal radiuslari uchun$r_1 = a + \varepsilon x,\ r_2 = a - \varepsilon x$ (5)formulalar ham o'rinli.

$x = \pm\frac{a}{\varepsilon}$tenglamalar bilan berilgan tog'ri chiziqlar ellipsning ***direktrisalari*** deyiladi (3-chizma).



3-chizma

1. Quyidagi tenglamalar bilan berilgan ellipslarning fokuslari,yarim o'qlari va eksentrisitetlari topilsin:

1) $\frac{x^2}{49}+\frac{y^2}{25}=1$; 2) $\frac{x^2}{121}+\frac{y^2}{81}=1$;3) $\frac{x^2}{169}+\frac{y^2}{144}=1$;

4) $2x^2+y^2=32$;5) $16x^2+25y^2=400$;6) $x^2+2y^2=18$.

2.Quyida berilganlarga asosan ellipsning tenglamasi tuzilsin:

1) $2c=8,\ \ b=3$;2) $a=6,\ \ \varepsilon=0{,}5$;

3) $a=5,\ \ c=4{,}8$;4) $a=5, c=4$.

3.Koordinata o'qlariga nisbatan simmetrik bo'lgan ellips$M(2;\sqrt{3})$ va $B(0;2)$ nuqtalardan o'tadi. Uning tenglamasi yozilsin va M nuqtadan fokuslargacha bo'lgan masofa topilsin. J: $\frac{x^2}{16}+\frac{y^2}{4}=1,\ \ \varepsilon=\frac{\sqrt{3}}{2}$,

$r_1=4-\sqrt{3}, r_2=4+\sqrt{3}$.

4.FokuslariOx o'qda yotuvchi ellips koordinata o'qlariga nisbatan simmetrik bo'lib$M(-4;-\sqrt{21})$ nuqtadan o'tadi va$\varepsilon=\frac{3}{4}$ eksentrisitetga ega. Ellips tenglamasi yozilsin va M nuqtaning fokal radiuslari topilsin. J: $\frac{x^2}{64}+\frac{y^2}{28}=1,\ r_1=11,\ r_2=5$.

5.$9x^2+25y^2=225$ ellipsda shunday$M(x,y)$ nuqta topilsinki undan o'ng fokusgacha bo'lgan masofa chap fokusgacha bo'lgan masofadan 4 marta katta bo'lsin. J: $\left(-\frac{15}{4};\ \pm\frac{\sqrt{63}}{4}\right)$.

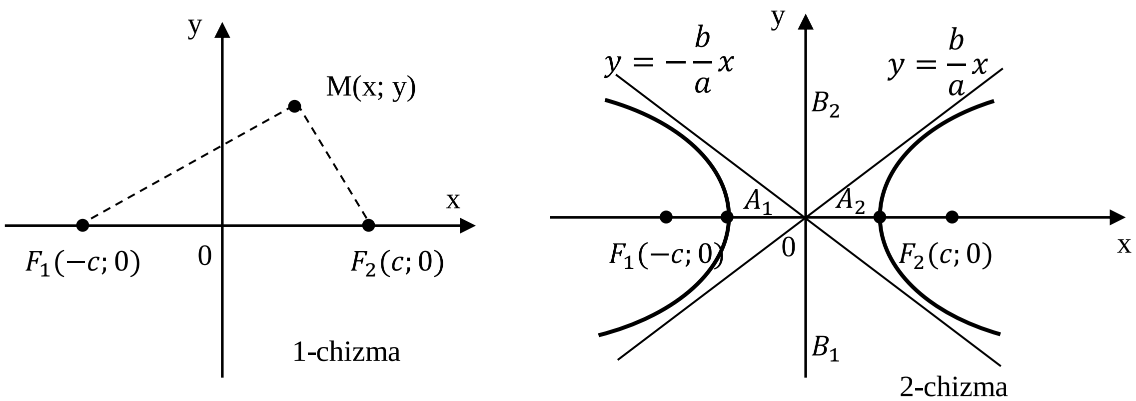
6.$x^2+2y^2=18$ ellipsning o'qlari orasidagi burchakni teng ikkiga bo'luvchi vatar uzunligi topilsin. J: $4\sqrt{3}$.

7.Ellips fokuslarining biridan katta o'qining uchlarigacha bo'lgan masofalar 5 va 1 ga teng. Uning tenglamasi yozilsin. J: $\frac{x^2}{9}+\frac{y^2}{5}=1,\ \frac{x^2}{5}+\frac{y^2}{9}=1.$

8. Koordinata o'qlariga nisbatan simmetrik ellips $M(2\sqrt{3};\sqrt{6})$ va $A(6;0)$ nuqtalardan o'tadi. Uning tenglamasi yozilsin, eksentrisiteti va M nuqtadan fokuslargacha masofalar topilsin. J: $\frac{x^2}{36}+\frac{y^2}{9}=1,\ \varepsilon=\frac{\sqrt{3}}{2},\ r_1=3,\ r_2=9.$

5-§. Giperbola

Har bir nuqtasidan fokuslar deb ataluvchi berilgan ikkita F_1 va F_2 nuqtalargacha masofalarning ayirmasi o'zgarmas 2a songa teng bo'lgan nuqtalarning geometrik o'rniga ***giperbola*** deb ataladi (1-chizma).



1-chizma

2-chizma

Ta'rifga asosan

$$|F_1M|-|F_2M|=\sqrt{(x+c)^2+y^2}-\sqrt{(x-c)^2+y^2}=2a \qquad (1)$$

tenglikni yozamiz. Uni soddalashtirib va $c^2-a^2=b^2$ (2) almashtirish qilib

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad (3)$$

tenglamani hosil qilamiz. Bunga giperbolaning ***kanonik*** tenglamasi deyiladi. Giperbola koordinata o'qlariga nisbatan simmetrik joylashgan bo'ladi (2-chizma).

Giperbolaning fokuslari orasidagi 2c masofani uning haqiqiy o'qi uzunligi $2a$ga nisbati giperbolaning eksentrisiteti deyiladi. Uni ε bilan belgilanadi. Demak,

$$\varepsilon=\frac{2c}{2a}=\frac{c}{a}=\frac{\sqrt{a^2+b^2}}{a}=\sqrt{1+\left(\frac{b}{a}\right)^2} \qquad (4)$$

$A_1(-a,0)$ va $A_2(a,0)$ nuqtalar giperbolaning ***uchlari*** deyiladi. $|A_1A_2| = 2a$ masofa giperbolaning ***haqiqiy o'qi*** deyiladi. BB_1 masofa giperbolaning ***mavhum o'qi*** deyiladi.

$y = \pm\frac{b}{a}x$ tenglamalar bilan berilgan chiziqlar giperbolaning ***asimptotalari*** deyiladi.

Tenglamalari $x = \pm\frac{a}{\varepsilon}$ bo'lgan ikkita b_1 va b_2 chiziqlar giperbolaning ***direktrisalari*** deyiladi.

Giperbolaning $M(x;y)$ nuqtasidan uning F_1 va F_2 fokuslarigacha bo'lgan $|F_1M|$ va $|F_2M|$ masofalar giperbolaning ***fokal radiuslari*** deyiladi.

Fokal radiuslar $r_1 = \pm(a+\varepsilon x)$, $r_2 = \pm(a-\varepsilon x)$ tenglamalar bilan ifodalanadi.

Agar (3) kanonik tenglamada $a = b$ bo'lsa, u holda giperbola ***teng yonli*** deyiladi.

1.Quyidagi tenglamalar bilan berilgan giperbolaning fokuslari, yarim o'qlari va eksentrisitetlari topilsin.

1) $\frac{x^2}{36} - \frac{y^2}{25} = 1$;   2) $\frac{x^2}{64} - \frac{y^2}{49} = 1$; 3) $\frac{x^2}{121} - \frac{y^2}{81} = 1$;

4) $x^2 - 4y^2 = 1$;   5) $x^2 - 4y^2 = 16$; 6) $9x^2 - 25y^2 = 225$.

2. Quyida berilganlarga asosan giperbolaning tenglamasi tuzilsin.

1) $2c = 6$; $\varepsilon = 1{,}5$;2) $2a = 4\sqrt{5}$, $\varepsilon = \frac{\sqrt{5}}{2}$;

3) $2c = 10$, $2a = 8$;4) $a = 2\sqrt{5}$, $\varepsilon = \sqrt{1{,}2}$.

3.$x^2 - 4y^2 = 16$ giperbola va uning asimptotalari yasalsin. Giperbolaning fokuslari, eksentrisitetlari va asimptotalari orasidagi burchak topilsin. J: $\varepsilon = \frac{\sqrt{5}}{2}$, 53°08'.

4. $x^2 - 4y^2 = 16$ giperbolada ordinatasi 1 ga teng M nuqta olingan undan fokuslarigacha bo'lgan masofalar topilsin. J: $r_1 = 1$; $r_2 = 9$.

5. Giperbola koordinata o'qlariga nisbatan simmetrik bo'lib $M(6;-2\sqrt{2})$ nuqtadan o'tadi ba $b = 2$ mavhum yarim o'qqa ega. Uning tenglamasiyozilsin

hamda M nuqtadan fokuslarigacha bo'lgan masofa topilsin. J: $\frac{x^2}{12} - \frac{y^2}{4} = 1$, $2\sqrt{3}$va $6\sqrt{3}$.

6. Uchlari $\frac{x^2}{25} + \frac{y^2}{9} = 1$ ellipisning fokuslarida, fokuslari esa uning uchlarida bo'lgan giperbolaning tenglamasi yozilsin. J: $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

7. $x^2 - 4y^2 = 16$ giperbolaga $A(0;-2)$ nuqtada o'tkazilgan urunmalarning tenglamalari yozilsin. J: $y + 2 = \pm\frac{\sqrt{2}}{2}x$.

8. Biror uchidan fokuslarigacha masofalari 9 va 1 ga teng bo'lgan giperbolaning kanonik tenglamasi yozilsin. J: $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

9. Markazi $x^2 - 3y^2 = 12$ giperbolaning o'ng fokusida bo'lgan va koordinatalar boshidan o'tuvchi aylana bilan shu giperbola asimptotalarining kesishgan nuqtalari topilsin. J: $(0;\ 0), (6;\ 2\sqrt{3})$.

10. $M\left(6; \frac{3}{2}\sqrt{5}\right)$nuqtadan o'tuvchi va koordinata o'qlariga nisbatan simmetrik bo'lgan giperbolaning haqiqiy yarim o'qi $a = 4$ ga teng. Giperbolaning chap fokusidan asimptotalariga tushirilgan perpendikulyarlarning tenglamalari yozilsin.J: $y = \pm\frac{4}{3}(x + 5)$.

11. FokuslariOxo'qida yotuvchi va oralaridagi masofa$10\sqrt{2}$ ga teng bo'lgan giperbola asimptotalarining tenglamalari $y = \pm\frac{3}{4}x$ dan iborat. Giperbolani kanonik tenglamasi yozilsin. J: $\frac{x^2}{37} - \frac{y^2}{18} = 1$.

12. Fokuslari Ox o'qida va $M(4;-2)$ nuqtadan o'tuvchi teng tomonli giperbolaning tenglamasi tuzilsin. J: $x^2 - y^2 = 12$.

13. Fokuslari Ox o'qida bo'lib $M(-10;8)$ nuqtadan o'tuvchi teng tomonli giperbolaning tenglamasi tuzilsin. J: $x^2 - y^2 = 36$.

14. FokuslariOy o'qida bo'lgan va $C(1;-3)$ nuqtadan o'tuvchi teng tomonli giperbolaning tenglamasi tuzilsin. J: $y^2 - x^2 = 8$.

15. Quyidagi tenglamalar bilan berilgan giperbolalarning markazi va yarim o'qlarini toping.

1) $9x^2-25y^2-18x-100y-316=0$; J: $C(1;\ -2),\ a=5,\ b=3.$

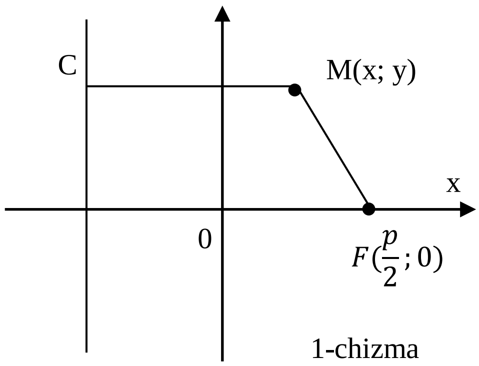
2) $5x^2-6y^2+10x-12y-31=0$;J: $C(-1;\ -1), a=\sqrt{6}, b=\sqrt{5}.$

3) $x^2-4y^2+2x+16y-7=0$;   J: $C(-1;\ 2),\ a=2,\ b=\sqrt{8}.$

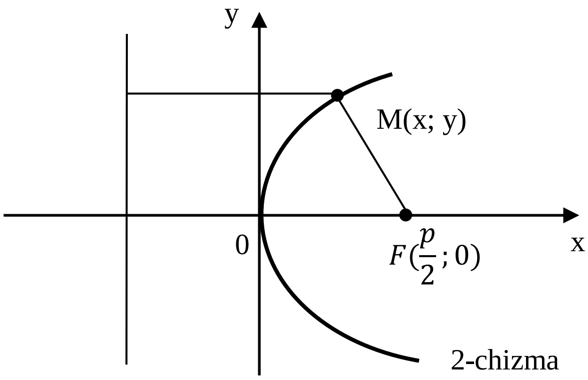
4) $3x^2-y^2+12x-4y-7=0.$ J: $C(-2;\ -2),\ a=2,\ b=2\sqrt{3}.$

6-§. Parabola

Fokus deb ataluvchi berilgan F nuqta va direktrisa deb ataluvchi l to'g'ri chiziqqacha masofalari o'zaro teng bo'lgan tekislikdagi nuqtalarning geometrik o'rniga ***parabola*** deb aytiladi (1-chizma).



1-chizma



2-chizma

Ta'rifga asosan$|MC|=|MF|$ yoki

$$\sqrt{(x-\frac{p}{2})^2+y^2}=x+\frac{p}{2} \quad (1)$$

Bu tenglamani soddalashtirib$y^2=2px$ (2) ni hosil qilamiz. (2) ga parabolaning ***kanonik tenglamasi*** deyiladi. Bu tenglamani qanoatlantiruvchi nuqtalarning geometrik o'rni paraboladan iborat(2-chizma).

$|MC|=d$ va $|MF|=r$(fokal radius) deb belgilasak, ta'rifga asosan $r=d=x+\frac{p}{2}$bo'ladi. Parabola uchun eksentrisitet$\varepsilon=\frac{r}{d}=1$ bo'ladi.

1. Quyidagi tenglamalar bilan berilgan parabolalarning fokuslari topilsin va direktrisalarining tenglamalari tuzilsin.

1)$y^2=8x$; 2) $y^2=12x$; 3) $y^2=24x$; 4) $x^2=-32y$;

5) $x^2=4y$; 6) $x^2=-4y$.

2. 1) $(0;0)$ va $(1;-3)$ nuqtalardan o'tuvchi vaOx o'qqa nisbatan simmetrik; 2) $(0;0)$ va $(2;-4)$nuqtalardan o'tuvchi va Oy o'qqa nisbatan simmetrik bo'lgan parabola tenglamasi tuzilsin. J: $y^2 = 9x$; $y = -x^2$.

3. $x^2 + y^2 + 4y = 0$ aylana va $x + y = 0$ to'g'ri chiziqning kesishgan nuqtalaridan o'tib, Oy o'qqa nisbatan simmetrik bo'lgan parabolaningtenglamasi yozilsin. J: $y = -\frac{x^2}{2}$.

4. $y^2 = 6x$ parabolada fokal radius- vektori 4,5 ga teng bo'lgan nuqta topilsin. J: $(3; 3\sqrt{2})$.

5. Koordinatalar boshidan va $x = 4$ to'g'ri chiziqdan teng uzoqlashgan nuqtalar geometrik o'rnining tenglamasi tuzilsin. Bu egri chiziqning koordinata o'qlari bilan kesishgan nuqtalari topilsin va egri chiziq yasalsin. J: $y^2 = 8(2 - x)$.

6. $y = x$ to'g'ri chiziq bilan $x^2 + y^2 + 6y = 0$ aylananing kesishgan nuqtalaridan o'tuvchi va Ox o'qqa nisbatan simmetrik bo'lgan parabolaning tenglamasi yozilsin.J: $y^2 = -3x$.

7. Uchi koordinatalar boshida va direktrisasining tenglamasi 2y+7=0 bo'ldan parabolaning tenglamasi tuzilsin. J: $x^2 = 14y$.

8. Uchi koordinatalar boshida va direktisasining tenglamasi x+3=0 bo'ldan parabolaning tenglamasi tuzilsin. J: $y^2 = 12x$.

9.Quyidagilarga asoslanib, parabolaning kanonik tenglamasi tuzilsin:

1) Fokusdan parabolaning uchigacha bo'lgan masofa 2 ga teng;

2) Fokusdan direktrisagacha bo'lgan masofa 6 ga teng;

3) Parabolaning uchidan direktrisagacha bo'lgan masofa 1 ga teng.

J: 1) $y^2 = 8x$, 2) $y^2 = 12x$, 3) $y^2 = 4x$.

10.Quyidagi parabolalarning tenglamalarini soddaroq ko'rinishga keltiring va uchlarini toping.

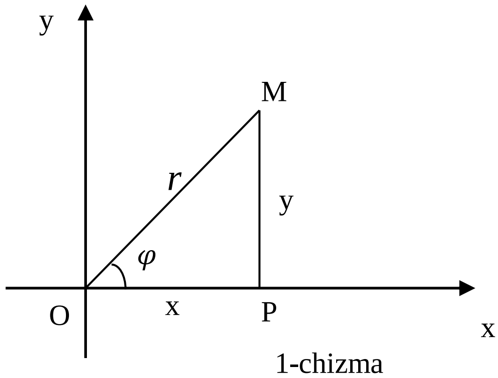
1)$y^2 - 2y - 2x - 5 = 0$; 2) $y^2 + 2x + 2y - 1 = 0$;

3) $x^2 - 4x - 2y + 10 = 0$; 4) $x^2 + 4x - y + 4 = 0$

J: 1) $(y-1)^2 = 2(x+3)$,$A\left(-\frac{5}{2}; 0\right)$; 2) $(y+1)^2 = -2(x-1)$, $C(1; -1)$; 3) $(x-2)^2 = 2(y-3)$,C(2; 3);4)$(x+2)^2$=y, $C(-2;0)$.

7-§. Qutb koordinatalar sistemasi

Tekislikda qutb deb ataluvchi O nuqta va qutb o’qi deb ataluvchi OP nur berilgan bo’lsin (1-chizma).



1-chizma

U holda tekislikdagi M nuqtaning o’rni

1) $\varphi = \angle MOP$ qutb burchagi;

2) $r = OM$ radius vektor bilan aniqlanadi. φ bilan r orasidagi bog’lanishni o’rganganda, qutb koordinatalari φ va r har qanday musbat va manfiy qiymatlar qabul qiladi deb qaraladi. Bunda manfiy φ burchak soat strelkasining harakati boyicha hisoblansa, manfiy r, nurning o’zi bo’yicha emas balki, qutbning ikkichi tomoniga ya’ni davomida joylashtiriladi. Agar qutbni Dekart koordinatalar sistemasining boshi, OP qutb o’qi esa, Ox o’qi deb qabul qilsak, u holda M nuqtaning Dekart koordinatalar sistemasidagi $(x;\ y)$ koordinatalari bilan uning $(\varphi; r)$ qutb koordinatalar sistemasi orasidagi bog’lanish

$$x = rcos\varphi,\ \ y = rs\Box n\varphi \qquad (1)$$

$$r = \sqrt{x^2 + y^2}, \quad tg\varphi = \frac{y}{x} (2)$$

tenglamalar bilan ifodalanadi.

Agar ellips, giperbola, parabola fokusini qutb deb olib, qutb o’qi esa qutbga eng yaqin uchiga qaratilgan yo’nalishga teskari yo’naltirilgan fokal simmetriya o’qini olsak, u holda bu egri chizqilarning qutb koordinatalar sistemasidagi tenglamalari bir xil $r = \frac{p}{1-\varepsilon cos\varphi}$ (3) ko’rinishda bo’ladi. Bunda ε – eksentrisitet, p – parametr, Ellips va giperbola uchun $p = \frac{b^2}{a}$.

1. Qutb koordinatalar sistemasida $A(0;3)$, $B\left(\frac{\pi}{4};2\right)$, $C\left(\frac{\pi}{2};3\right)$, $D(\pi;2)$, $E\left(\frac{3\pi}{2};3\right)$ nuqtalar tasvirlansin.

2. Qutb koordinatalar sistemasida $A\left(\frac{\pi}{2};-2\right)$, $B\left(-\frac{\pi}{2};3\right)$, $C\left(-\frac{\pi}{4};-4\right)$, $D\left(\frac{2\pi}{3};-3\right)$ nuqtalar aniqlansin.

3. $r = 2 + 2cos\varphi$ chiziq yasalsin.

4. Ushbu 1) $x^2 - y^2 = a^2$; 2) $x^2 + y^2 = a^2$;3)$y = x$; 4) $xcos\alpha + ysin\alpha - p = 0$;5) $x^2 + y^2 = ax$; 6) $(x^2 + y^2)^2 == a^2(x^2 - y^2)$chiziqlarning tenglamalari qutb koordinatalaridagi tenglamalari bilan almashtirilsin.

<u>Ko'rsatma</u>: $x = \rho cos\varphi$, $y = \rho sin\varphi$ larni berilgan tenglamalarga qo'yib soddalashtirilsin.

5. Ushbu 1) $rcos\varphi = a$;   2) $r = 2asin\varphi$;   3) $r^2 sin2\varphi = 2a^2$;

4) $rsin\left(\varphi + \frac{\pi}{4}\right) = a\sqrt{2}$;   5) $r = a(1 + cos\varphi)$

chiziqlarning tenglamalari Dekart koordinatalaridagi tenglamalari bilan almashtirilsin.

<u>Ko'rsatma.</u>$\rho = \sqrt{x^2 + y^2}$; $tg\varphi = \frac{y}{x}$formulalardan foydalanilsin.

6. Quyidagi:1) $r = \frac{9}{5-4cos\varphi}$; 2) $r = \frac{9}{4-5cos\varphi}$;   3) $r = \frac{3}{1-cos\varphi}$ ikkinchi tartibli egri chiziqlarning kanonik tenglamalari yozilsin.

J: 1) $\frac{x^2}{25} + \frac{y^2}{9} = 1$; 2) $\frac{x^2}{16} - \frac{y^2}{9} = 1$; 3) $y^2 = 6x$.

7. Ushbu 1) $r = \frac{9}{2-\sqrt{3}cos\varphi}$;   2) $r = \frac{9}{2-\sqrt{5}cos\varphi}$;   3) $r = \frac{3}{2-2cos\varphi}$ ikkinchi tartibli egri chiziqlarning kanonik tenglamalari yozilsin.

J: 1) $\frac{x^2}{4} + y^2 = 1$; 2) $\frac{x^2}{4} - y^2 = 1$; 3) $y^2 = x$.

8. Dekart koordinatalar sistemasida berilgan quyidagi nuqtalarni qutb koordinatalarini toping: 1) $(-1;\sqrt{3})$,   2) $(2;\sqrt{2})$,   3) $(\sqrt{3};\sqrt{5})$;   4) $(2;\sqrt{6})$.

9. Dekart koordinatalar sistemasida $M_1(-1;1)$ va $M_2(1;\sqrt{3})$ nuqtalar berilgan. Ularni qutb koordinatalari aniqlansin.

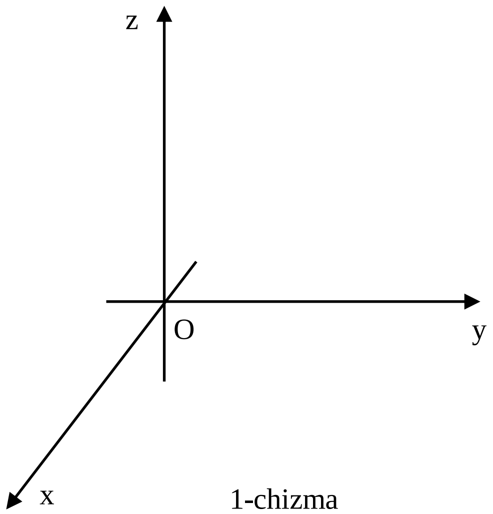
10. Ellipsning tenglamasini $\begin{cases} x = acos\theta \\ y = bsin\theta \end{cases}$ $(-\infty < и < \infty)$ ko'rinishda yozish mumkinligini isbotlang.

11. $r = 3 - 2sin2\varphi$; $r = 2 + cos3\varphi$; $r = 1 - sin3\varphi$ chiziqlar yasalsin.

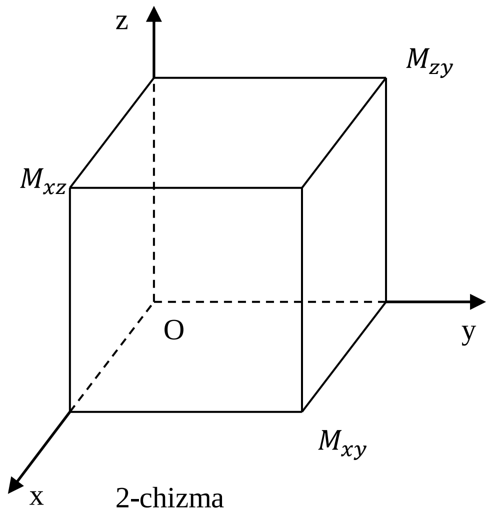
Ko'rsatma. Oldin r_{max} va r_{min} larni beradigan burchaklar aniqlansin.

8-§. Fazodagi analitik geometriyaning asosiy tushunchalari va masalalari

Masshtab birligi bilan ta'minlangan o'zaro perpendikulyar hamda bitta O nuqtada kesishuvchi Ox, Oy, Oz to'g'ri chiziqlar bilan hosil qilingan sistema fazodagi to'g'ri burchakli Dekart koordinatalar sistemasi deyiladi(1-chizma).



1-chizma



2-chizma

O nuqta koordinatalar boshi, Ox - absissalar o'qi, Oy – ordinatalar o'qi, Oz – applikatalar o'qi deyiladi.

Fazodagi M nuqtaning holati uni Ox, Oy, Oz o'qlarga proyeksiyalari – (x, y, z) uchlik bilan aniqlanadi (2-chizma). (x, y, z) uchlik M nuqtaning koordinatalari deb ataladi va uni $M(x, y, z)$ kabi yoziladi.

Fazoda Dekart koordinatalar sistemasi va $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$ nuqtalar berilgan bo'lsa, u holda ular orasidagi masofa quyidagi formula bilan hisoblanadi:

$$M_1M_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z)^2} \quad (1)$$

Fazodagi $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$ nuqtalarni tutashtiruvchi M_1M_2 kesmani λ (biror son) nisbatda bo'luvchi N nuqtaning koordinatalari

$$N_x = \frac{x_1 + \lambda x_2}{1 + \lambda}, \quad N_y = \frac{y_1 + \lambda y_2}{1 + \lambda}, \quad N_z = \frac{z_1 + \lambda z_2}{1 + \lambda} \qquad (2)$$

formulalardan topiladi.

Agar$\lambda = 1$,ya’ni$\frac{M_1N}{NM_2} = 1$bo’lsa, u holda

$$N_x = \frac{x_1 + x_2}{2}, \quad N_y = \frac{y_1 + y_2}{2}, \quad N_z = \frac{z_1 + z_2}{2} \qquad (3)$$

bo’lib, bu formulalarga kesma o’rtasining koordinatalarini topish formulalari deyiladi.

1. $M_1(2;3;4)$; $M_2(-3;4;4)$; $M_3(3;-4;4)$; $M_4(2;3;-5)$; $M_5(0;2;3)$; $M_6(2;0;4)$, $M_7(3;4;-4)$ nuqtalarni tasvirlang.

2. Quyida berilgan nuqtalar orasidagi masofalar topilsin:

1) $M_1(2;3;4)$va$M_2(6;6;4)$; J: 5.

2) $M_3(3;6;-3)$va $M_4(6;9;3)$; J: $3\sqrt{6}$.

3) $M_5(-3;-4;-6)$ va $M_6(6;-4;-6)$. J: 9.

3. Quyida berilgan nuqtalarni birlashtiruvchi kesmalar o’rtasining koordinatalari topilsin.

1) $M_1(6;4;8)$va$M_2(8;6;10)$. J: $N_1(7;5;9)$;

2) $M_3(-4;-6;0)$va $M_4(6;0;-10)$. J: $N_2(1;-3;-5)$;

3) $M_5(0;0;7)$ va $M_6(6;10;-3)$.J: $N_3(3;5;2)$.

4.$A(2;1;-3)$ va $B(6;-9;3)$ nuqtalarni birlashtiruvchi kesmani $2:3$ nisbatda bo’luvchi C nuqtaning koordinatalari topilsin.

J: $C(3{,}6;\ -3;\ -0{,}6)$.

5.Uchlari $A(2;-1;3)$, $B(1;1;1)$ va $C(0;0;5)$ nuqtalarda bo’lgan ABC uchburchakning perimetri va yuzi topilsin. J: $6 + 3\sqrt{2}$; 4,5.

6. Parallelogrammning ketma-ket uchta $A(1;-2;3)$, $B(3;2;1)$ va $C(6;4;4)$ uchlari berilgan. Uning to’rtinchi uchi D topilsin.J: D(4;0;6).

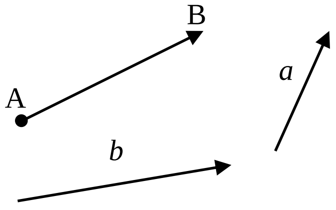
7. Uchlari $A(2;3;1)$, $B(0;0;0)$ va $C(4;-1;3)$ nuqtalarda bo’lgan uchburchak tomonlarining uzunliklari topilsin. J: $\sqrt{14}$; $\sqrt{26}$; $2\sqrt{6}$.

8. ZOX tekisligida shunday nuqta topilsinki, undan $A(0;0;4)$, $B(2;2;2)$ va $C(0;2;1)$ nuqtalargacha bo’lgan masofalar teng bo’lsin.

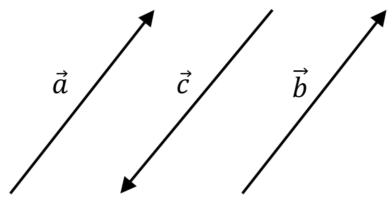
J: $D\left(\frac{8}{3}; 0; \frac{11}{6}\right)$.

9-§. Tekislikda va fazoda vektorlar

Yo'naltirilgan kesma **vektor** deyiladi va u $\overrightarrow{AB}$ yoki $\vec{a}, \vec{b}$ kabi belgilanadi(1-chizma).



1-chizma



2-chizma

Yo'naltirilgan $\overrightarrow{AB}$ kesmaning A nuqtasi vektorning ***boshi***, B esa ***oxiri*** deyiladi. $\overrightarrow{AB}$kesmaning uzunligiga vektorning ***uzunligi***deyiladi.

Boshlang'ich va oxirgi nuqtalari ustma-ust tushgan vektor ***nol vektor*** deyiladi va u $\vec{O}$ kabi belgilanadi.

Bitta to'g'ri chiziqda yoki parallel to'g'ri chiziqlarda yotgan vektorlar ***kollinear vektorlar*** deyiladi.

Yo'nalishlari bir xil va uzunliklari teng bo'lgan ikkita $\vec{a}$va$\vec{b}$ vektorlar ***teng vektorlar*** deyiladi va $\vec{a} = \vec{b}$ kabi yoziladi.

2- chizmada $\vec{a} = \vec{b}$, $\vec{a} \neq \vec{c}$, $\vec{b} \neq \vec{c}$.

Bitta yoki parallel tekisliklarda joylashgan uch va undan ortiq vektorlar ***komplanar***vektorlar deyiladi.

$\vec{a}$ vektorni λ songa ko'paytmasi deb, quyidagi uchta shartni qanoatlantiruvchi yangi bir $\vec{c}$ vektorga aytiladi:

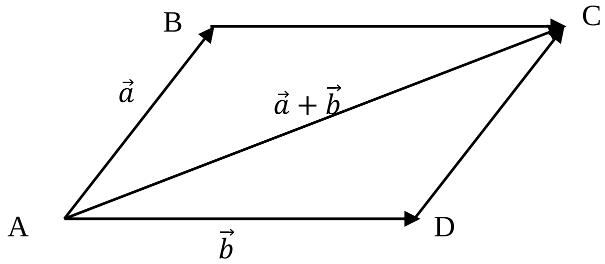
1. $|\vec{c}| = |\square| \cdot |\vec{a}|$;
2. $\vec{c} \parallel \vec{a}$;
3. $\lambda > 0$ bo'lganda $\vec{a}$ va $\vec{c}$ vektorlar bir hil yo'nalgan, $\lambda < 0$ da esa $\vec{a}$va$\vec{c}$ vektorlar qarama- qarshi yo'nalgan bo'ladi.

Vektorni songa ko'paytmasi $\lambda\vec{a}$ kabi yoziladi va u quyidagi xossalarga ega:

1. $\lambda(\beta\vec{a}) = \beta(\lambda\vec{a})$. 2. $(\lambda \pm \beta)\vec{a} = \lambda\vec{a} \pm \beta\vec{a}$. 3. $0 \cdot \vec{a} = 0$.

$(-1)\vec{a}$ vektor $\vec{a}$ vektorga qarama-qarshi vektor deyiladi va $-\vec{a}$ kabi belgilanadi.

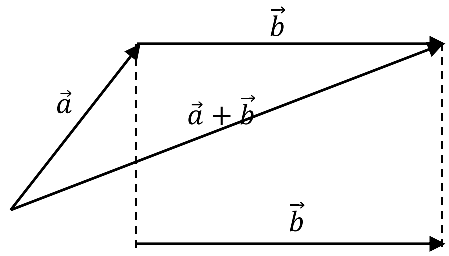
$\vec{a}$ va $\vec{b}$ vektorlarning yig'indisi deb $ABCD$ parallelogrammning A uchidan chiquvchi dioganalidan hosil qilingan $\overrightarrow{AC}$ vektorga aytiladi va $\vec{a}+\vec{b}$ kabi belgilanadi(3 – chizma).



3-chizma

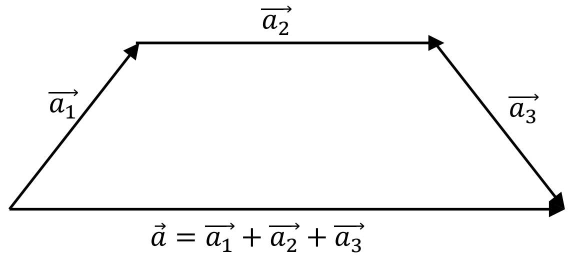
Vektorlar yig'indisini bu usulda aniqlash ***parallelogramm qoidasi*** deyiladi. Bu yig'indini***uchburchak qoidasi*** deb ataluvchi quyidagi usul bilan ham topish mumkin. Bunda dastlab parallel ko'chirish orqali $\vec{b}$vektorning boshi $\vec{a}$ vektorning uchi ustiga keltiriladi(4 – chizma). So'ngra $\vec{a}$ boshidan chiqib, $\vec{b}$ ni uchida tugaydigan vektor hosil qilinadi va u $\vec{a}+\vec{b}$ yig'indini ifodalaydi.

Ikkita $\vec{a}$ va $\vec{b}$ vektorlarning yig'indisini topishda har ikkala usuldan foydalanish mumkin.



4-chizma

a_1, a_2, a_3 vektorlarning yig'indisi parallelogramm qoidasini ketma-ket uch marta yoki ko'pburchak qoidasi bilan aniqlanadi(5 – chizma).



5-chizma

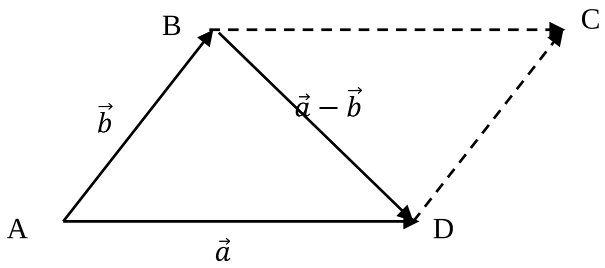
Vektorlarni qo'shish amali quyidagi hossalarga ega:

$$1. \vec{a} + \vec{b} = \vec{b} + \vec{a}. \; 2. \left(\vec{a} + \vec{b}\right) + \vec{c} = \vec{a} + \left(\vec{b} + \vec{c}\right).$$

$$3. \lambda\left(\vec{a} + \vec{b}\right) = \lambda\vec{a} + \lambda\vec{b}. \; 4. \vec{a} + \vec{0} = \vec{a}.$$

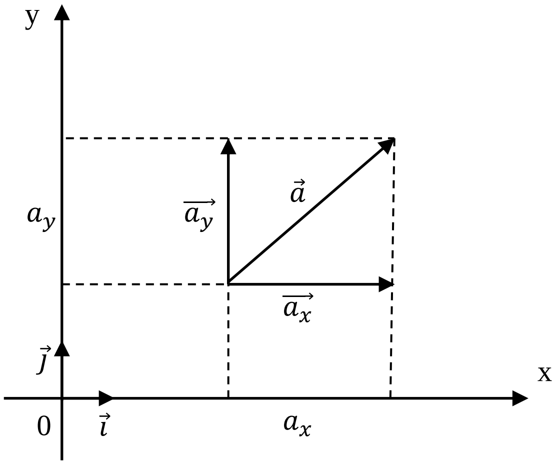
$\vec{a}$va$\overrightarrow{b}$ vektorlarning ayirmasi deb $\vec{a}$ va $-\vec{b}$ vektorlarning yig'indisiga aytiladi.

$\vec{a}$va$\overrightarrow{b}$ vektorlarning ayirmasi $\vec{a} - \vec{b}$ kabi belgilanadi va u bu vektorlardan hosil qilingan $ABCD$ parallelogrammning B uchidan chiquvchi $\overrightarrow{BD}$ dioganalidan iborat bo'ladi(6 – chizma).



6-chizma

Tekislikda $\vec{a}$ vektor hamda Ox va Oy koordinata o'qlarida musbat yo'nalishga ega va uzunliklari birga teng $\vec{\imath}$ va$\vec{\jmath}$ (ortlar) vektorlarni kiritamiz(7 – chizma).



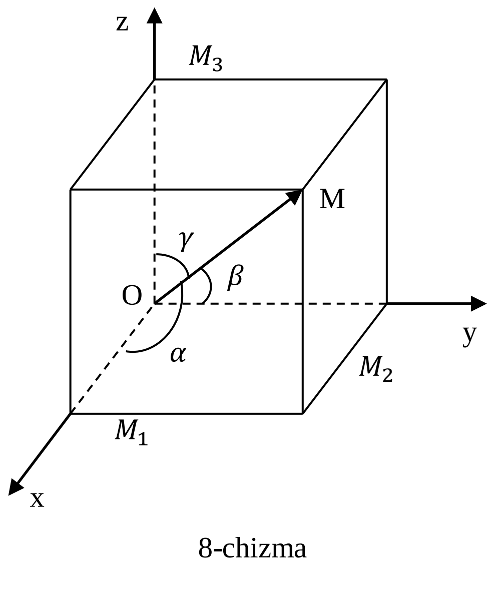
7-chizma

a_x va a_y vektorlar $\vec{a}$ vektorning Ox va Oy***o'qlardagi proyeksiyalari*** deyiladi. a_x va a_y proyeksiyalarni $a_x = \pm|a_x|\vec{i}$va $a_y = \pm|a_y|\vec{j}$ deb yozish mumkin. Bunda $\vec{a} = \overrightarrow{a_x} + \overrightarrow{a_y} = (\pm|a_x|)\vec{i} + (\pm|a_y|)\vec{j} = x\vec{i} + y\vec{j}$ (1)

(1) tenglik $\vec{a}$ vektorning ortlar bo'yicha ***yoyilmasi***,x va y sonlar esa uning koordinatalari deyiladi.

Fazoda to'g'ri burchakli Dekart koordinatalar sistemasi va unda M nuqtani qaraymiz. Bu nuqtaning radius vektori $\overrightarrow{OM} = r$ ning o'qlardagi $OM_1 = x, OM_2 =$ □ va $OM_3 = z$ proyeksiyalari nuqtaning yoki $\overrightarrow{OM} = r$ vektorning koordinatalari deyiladi(8- chizma).

Demak, $r\{x, y, z\}$ deb yozish mumkin.



8-chizma

$\overrightarrow{OM} = r$ radius vektorning moduli (uzunligi)

$$r = \sqrt{x^2 + y^2 + z^2} \quad (2)$$

formula bilan aniqlanadi. Koordinata oqlaridagi $\vec{\imath}, \vec{\jmath}, \vec{k}$ birlik vektorlar ***ortlar*** deyiladi. Radius-vektor ortlar orqali quyidagicha aniqlanadi.

$$r = x\vec{\imath} + y\vec{\jmath} + z\vec{k} \quad (3)$$

Boshi $A(x_1, y_1, z_1)$ va oxiri $B(x_2, y_2, z_2)$ nuqtaarda bo'lgan $u = \overrightarrow{AB}$ vektor koordinata o'qlaridagi proyeksiyalari bo'yicha quyidagicha ifodalanadi.

$$\vec{u} = \overrightarrow{AB}(x_2 - x_1; y_2 - y_1; z_2 - z_1) \quad (4)$$

Agar $\vec{u} = \overrightarrow{AB}$ vektor koordinata o'qlari bilan α, β va γ burchaklar tashkil etsa, u holda

$$cos\alpha = \frac{x}{r}, \qquad cos\beta = \frac{y}{r}, \qquad cos\gamma = \frac{z}{r} \qquad (5)$$

va shu bilan birga

$$cos^2\alpha + cos^2\beta + cos^2\gamma = 1 \qquad (6)$$

bo'ladi. Bu yerda $cos\alpha, cos\beta$ va $cos\gamma$ lar ***yo'naltiruvchi kosinuslar*** deyiladi.

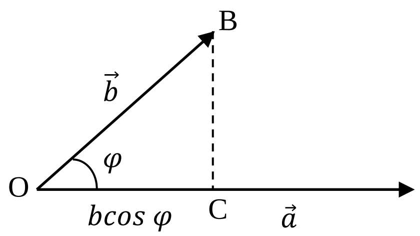
Ikki $\vec{a}$va$\vec{b}$ vektorlarning ***skalyar ko'paytmasi*** deb shu vektorlar uzunliklarining ular orasidagi burchak kosinusi bilan ko'paytmasiga aytiladi. Skalyar ko'paytma $\vec{a}\vec{b}$ kabi belgilanadi. Demak, ta'rifga asosan

$$\vec{a}\vec{b} = |\vec{a}||\vec{b}| \cdot cos\varphi \quad (7)$$

Skalyar ko'paytmaniboshqacha

$$\vec{a}\vec{b} = a \cdot np_a b = b \cdot np_b a \quad (8)$$

ko'rinishda ham yozish mumkin(9 – chizma).



9-chizma

Skalyar ko'paytma quyidagi xossalarga ega:

1. $\vec{a}\vec{b} = \vec{b}\vec{a}$. 2. $\vec{a}(\vec{b} + \vec{c}) = \vec{a}\vec{b} + \vec{a}\vec{c}$.

3. Agar $a \parallel b$ bo'lsa, $\vec{a}\vec{b} = \pm|\vec{a}| \cdot |\vec{b}|$.Xususiy holda $\vec{a}\vec{a} = |a|^2$.

4. $\lambda(\vec{a}\vec{b}) = (\lambda\vec{a}) \cdot \vec{b} = \vec{a}(\lambda\vec{b})$.   5. Agar $\vec{a} \perp \vec{b}$ bo'lsa, $\vec{a}\vec{b} = 0$.

6. $\vec{i} \cdot \vec{j} = 0, \quad \vec{j} \cdot \vec{k} = 0, \quad \vec{i} \cdot \vec{k} = 0, \quad \vec{i} \cdot \vec{i} = 1, \quad \vec{j} \cdot \vec{j} = 1, \quad \vec{k} \cdot \vec{k} = 1.$

($\vec{i}, \vec{j}, \vec{k}$ −birlik vektorlar va $|\vec{i}| = |\vec{j}| = |\vec{k}| = 1$).

7. Agar $\vec{a}\{a_x, a_y, a_z\}$ va $\vec{b}\{b_x, b_y, b_z\}$ bo'lsa, u holda

$$\vec{a}\vec{b} = a_x b_x + a_y b_y + a_z b_z \qquad (9)$$

Ikki $\vec{a}$va $\vec{b}$ vektor orasidagi burchak:

$$cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|a| \cdot |b|} = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} \cdot \sqrt{b_x^2 + b_y^2 + b_z^2}}$$

Ikki vektorning parallelik sharti:

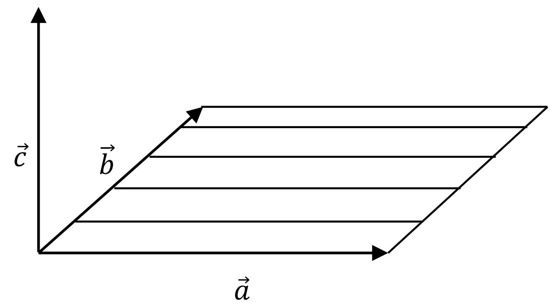
$$\vec{b} = m\vec{a} \text{ yoki } \frac{b_x}{a_x} = \frac{b_y}{a_y} = \frac{b_z}{a_z} = m \qquad (11)$$

Ikki vektorning perpendikulyarlik sharti:

$$\vec{a}\vec{b} = 0 \text{ yoki } a_{\square} b_x + a_y b_y + a_z b_z = 0 \qquad (12)$$

$\vec{a}$va $\vec{b}$ vektorlarning vektor ko'paytmasi deb quyidagi shartlarni qanoatlantiruvchi uchinchi $\vec{c}$ vektorga aytiladi:

1) $\vec{c}$ vektorning uzunligi $\vec{a}$va $\vec{b}$ vektorlarga qurilgan parallelogrammning yuziga teng bo'lib, $|\vec{c}| = |\vec{a}||\vec{a}| \cdot sin\varphi$ formula bilan topiladi (10 – chizma).



10-chizma

2) $\vec{c}$ vektor $\vec{a}$va $\vec{b}$ vektorlar yotgan parallelogramm tekisligiga perpendikulyar, ya'ni$\vec{c} \perp \vec{a}$, $\quad \vec{c} \perp \vec{b}$.

3) $\vec{c}$ vektor shunday yo'nalganki uning uchidan qaralganda $\vec{a}$ vektordan$\vec{b}$vektorga eng qisqa burilish soat mili harakatiga teskari bo'ladi.

Vektor ko'paytma $\vec{a} \times \vec{b}$ kabi yoziladi.

Vektor ko'paytma quyidagi xossalarga ega:

1. $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$. 2. $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

3. Agar $\vec{a} \parallel \vec{b}$ bo'lsa, $\vec{a} \times \vec{b} = 0$. Xususiy holda $\vec{a} \times \vec{a} = 0$.

$\vec{i}, \vec{j}, \vec{k}$ ortlarning vektorial ko'paytmalari quyidagicha:

$$\vec{i} \times \vec{j} = \vec{k}, \quad \vec{j} \times \vec{k} = \vec{i}, \quad \vec{i} \times \vec{k} = \vec{j}, \qquad \vec{j} \times \vec{k} = -\vec{i}, \quad \vec{k} \times \vec{j} = -\vec{i}.$$

Agar $\vec{a}\{a_x, a_y, a_z\}$ va $\vec{b}\{b_x, b_y, b_z\}$bo'lsa, u holda

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} \quad (13)$$

$\vec{a}$va$\vec{b}$ vektorlarda yasalgan parallelogrammning yuzi:

$$S = |\vec{a} \times \vec{b}|$$

$\vec{a}$va$\vec{b}$ vektorlarda yasalgan uchburchakning yuzi:

$$S = \frac{1}{2}|\vec{a} \times \vec{b}|$$

$\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlarning aralash ko'paytmasi deb dastlabki ikkita vektorlarning vektor ko'paytmasini uchinchi $\vec{c}$ vektorga skalyar ko'paytmasiga, ya'ni $[\vec{a} \times \vec{b}] \times \vec{c}$ ga aytiladi.

Aralash ko'paytma $\vec{a}\vec{b}\vec{c}$ kabi belgilanadi. Demak, $\vec{a}\vec{b}\vec{c} = [\vec{a} \times \vec{b}]\vec{c}$

Aralash ho'paytma quyidagi xossalarga ega:

1. Aralash ko'paytmaning istalgan ikkita ko'paytuvchisinung o'rinlari o'zaro almashtirilsa, ko'paytmaning ishorasi o'zgaradi:

$$(\vec{a} \times \vec{b})\vec{c} = -(\vec{a} \times \vec{c})\vec{b} = -(\vec{c} \times \vec{b})\vec{a}.$$

2. Aralash berilgan uchta vektordan ikkitasi o'zaro teng yoki parallel bo'lsa, aralash ko'paytma nolga teng bo'ladi.

3. Aralashko'paytmada vektorial va skalyar ko'paytma amallari o'rnini almashtirish mumkin, ya'ni $(\vec{a}\times\vec{b})\vec{c}=\vec{a}(\vec{b}\times\vec{c})$.

4. Aralash ko'paytmada ko'paytuvchilar o'rnini soat miliga teskari yo'nalish bo'yicha doiraviy ravishda almashtirilsa, uning qiymati o'zgarmaydi, ya'ni

$$\vec{a}\vec{b}\vec{c}=\vec{c}\vec{a}\vec{b}=\vec{b}\vec{c}\vec{a}=\vec{a}\vec{b}\vec{c}$$

Agar $\vec{a}\{a_x,a_y,a_z\}$, $\vec{b}\{b_x,b_y,b_z\}$ va $\vec{c}\{c_x,c,c_z\}$ bo'lsa,

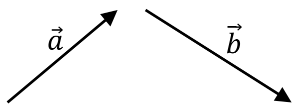
$$\vec{a}\vec{b}\vec{c}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} \quad (14)$$

$\vec{a},\vec{b}$va$\vec{c}$ vektorlarda yasalgan parallelepipedning hajmi:

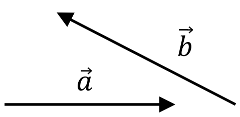
$$V=\left|\vec{a}\vec{b}\vec{c}\right|=\pm\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} \quad (15)$$

$\vec{a},\vec{b},\vec{c}$ vektorda yasalgan piramidaning hajmi:

$$V_{pir}=\pm\frac{1}{6}\vec{a}\vec{b}\vec{c}=\pm\frac{1}{6}\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix} \quad (16)$$

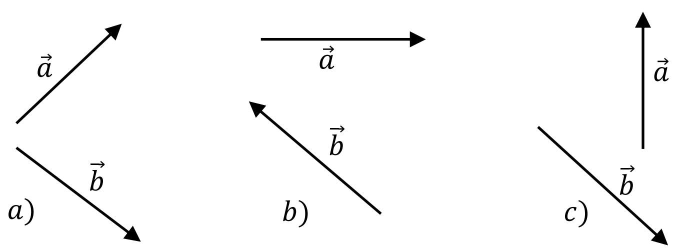
1. 

Chizmada berilgan $\vec{a}$va$\overrightarrow{b}$ vektorlar yig'indisi topilsin:

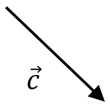
2. 

Chizmada berilgan $\vec{a}$va$\overrightarrow{b}$ vektorlar yig'indisi topilsin:

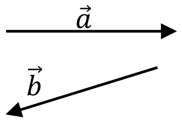
3. Chizmada berilgan vektorlar uchun:



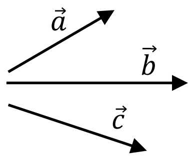
1) $\vec{a}-\vec{b}$;2) $\vec{b}-\vec{a}$; 3) $\vec{a}\cdot\vec{b}$ lar topilsin.

4. 

Berigan $\vec{c}$ vektor asosida $4\vec{c}$; -$4\vec{c}$; 1,5$\vec{c}$ vektorlar yasalsin.

5. 

Berilgan $\vec{a}$va $\overrightarrow{b}$ vektorlar bo’yicha $3\vec{a} + 2\vec{b}$ vektor yasalsin.

6. 

Chizmada berilgan vektorlarga asosan $\vec{d} = 2\vec{a} + 3\vec{b} - -5\vec{c}$ vektor yasalsin.

7. $A(3;2), B(-1;5)$ va $C(0;3)$ nuqtalar berilgan $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{BC}$ vektorlarning koordinatalari topilsin.

8. $\vec{a}\{3;5\}, \vec{b}\{2;-7\}$ bo’lsa, $\vec{a} + \vec{b}$; $\vec{a} - \vec{b}$; $4\vec{a}$; $-0{,}5\vec{b}$ lar topilsin.

9. $\overrightarrow{a_1}\{-2;4\}, \overrightarrow{a_2}\{3;1\}$ bo’lsa, $\overrightarrow{a_1} + \overrightarrow{a_2}$, $\overrightarrow{a_1} - \overrightarrow{a_2}$, $3\overrightarrow{a_1}$ va $5\overrightarrow{a_2}$ lar topilsin.

10. $A(4;0)$, $B(-1;3)$, $C(5;7)$lar berilgan $\overrightarrow{AC}$,$\overrightarrow{AB}$,$\overrightarrow{BC}$,$\overrightarrow{AB}$+$\overrightarrow{BC}$,$\overrightarrow{AB} - \overrightarrow{BC}$, $\vec{m} = -3\overrightarrow{AB} + 2\overrightarrow{BC} - 5\overrightarrow{AC}$ lar topilsin.

11. Quyidagi vektorlarning uzunliklari topilsin.

1) $\vec{a}\{5;2\sqrt{6}\}$; 2) $\vec{b}\{-5;7\}$; 3) $\vec{c}\{-6;8\}$; 4) $\vec{d}\{7;-7\}$.

12. $\overrightarrow{AB}$ vektorning uzunligi topilsin:1) $A(5;2)$,$B(8;-2)$;2) $A(3;5)$, $B(-3;3)$, 3) $A(6;8)$, $B(4;9)$;4) $A(2\sqrt{2};3\sqrt{2})$, $B(6\sqrt{2};8\sqrt{2})$.

13. $A(3;5)$, $B(-3;3)$, $C(5;-8)$ nuqtalar berilgan. $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{AC}$ vektorlarning uzunliklari topilsin.

14. $\vec{a}\{1;-3;-2\}$, $\vec{b}\{3;6;-1\}$ bo’lsa, $\vec{a} + \vec{b}$, $\vec{a} - \vec{b}$, $3\vec{a} + 5\vec{b}$ lar topilsin.

15. Boshi $A(3;5;7)$ va oxiri $B(2;3;-1)$ nuqtada bo’lgan $\overrightarrow{AB}$vektor berilgan. $3\overrightarrow{AB}$; $-0{,}5\overrightarrow{AB}$ lar topilsin.

16. $A(3;5;7)$, $B(-1;4;2)$, $C(0;-3;5)$, $D(6;-7;8)$ nuqtalar berilgan. $\overrightarrow{AB} + \overrightarrow{BC}$; $\overrightarrow{AC} - \overrightarrow{DC}$; $2\overrightarrow{AB}$; $-3\overrightarrow{CD}$; $3\overrightarrow{AB} + 2\overrightarrow{BC} - 4\overrightarrow{AD}$ lar topilsin.

17. $\vec{a}\{5;-3;\sqrt{2}\}$, $\vec{b}\{-2;3;1\}$, $\vec{c}\{0;12;5\}$, $\vec{d}\{-5;7;2\}$ vektorlarning uzunliklari topilsin.

18. Tomoni 6 ga teng bo’lgan ABC teng tomonli uchburchak berilgan: □) $\overrightarrow{AB}$ va $\overrightarrow{AC}$; $b)$ $\overrightarrow{AB}$ va $\overrightarrow{BC}$ vektorlarning skalyar ko’paytmalari topilsin. J: 18; -0,5.

19. Katetlari 5 ga teng bo’lgan teng yonli to’g’ri burchakli uchburchak berilgan. Agar $\angle C = 90°$ bo’lsa, $\overrightarrow{AC}$ va $\overrightarrow{AB}, \overrightarrow{CA}$ va $\overrightarrow{CB}$ vektorlarning skalyar ko’paytmasi topilsin. J: 25; 0.

20. Uzunliklari 2 va 7ga teng bo’lgan $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $30°$ ga teng. $\left(3\vec{a} + \vec{b}\right)\left(\vec{a} + 3\vec{b}\right)$ topilsin. J: $34 + 15\sqrt{2}$.

21. Uzunliklari 5 va 3 ga teng bo’lgan $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $45°$ ga teng. $(\vec{a} + \vec{b})^2$topilsin. J: 37; -6.

22. Uzunliklari 3 va 4 ga teng bo’lgan $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $60°$ ga teng. $(\vec{a} + \vec{b})^2, \left(3\vec{a} - 2\vec{b}\right) \cdot 2\vec{a}$topilsin.

23. Quyida berilgan$\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko’paytmasi topilsin:

1) $\vec{a}\{5; 7\}$, $\vec{b}\{4; 3\}$;2) $\vec{a}\{-3; 5\}$, $\vec{b}\{16; 1\}$;3) $\vec{a}\{2; 0\}$, $\vec{b}\{-3; -7\}$;

4) $\vec{a}\{-3; 1\}$, $\vec{b}\{1; -3\}$;5) $\vec{a}\{5; -7\}$, $\vec{b}\{7; 5\}$;6)$\vec{a}\{2; 0\}$, $\vec{b}\{0; -3\}$;

24. Quyida berilgan $\vec{a}$ va $\vec{b}$vektorlar orasidagi burchak topilsin:

1) $\vec{a}\{4; 0\}$, $\vec{b}\{2; -2\}$; 2) $\vec{a}\{6; -2\}$, $\vec{b}\{9; -12\}$;

3) $\vec{a}\{-2; 3\}$, $\vec{b}\{4; -1\}$; 4) $\vec{a}\{4; 0\}$, $\vec{b}\{2; -2\}$.

25. $\overrightarrow{AB}\{-3; 2; 6\}$ vektorning boshi $A(-1; 0; 4)$ nuqtada joylashgan. Uning oxiri bo’lgan B nuqtaning koordinatalari topilsin.

26. $\vec{a} = 2\vec{\imath} + \vec{\jmath}$ va $\vec{b} = -2\vec{\jmath} + \vec{k}$ vektorlarda yasalgan parallelogramm diagonallari orasidagi burchak topilsin. J: 90°.

27. $\vec{a} = \vec{\imath} + \vec{\jmath} + 2\vec{k}$ va $\vec{b} = \vec{\imath} - \vec{\jmath} + 4\vec{k}$ vektorlar berilgan. $np_b a$ va $np_a b$ lar topilsin.

28. 1) m va n o’zaro $30°$ burchak tashkil etuvchi birlik vektorlar bo’lsa, $(m + n)^2$ hisoblansin; 2) agar $a = 2\sqrt{2}$, $b = 4$ va ular orasidagi burchak $135°$ bo’lsa, $(a - b)^2$ hisoblansin. J: 1) $2 + \sqrt{3}$, 2) 40.

29. 1) $(\vec{a}+\vec{b})^2$; 2) $(\vec{a}+\vec{b})^2+\ (\vec{a}-\vec{b})^2$ifodalardagi qavslar ochilsin va hosil bo'lgan ifodalarning geometrik ma'nosi aniqlansin. J: $(\vec{a}+\vec{b})^2=\vec{a}^2+\vec{b}^2-2\vec{a}\vec{b}\cos\varphi$ -kosinuslar teoremasi, $(\vec{a}+\vec{b})^2\ +\ +(\vec{a}-\vec{b})^2$=$2a^2+2b^2$-parallelogramm dioganallarining xossasi.

30. m va n lar o'zaro $120°$ burchak tashkil etuvchi birlik vektorlar bo'lsa, $a=2m+4n,\ b=m-n$ vektorlar orasidagi burchak topilsin. J: 120°.

31. $\vec{a}=-6\vec{i}+3\sqrt{3}\vec{j}+\vec{k}$ vektor yo'nalishidagi birlik vektor koordinatalari topilsin. J: $\vec{a}_0=\left\{-\frac{3}{4};\frac{3\sqrt{3}}{8};\frac{1}{8}\right\}$.

32. $\vec{a}\{6;1;-4\},\vec{b}\{3;2;-1\}$ va $\vec{c}\{5;5;0\}$ bo'lsa, $2\overrightarrow{a^2}-3\vec{a}\vec{b}+4|\vec{c}|^2$ ifodaning qiymati topilsin. J: 234.

33. $\vec{a}\{1;-3;z\}$ va $\vec{b}\{5;4;-3\}$ vektorlarning skalyar ko'paytmasi 6 ga teng bo'lsa, z ni toping. J: $-\frac{13}{3}$.

34. Quyidagi vektorlar orasidagi burchakni hisoblang:

1) $\vec{a}\{1;-4;3\}$ va $\vec{b}\{-3;-1;4\}$. J: 60°.

1) $\vec{p}\{1;4;-2\}$ va $\vec{q}\{2;-3;5\}$. J: $\cos\alpha=-\frac{20}{\sqrt{798}}$.

2) $\vec{a}=2\vec{i}-\vec{j}+\vec{k}$ va $\vec{b}=\vec{i}+\vec{j}+2\vec{k}$. J: 60°.

35. Uchlari $A(-9;-3;0),\ B(-4;2;1),\ C(-2;8;-1)$ nuqtalarda bo'lgan uchburchakning BC tomoni bilan AD medianasi orasidagi burchakni toping.J: $\cos\alpha=\frac{3}{\sqrt{11}}$.

36. $ABCDEF$ tomoni 2 ga teng bo'lgan muntazam oltiburchak $\overrightarrow{AB}$ va $\overrightarrow{CD}$ vektorlarning skalyar ko'paytmasi topilsin. J: -2

37. $\vec{p}=3\vec{a}+2\vec{b}$ va $\vec{q}=\vec{a}+5\vec{b}$ vektorlar orasidagi burchak topilsin.Bu yerda $\vec{a}$ va $\vec{b}$ ozaro perpendikulyar birlik vektorlar. J: $\frac{\pi}{4}$.

38. $\vec{a}\{4;-2;-4\}$ va $\vec{b}\{6;-3;2\}$ vektorlar berilgan. $2\vec{a}-3\vec{b}$ va $\vec{a}+2\vec{b}$ vektorlarning skalyar ko'paytmasi topilsin. J: -200.

39. $\vec{a}\{2;-3;\alpha\}$ va $\vec{b}\{\beta;1;2\}$ vektorlar α va β ning qanday qiymatlarida kolleniyar bo’ladi? J: $\alpha=-6,\beta=\frac{2}{3}$.

40. Uchlari $A(1;-2;8),B(0;0;4)$ va □$(6;2;0)$ nuqtalarda bo’lgan uchburchak yasalsin. Uning yuzi va BD balandligi hisoblansin. J: $7\sqrt{5}$ kv.b, $BD=\frac{2\sqrt{21}}{3}$.

41. $\overrightarrow{AB}=\vec{m}+2\vec{n}$ va $\overrightarrow{AD}=\vec{m}-3\vec{n}$ vektorlarda yasalgan parallelogrammning yuzi topilsin. Bu yerda $|\vec{m}|=5,\ |\vec{n}|=3$ va$\vec{m}$^$\vec{n}=\frac{\pi}{6}$. J: $\frac{75}{2}$.

42. $\vec{a}=\vec{\imath}-2\vec{\jmath}+5\vec{k}$ va $\vec{b}=5\vec{\jmath}-7\vec{k}$ vektorlardan yasalgan uchburchakning yuzi topilsin. J: $S=\frac{\sqrt{195}}{2}$.

43. $\vec{a}=\vec{\imath}-\vec{\jmath},\ \vec{b}=\vec{\imath}+\vec{k}$ va $\vec{c}=\vec{\jmath}-\vec{k}$ vektorlar berilgan. $\vec{a}\times(\vec{b}\times\vec{c})$ topilsin. J: $-\vec{\imath}-\vec{\jmath}$.

44. $|\vec{a}|=4,\ |\vec{b}|=2,\ (\vec{a}$^$\vec{b})=\frac{\pi}{6}$bo’lsa, $|a\times b|$ ni toping. J: 4

45. Quyidagi vektorlar vektor ko’paytmasining koordinatalarini va modulini toping:

1) 1) $\vec{a}=\vec{\imath}+2\vec{\jmath}-\vec{k}$ va $\vec{b}\{1;0;5\}$. J: $\vec{a}\times\vec{b}=\vec{P}_1\{10;\ -6;\ -2\},|P_1|=\sqrt{140}$.

2) $\vec{a}=3\vec{\imath}-2\vec{\jmath}+\vec{k}$ va $\vec{b}=\vec{\jmath}+\vec{k}$. J: $\vec{a}\times\vec{b}$=$\vec{P}_2${-3;-3;3}, $|P_2|=3\sqrt{3}$.

46. Agar 1) $\vec{a}=3\vec{\imath}$ va $\vec{b}=2\vec{k}$; 2) $\vec{a}=\vec{\imath}+\vec{\jmath}$ va $\vec{b}=\vec{\imath}-\vec{\jmath}$; 3) $\vec{a}=2\vec{\imath}++3\vec{\jmath}$ va $\vec{b}=3\vec{\jmath}+2\vec{k}$ bo’lsa, $\vec{c}=\vec{a}\times\vec{b}$ vektor aniqlansin va yasalsin. Har bir hol uchun berilgan vektorlarda yasalgan parallelogrammning yuzi topilsin. J: 1) $-6\vec{\jmath},\ S_1=6$; 2) $-2\vec{k},\ S_2=6$;3) $6\vec{\imath}-4\vec{\jmath}+6\vec{k},\ S_3=2\sqrt{22}$.

47. Uchlari $A(7;3;4),B(1;0;6)$ va $C(4;5;-2)$ nuqtalarda bo’lgan uchburchakning yuzi topilsin. J: 24,5.

48. $\vec{a}=2\vec{\jmath}+\vec{k}$ va $\vec{b}=\vec{\imath}+2\vec{k}$ vektorlarda parallelogramm yasalsin hamda uning yuzi va balandligi aniqlansin. J: $\sqrt{21}$ kv.b, $h=\sqrt{4{,}2}$.

49. Ushbu ifodalar soddalashtirilsin.

1) $\vec{\imath}\times(\vec{\jmath}+\vec{k})-\vec{\jmath}\times(\vec{\imath}+\vec{k})+\vec{k}\times(\vec{\imath}+\vec{\jmath}+\vec{k})$. J: $2(\vec{k}-\vec{\imath})$.

2) $(\vec{a}+\vec{b}+\vec{c})\times\vec{c}+(\vec{a}+\vec{b}+\vec{c})\times\vec{b}+(\vec{b}-\vec{c})\times\vec{a}$. J: $2\vec{a}\times\vec{c}$.

3) $(2\vec{a}+\vec{b})\times(\vec{c}-\vec{a})+(\vec{b}+\vec{c})\times(\vec{a}+\vec{b})$. J: $\vec{a}\times\vec{c}$.

4) $2\vec{i}\cdot(\vec{j}\times\vec{k})+3\vec{j}\cdot(\vec{i}\times\vec{k})+4\vec{k}\cdot(\vec{i}\times\vec{j})$. J: 3.

50. $\vec{a}=3\vec{k}-2\vec{j}$, $\vec{b}=3\vec{i}-2\vec{j}$ va $\vec{c}=\vec{a}\times\vec{b}$ vektorlar yasalsin. $\vec{c}$ vektorning moduli hamda $\vec{a}$ va $\vec{b}$ vektorlarda yasalgan uchburchak yuzi hisoblansin. J: $3\sqrt{17}$, $S=\frac{3\sqrt{17}}{2}$ kv.b.

51. $\vec{a}=3\vec{i}+4\vec{j}, \vec{b}=-3\vec{j}+\vec{k}$ va $\vec{c}=2\vec{j}+5\vec{k}$ vektorlarda parallelepiped yasalsin hamda uning hajmi hisoblansin. J:$V=51$ kub b.

52. Uchlari $O(0;0;0), A(5;2;0), B(2;5;0)$ va $C(1;2;4)$ nuqtalarda bo'lgan piramida yasalsin hamda uning hajmi, ABCyog'ining yuzi va shu yoqqa tushirilgan balandligi topilsin. J: $V=14$, $H=\frac{7\sqrt{3}}{3}$.

53. $A(2;-1;-2)$, $B(1;2;1)$, $C(2;3;0)$ va $D(5;0;-6)$ nuqtalarning bir tekislikda yotishi ko'rsatilsin.

54. $\vec{a}=-\vec{i}+3\vec{j}+2\vec{k}$, $\vec{b}=2\vec{i}-3\vec{j}-4\vec{k}$ va $\vec{c}=-3\vec{i}+12\vec{j}+6\vec{k}$vektorlarning o'zaro komplanar ekanligi ko'rsatilsin.

55. Uchlari $A(2;0;0), B(0;3;0), C(0;0;6)$ va $D(2;3;8)$ nuqtalarda bo'lgan piramida yasalsin hamda uning hajmi va ABC yog'iga tushurilgan balandligi hisoblansin. J: $V=14, H=\sqrt{14}$.

56. $\vec{a}=\vec{i}+\vec{j}+4\vec{k}$, $\vec{b}=\vec{i}-2\vec{j}$ va $\vec{c}=3\vec{i}-3\vec{j}+4\vec{k}$ vektorlarning o'zaro komplanar ekanligi isbotlansin.

57. Quyidagi vektorlarning aralash ko'paytmasi topilsin.

1) $\vec{a}\{1;0;3\}$, $\vec{b}\{1;-3;4\}$, $\vec{c}\{-2;1;0\}$. J: -19.

2) $\vec{a}\{5;-1;0\}$, $\vec{b}\{-2;3;1\}$, $\vec{c}\{1;0;3\}$. J: 40.

58. $\vec{a}\{-2;1;5\}$, $\vec{b}\{3;0;2\}$ va $\vec{c}\{x;4;2\}$ vektorlarning aralash ko'paytmasi 68 ga teng bo'lsa, x ning qiymati topilsin. J: -1.

59. $\vec{a}\{4;-34;-3\}$, $\vec{b}\{3;-6;b_3\}$ va $\vec{c}\{4;-4;2\}$ vektorlar b_3 ning qanday qiymatida komplanar bo'ladi?

60. $A(1;-2;0), B(3;-1;5), C(0;1;1), D(2;1;5)$ nuqtalarning bir tekislikda yotishini isbotlang.

<u>Ko’rsatma:</u>$\overrightarrow{AB}\overrightarrow{AC}\overrightarrow{AD} = 0$ ekanligi ko’rsatiladi.

61. Qirralari quyidagi vektorlardan iborat bo’lgan parallelepiped hajmini toping:

1) $\overrightarrow{a_1}\{5;3;-2\}$, $\overrightarrow{b_1}\{1;-4;2\}$, $\overrightarrow{c_1}\{3;1;4\}$ J: $V = 110$.

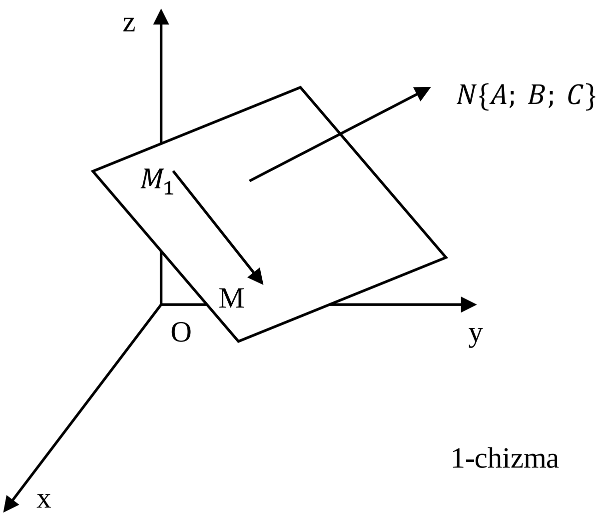
2) $\overrightarrow{a_2}\{4;3;-1\}$, $\overrightarrow{b_2}\{2;1;2\}$, $\overrightarrow{c_2}\{-3;-2;5\}$ J: $V = 11$.

62. Uchlari $O(0;0;0)$, $A(1;0;0)$, $B(0;1;0)$, $C(0;0;1)$ nuqtalarda bo’lgan tetraedr hajmi va OH balandligi topilsin. J: $V = \frac{1}{6}, OH = \frac{\sqrt{3}}{3}$.

63. Uchlari $A(2;0;0), B(0;3;0), C(0;0;6)$ va $D(2;3;8)$ nuqtalarda bo’lgan piramida yasalsin, hamda uning hajmi va ABC yog’iga tushirilgan balandligi hisoblansin.

10-§. Fazoda tekislik va uning tenglamasi

$M_1(x_1, y_1, z_1)$ nuqtadan o’tuvchi va $N(A, B, C)$ vektorga perpendikulyar bo’lgan tekislik tenglamasi quyidagicha(1-chizma):



1-chizma

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0 \quad (1).$$

Agar bu tenglamadagi qavslarni ochib va $-Ax_1 - By_1 - Cz_1 = D$ belgilash qilsak, u holda tekislikning umumiy tenglamasi deb ataluvchi quyidagi tenglama hosil qilinadi:

$$Ax + By + Cz + D = 0 \quad (2)$$

Bu yerda $A^2 + B^2 + C^2 \neq 0$ (3) shart bajarilishi kerak.

A, B, C, D larning qiymatlariga qarab tekislik turlicha holatda bo’lishi mumkin:

1. $D = 0$ bo’lsa, $Ax + By + Cz = 0$ bo’lib, bu tenglama bilan aniqlangan tekislik koordinata boshidan o’tadi.

2. $A = 0$ bo’lsa, $By + Cz + D = 0$ bo’lib,bu holda tekislik OX o’qiga parallel bo’ladi.

3. $B = 0$ bo’lsa, $Ax + Cz + D = 0$ bo’lib, bu holda tekislik OY o’qiga parallel bo’ladi.

4. $C = 0$ bo’lsa, $Ax + By + D = 0$ bo’lib, bu holda tekislik OZ o’qiga parallel bo’ladi.

5. $A = 0, D = 0$ bo’lsa, $By + Cz = 0$ bo’lib, bu holda tekislik OX o’qidan o’tadi.

6. $B = 0, D = 0$ bo’lsa, $Ax + Cz = 0$ bo’lib, bu holda tekislik OY o’qidan o’tadi.

7. $C = 0, D = 0$ bo’lsa, $Ax + By = 0$ bo’lib, bu holda tekislik OZ o’qidan o’tadi.

8. $A = 0, B = 0$ bo’lsa, $Cz + D = 0$ bo’lib, bu holda tekislik XOY tekisligiga parallel bo’ladi.

9. $A = 0, C = 0$ bo’lsa, $By + D = 0$bo’lib, bu holda tekislik XOZ tekisligiga parallel bo’ladi.

10. $B = 0,\ C = 0$ bo’lsa, $Ax + D = 0$ bo’lib, bu holda tekislik YOZ tekisligiga parallel bo’ladi.

11. $A = 0,\ B = 0,\ D = 0$ bo’lsa, $Cz = 0$ $(\square = 0)$ bo’lib, bu holda tekislik XOY tekisligi bilan ustma-ust tushadi.

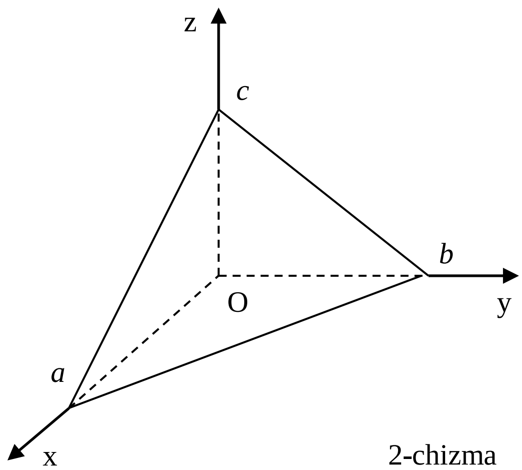
12. $B = 0,\ C = 0,\ D = 0$ bo’lsa, $Ax = 0$ $(x = 0)$ bo’lib, bu holda tekislik YOZ tekisligi bilan ustma-ust tushadi.

13. $A = 0,\ C = 0,\ D = 0$ bo’lsa, $By = 0$ $(y = 0)$ bo’lib, bu holda tekislik XOZ tekisligi bilan ustma-ust tushadi.

Agar (2)umumiy tenglamada $A \neq 0,\ B \neq 0,\ C \neq 0,\ D \neq 0$ bo'lsa, u holda (2)tenglamani

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \quad (3)$$

ko'rinishga keltirish mumkin. Bu yerda, $a = -\frac{A}{D},\ b = -\frac{B}{D},\ c = -\frac{C}{D}$. (3) tenglamani tekislikning kesmalar bo'yicha tenglamasi deyiladi
(2-chizma).



2-chizma

Agar $A_1x + B_1y + C_1z + D_1 = 0$ va $A_2x + B_2y + C_2z + D_2 = 0$ tekisliklar berilgan bo'lib, ular uchun

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \quad (4)$$

bo'lsa, tekisliklar parallel

$$A_1A_2 + B_1B_2 + C_1C_2 = 0 \quad (5)$$

bo'lsa, tekisliklar perpendikulyar bo'ladi.

Ikki tekislik orasidagi burchak

$$cos\varphi = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}} \quad (6)$$

formuladan topiladi.

$M_0(x_0, y_0, z_0)$ nuqtadan $Ax + By + Cz + D = 0$ tekislikkacha bo'lgan masofa

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} \quad (7)$$

formuladan topiladi.

Berilgan uchta $M_1(x_1, y_1, z_1)$, $M_2(x_2, y_2, z_2)$ va $M_3(x_3, y_3, z_3)$ nuqtalardan o'tuvchi tekislik tenglamasi

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \qquad (8)$$

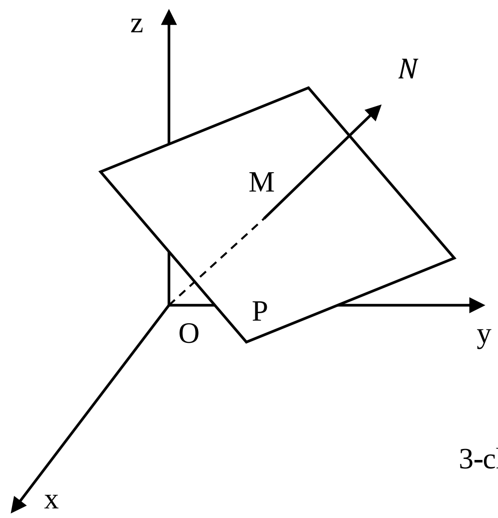
formula yordamida tuziladi.

$xcos\alpha + ycos\beta + zcos\gamma - p = 0$ (9) tenglamaga tekislikning normal tenglamasi deyiladi. Bu yerda α, β, γ lar tekislikka normal vektorning koordinata o'qlari bilan hosil qilgan burchaklari p normal vektorning tekislikkacha bo'lgan qismi uzunligi (3-chizma).

Agar tekislik tenglamasi normal bo'lmasa, u holda tenglamani normallovchi ko'paytuvchi deb ataluvchi

$$\mu = \pm \frac{1}{\sqrt{A^2 + B^2 + C^2}}$$

ga ko'paytiriladi.



3-chizma

1. Quyidagi tekisliklar yasalsin:

1) $5x - 2y + 3z - 10 = 0$; 2) $2x + y - z + 6 = 0$;

3) $3x + 2y - z = 0$; 4) $3x + 2z = 0$;5) $2z - 7 = 0$;

6) $2x - 5 = 0$; 7) $x + z = 1$; 8) $x + z = 1$.

2. $2x+3y+6z-12=0$ tekislik yasalsin va unga normal vektorning koordinata o'qlari bilan tashkil etgan burchaklari topilsin. J: $\cos\alpha=\frac{2}{7},\ \cos\beta=\frac{3}{7}$, $\cos\gamma=\frac{6}{7}$.

3. $M_1(0;-1;3)$ va $M_2(1;3;5)$ nuqtalar berilgan. M_1 nuqtadan o'tuvchi va $N=\overrightarrow{M_1M_2}$ vektorga perpendikulyar tekislik tenglamasi yozilsin.

J: $x+4y-2z-2=0$.

4. $M(a;a;0)$ nuqtadan o'tuvchi va $\overrightarrow{OM}$ vektorga perpendikulyar tekislik tenglamasi yozilsin. J: $x+y-2a=0$

5. $2x-2y+z-6=0$ tekislik yasalsin va unga normal vektorning koordinata o'qlari bilan tashkil etgan burchaklar topilsin. J: $\cos\alpha=\frac{2}{3},\ \cos\beta=-\frac{2}{3}$, $\cos\gamma=\frac{1}{3}$.

6. $M(2;-1;3)$ nuqtadan o'tuvchi va koordinata o'qlaridan teng kesmalar ajratuvchi tekislik tenglamasi yozilsin. J: $x+y+z-4=0$.

7. $M_1(-4;0;4)$ nuqtadan o'tib, OX va OY o'qlardan a=4 va b=3 kesmalar ajratuvchi tekislikning tenglamasi yozilsin.

J: $3x+4y+6z+2=0$.

8. $M(1;-3;5)$ nuqtadan o'tib, OY va OZ o'qlardan OXo'qdagidan ko'ra ikki marta katta kesma ajratuvchi tekislik tenglamasi tuzilsin.

J: $2x+y+z-4$=0.

9. Quyida berilgan tekisliklar orasidagi burchaklar topilsin:

1) $x-2y+2z-8=0$ va $x+z-6=0$. J: 45°.

2) $x+2z-6=0$ va $x+2y-4=0$. J: 78°30'.

3) $x+y-3=0$ va $2x-2z+1=0$. J: 60°.

4) $2x-y+3z=0$ va $x+4y-6z=0$. J: $\cos\varphi=-\frac{10\sqrt{722}}{371}$.

10. $(5;1;-1)$ nuqtadan $x-2y-2z+4=0$ tekislikkacha bo'lgan masofa topilsin. J: 3.

11. $M_1(1;-1;2)$, $M_2(2;1;2)$ va $M_3(1;1;4)$ nuqtalardan o'tuvchi tekislikning tenglamasi tuzilsin. J: $2x-y+z-5=0$.

12. $4x + 3y - 5z - 8 = 0$ va $4x + 3y - 5z + 12 = 0$ parallel tekisliklar orasidagi masofa topilsin. J: $2\sqrt{2}$.

13. $(2; 2; -2)$ nuqtadan o'tuvchi va $x - 2y - 3z = 0$ tekislikka parallel bo'lgan tekislik tenglamasi tuzilsin. J: $x - 2y - 3z - 4 = 0$.

14. $2x - y + z - 4 = 0, x + y - z - 2 = 0, 2x - y + 3z - 6 = 0$ tekisliklarning bir nuqtada kesishishini ko'rsating. J: $M_0(2; 2; 1)$.

15. 1) $M_0(3; -2; 5)$ nuqtadan o'tib, $2x - y - z + 3 = 0$ tekislikka;

2) koordinatalar boshidan o'tib, $x - y + 3z - 5 = 0$ tekislikka;

3) $M_0(1; -1; 3)$ nuqtadan o'tib, $2x - y + z + 5 = 0$ tekislikka parallel bo'lgan tekisliklarning tenglamalari tuzilsin. J: 1) $2x - y - z - 3 = 0$; 2) $x - y + 3z = 0$

16.$x - y = 0, x + y - 2z + 1 = 0, 2x + z - 4 = 0$ tekisliklarning kesishgan nuqtasi hamda $M(2; 1; 7)$ va $O(0; 0; 0)$ nuqtalardan o'tgan tekislik tenglamasi tuzilsin. J: $39x - 29y - 7z = 0$.

17. Koordinata boshidan tekislikka tushirilgan perpendikulyarning asosi $M(-1; 2; -3)$ nuqtada. Shu tekislik tenglamasi tuzilsin. J: $x - 2y + 3z + 14 = 0$

18. $M(1; 2; -1)$ nuqtadan o'tuvchi va $N\{1; 1; 2\}$ vektorga perpendikulyar bo'lgan tekislik tenglamasi tuzilsin. J: $x+y+2z-1=0$.

19. $4x + 3y - 5z - 8 = 0$ va $4x + 3y - 5z - 12 = 0$ parallel tekisliklar orasidagi masofa topilsin.

20. $kx - 2y + 5z + 10 = 0$ va $6x - (1 + k)y + 10z - 2 = 0$ tekisliklar k ning qanday qiymatida parallel bo'ladi.

21. $\frac{2}{3}x - \frac{1}{3}y + \frac{1}{3}z - 3 = 0$ tenglama normal tenglama ko'rinishiga keltirilsin. J: $\frac{2}{\sqrt{6}}x - \frac{1}{\sqrt{6}}y + \frac{1}{\sqrt{6}}z - \frac{9}{\sqrt{6}} = 0$.

22. $x - 4y - 8z + 5 = 0$ tekislikdan 4 birlik masofada yotuvchi va unga parallel bo'lgan tekislik tenglamasi tuzilsin.

J: $x - 4y - 8z - 31 = 0$, $x - 4y - 8z + 41 = 0$.

23. $6x - 3y + 2z - 14 = 0$ tekislikdan 3 birlik masofada yotuvchi nuqtalar to'plamining tenglamasi tuzilsin. J: $6x - 3y + 2z - 35=0$, $6x - 3y + 2z + 7=0$

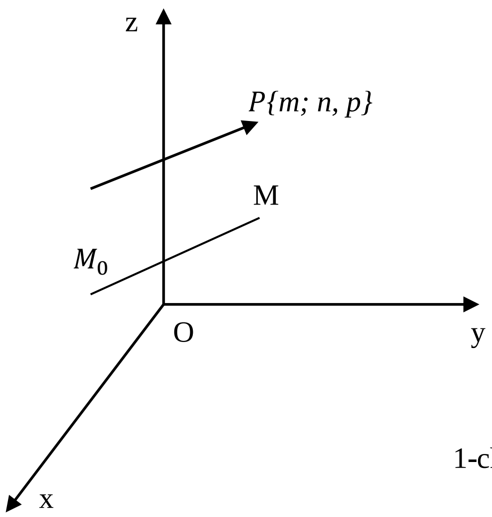
24. $2x - y + 3z - 9 = 0, x + 2y + 2z - 3 = 0$ va $3x + y - 4z + +6 = 0$ tekisliklarning kesishgan nuqtasi topilsin. J: $(1;\ -1; 2)$.

11-§. Fazoda to'g'ri chiziq va uning tenglamasi

Fazodagi $M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi va $P\{m, n, p\}$ vektorga parallel bo'lgan to'g'ri chiziq tenglamasi

$$\frac{x - x_0}{m} = \frac{y - y_0}{n} = \frac{z - z_0}{p} \qquad (1)$$

dan iborat. Bu tenglamani fazodagi to'g'ri chiziqning ***kanonik tenglamasi*** deyiladi. $P\{m, n, p\}$ vektor to'g'ri chiziqning ***yo'naltiruvchi vektori***deyiladi(1-chizma).



1-chizma

(1)tenglamadagi har bir nisbatni biror t parametrga tenglab, ulardan $x,\ y$va z larni topib quyidagi tengliklarni hosil qilamiz:

$$\begin{cases} x = mt + x_0 \\ y = nt + y_0 \\ z = pt + z_0 \end{cases} \qquad (2)$$

Bu tenglamaga to'g'ri chiziqning ***parametrik tenglamasi*** deyiladi.

Fazodagi ikkita $M_1(x_1, y_1, z_1)$ va $M_2(x_2, y_2, z_2)$ nuqtalar orqali o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \qquad (3)$$

formula yordamida tuziladi.

Fazodagi to'g'ri chiziqni ikkita tekislikning kesishish chizig'i (agar ular parallel bo'lmasa) deb qarash mumkin. Bu to'g'ri chiziqni ushbu

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0 \\ A_2x + B_2y + C_2z + D_2 = 0 \end{cases} \qquad (4)$$

sistemaning yechimlari to'plamidan iborat deb qarash mumkin.

(4) sistema bilan aniqlangan tenglamaga fazodagi to'g'ri chiziqning ***umumiy tenglamasi*** deyiladi.

$\frac{x-x_1}{m_1} = \frac{y-y_1}{n_1} = \frac{z-z_1}{p_1}$ va $\frac{x-x_2}{m_2} = \frac{y-y_2}{n_2} = \frac{z-z_2}{p_2}$

tenglamalar bilan berilgan fazodagi to'g'ri chiziqlar orasidagi burchak

$$cos\varphi = \frac{m_1m_2 + n_1n_2 + p_1p_2}{\sqrt{m_1^2 + n_1^2 + p_1^2} \cdot \sqrt{m_2^2 + n_2^2 + p_2^2}} \qquad (5)$$

formuladan topiladi.

Agar to'g'ri chiziqlar o'zaro perpendikulyar bo'lsa, u holda

$$m_1m_2 + n_1n_2 + p_1p_2 = 0 \quad (6)$$

bo'ladi.

Agar to'g'ri chiziqlar parallel bo'lsa, u holda

$$\frac{m_1}{m_2} = \frac{n_1}{n_2} = \frac{p_1}{p_2} \qquad (7)$$

bo'ladi.

1. $M_0(-2; 1; -1)$ nuqtadan o'tuvchi $\vec{N}\{1; -1; 2\}$ vektorga parallel bo'lgan to'g'ri chiziqning kanonik va parametrik tenglamasi tuzilsin.

2. $M_0(-1; 3; 1)$ nuqtadan o'tib, $\vec{U}\{3; 1; -2\}$ vektorga parallel bo'lgan to'g'ri chiziqning kanonik va parametrik tenglamasi tuzilsin.

J: $\frac{x+1}{3} = \frac{y-3}{-1} = \frac{z-1}{-2}$; $\begin{cases} x = 3t - 1 \\ y = -t + 3 \\ z = -2t + 1 \end{cases}$.

3. $A(4; 3; 0)$ nuqtadan o'tuvchi va $P\{-1; 1; 1\}$ vektorga parallel bo'lgan to'g'ri chiziq tenglamasi tuzilsin. J: $\frac{x-4}{-1} = \frac{y-3}{1} = \frac{z}{1}$.

4. Quyidagi berilgan nuqtalardan o'tuvchi to'g'ri chiziq tenglamalari tuzilsin.

1) $A(-1; 2; 3)$ va $B(2; 6; -2)$;

2) $A(2;-1;3)$ va $B(2;3;3)$;

3) $A(3;4;5)$ va $B(-6;-2;-10)$.

5. Quyida berilgan to'g'ri chiziq tenglamalarini: 1) proyeksiyalar bo'yicha; 2) kanonik ko'rinishda yozilsin:

1) $\begin{cases} x+2y+3z-13=0 \\ 3x+y+4z-14=0 \end{cases}$; 2) $\begin{cases} 2x+y+8z-16=0 \\ x-2y-z+2=0 \end{cases}$;

3) $\begin{cases} x-y+2z+4=0 \\ 3x+y-5z-8=0 \end{cases}$.

6. $M(-2;1;-1)$ nuqtadan o'tuvchi va $P\{1;-2;3\}$ vektorga parallel bo'lgan to'g'ri chiziq tenglamasi yozilsin.

7. Fazodagi to'g'ri chiziq quyidagi umumiy tenglamalar bilan berilgan. Ularning kanonik tenglamalari yozilsin.

1) $\begin{cases} x-y+2z+4=0 \\ 3x+y-5z-8=0 \end{cases}$; 2) $\begin{cases} x-2y+3z-4=0 \\ 3x+2y-5z-4=0 \end{cases}$;

3) $\begin{cases} x+2y+z-1=0 \\ x+y+1=0 \end{cases}$.

8. Quyidagi nuqtalardan o'tuvchi to'g'ri chiziqlarning tenglamalari yozilsin:

1) $M_1(-3;5;1)$ va $M_2(1;0;-2)$; 2) $M_1(4;6;8)$ va $M_2(2;4;6)$;

3)$M_1(-2;-4;-6)$ va $M_2(2;4;6)$; 4) $M_1(3;5;7)$ va $M_2(5;7;9)$.

9. Quyida berilgan to'g'ri chiziqlar orasidagi burchak topilsin:

1) $\begin{cases} x-y+z-4=0 \\ 2x+y-2z+5=0 \end{cases}$ va $\begin{cases} x+y+z-4=0 \\ 2x+3y-z-6=0 \end{cases}$

2) $\begin{cases} 2x-y+7=0 \\ 2x-z+5=0 \end{cases}$ va $\begin{cases} 3x-2y+8=0 \\ z=3x \end{cases}$.

10. Uchlari $M_1(3;6;-7)$, $M_2(-5;2;3)$ va $M_3(4;-7;-2)$ nuqtalarda bo'lgan uchburchakning M_3 nuqtasigan o'tkazilgan medianasining parametrik tenglamasi tuzilsin. J: $\begin{cases} x=5t+4 \\ y=-11t-7 \\ z=-2 \end{cases}$.

11. $\begin{cases} x-y-4z-5=0 \\ 2x+y-2z-4=0 \end{cases}$ va $\begin{cases} x-6y-6z+2=0 \\ 2x+2y+9z-1=0 \end{cases}$ to'g'ri chiziqlar orasidagi φ burchak kosinusi topilsin. J: $\cos\alpha = \pm\frac{4}{21}$.

12. 1) $\begin{cases} x = z + 5 \\ y = 4 - 2z \end{cases}$ va 2) $\frac{x-3}{1} = \frac{y-2}{2} = \frac{z-3}{1}$ to'g'ri chiziqlarning XOY va XOZ tekisliklardagi izlari topilsin.

<u>Ko'rsatma</u>: To'g'ri chiziqning teglamalarida 1) $z = 0$; 2) $y = 0$ deb faraz qilish kerak. J: 1) (5; 4; 0) va (7; 0; 2) 2) (0; -4; 0) va (2; 0; 2).

13. $\frac{x}{2} = \frac{y}{3} = \frac{z}{1}$ to'g'ri chiziqning $x = z + 1, y = 1 - z$ to'g'ri chiziqqa perpendikulyar ekanligi ko'rsatilsin.

14. $(-4; 3; 0)$ nuqtadan o'tuvchi va $\begin{cases} x - 2y + z = 4 \\ 2x + y - z = 0 \end{cases}$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziqtenglamasi yozilsin. J: $\frac{x+4}{1} = \frac{y-3}{3} = \frac{z-0}{5}$.

15. $M_0(1; -3; 4)$ nuqtadan o'tib, $\begin{cases} 2x - y + z - 3 = 0 \\ x + 3y - z - 1 = 0 \end{cases}$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziqning parametrik tenglamasi tuzilsin.

J: $\begin{cases} x = 1 - 2t \\ y = 3t - 3 \\ z = 7t + 4 \end{cases}$.

16. $2x - y + z + 1 = 0$ tekislik bilan $M_1(3; 2; 0)$, $M_2(1; -1; 1)$ va $M_3(1; -3; 2)$ nuqtalardan o'tuvchi tekislikning kesishishidan hosil bo'lgan to'g'ri chiziq tenglamasi yozilsin. J: $\begin{cases} 2x - y + z + 1 = 0 \\ x - 2y - 4z + 1 = 0 \end{cases}$.

12-§. Fazodagi to'g'ri chiziq va tekislik

Fazodagi $\frac{x-x_0}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p}$ to'g'ri chiziq va $Ax+By+Cz+D=0$ tekislik orasidagi burchak quyidagi formuladan topiladi:

$$sin\alpha = \frac{Am + Bn + Cp}{\sqrt{A^2 + B^2 + C^2} \cdot \sqrt{m^2 + n^2 + p^2}} \qquad (1)$$

Agar to'g'ri chiziq va tekislik o'zaro parallel bo'lsa, u holda

$Am + Bn + Cp = 0$ (2)tenglik o'rinli bo'ladi.

Agar to'g'ri chiziq va tekislik o'zaro perpendikulyar bo'lsa, u holda

$$\frac{A}{m} = \frac{B}{n} = \frac{C}{p} \qquad (3)$$

tenglik o'rinli bo'ladi.

Berilgan $M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi va l_1 hamda l_2 to'g'ri chiziqlarga parallel bo'lgan tekislik tenglamasi quyidagicha:

$$\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix} = 0 \qquad (4)$$

$M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi l chiziqqa perpendikulyar bo'lgan tekislik tenglamasi quyidagi ko'rinishda bo'ladi:

$$m(x - x_0) + n(y - y_0) + p(z - z_0) = 0 \quad (5)$$

Berilgan l to'g'ri chiziq va p tekislikning kesishish nuqtasi

$$\begin{cases} Ax + By + Cz + D = 0 \\ x = x_0 + mt \\ y = y_0 + nt \\ z = z_0 + pt \end{cases} \qquad (6)$$

sistemadan topiladi.

$$\frac{x-x_1}{m_1} = \frac{y-y_1}{n_1} = \frac{z-z_1}{p_1} \text{ va} \frac{x-x_2}{m_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{m_2}$$

to'g'ri chiziqlarni bir tekislikda yotish sharti quyidagicha:

$$\begin{vmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix} = 0 \qquad (7)$$

1. $3x + 2y - 5z - 1 = 0$ tekislik bilan $\begin{cases} x = 4t + 2 \\ y = -3t + 2 \\ z = 2t + 1 \end{cases}$ to'g'ri chiziqning kesishish nuqtasini toping. J: (6; -1; 3).

2. $\frac{x+6}{-2} = \frac{y-1}{3} = z - 1$ to'g'ri chiziq bilan $2x - 5y + 6z - 1 = 0$ tekislikning kesishish nuqtasi topilsin.

3. $y = 3x - 1,\ 2z = -3x + 2$ to'g'ri chiziq bilan *2x+y+z-4=0* tekislik orasidagi burchak topilsin. J: $\sin\varphi = \frac{1}{\sqrt{6}}$.

4. $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z-1}{3}$ to'g'ri chiziq $2x + y - z = 0$ tekislikka parallel ekanligini isbotlang.

5. *x=2t-1*, *y=t+2*, *z=1-t* to'g'ri chiziqning *3x-2y+z-3=0* tekislik bilan kesishgan nuqtasi topilsin. J: (5; 5; -2).

6. $\frac{x}{2}=\frac{y-1}{1}=\frac{z+1}{2}$ to'g'ri chiziqning $x+2y+3z-29=0$ tekislik bilan kesishish nuqtasi topilsin. J: (6; 4; 5).

7. $\begin{cases} x=2t+1 \\ y=3t-2 \\ z=-6t+1 \end{cases}$ va $\begin{cases} 2x+y-4z+2=0 \\ 4x-y-5z+4=0 \end{cases}$ to'g'ri chiziqlarning perpendikulyarligi isbotlansin.

8. $\frac{x+7}{3}=\frac{y+4}{4}=\frac{z+3}{-2}$ va $\frac{x-21}{6}=\frac{y+5}{-4}=\frac{z-2}{-1}$ to'g'ri chiziqlar orasidagi eng qisqa masofani toping. J: 13.

9. $\frac{x+2}{2}=\frac{y}{-3}=\frac{z-1}{4}$ va $\frac{x-3}{l}=\frac{y-1}{4}=\frac{z-7}{2}$ to'g'ri chiziqlar l ning qanday qiymatlarida kesishadi? J: 3.

10. $\begin{cases} x=2t-1 \\ y=t+2 \\ z=1-t \end{cases}$ to'g'ri chiziqning $3x-2y+z-3=0$ tekislik bilan kesishgan nuqtasi topilsin. J: (5; 5; -2).

11. $\frac{x+3}{1}=\frac{y+1}{2}=\frac{z+1}{1}$ va $\begin{cases} x=3z-4 \\ y=z+2 \end{cases}$ to'g'ri chiziqlarning kesishuvchi ekanligi ko'rsatilsin va kesishish nuqtasi topilsin.

12. Kanonik tenglamalari $\frac{x-1}{a}=\frac{y-3}{-3}=\frac{z+4}{\sqrt{2}}$ va $\frac{x+2}{2}=\frac{y+13}{a}=\frac{z-6}{\sqrt{2}}$ bo'lgan to'g'ri chiziqlar a parametrning qanday qiymatlarida o'zaro perpendikulyar bo'ladi? J: 2.

13. Kanonik tenglamalari $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z}{\sqrt{2}}$ va $\frac{x+2}{1}=\frac{y-3}{1}=\frac{z+5}{\sqrt{2}}$ bo'lgan to'g'ri chiziqlar orasidagi burchak topilsin. J: 60°.

14. $\frac{x+1}{1}=\frac{y-2}{\sqrt{2}}=\frac{z-1}{1}$ to'g'ri chiziq bilan $x+y\sqrt{2}-z+1=0$ tekislik orasidagi burchak topilsin. J: 30°.

15. $\frac{x+1}{3}=\frac{y-2}{n}=\frac{z+3}{-2}$ to'g'ri chiziq n ning qanday qiymatida $x-3y+6z+$ $+7=0$ tekislikka parallel bo'ladi.

16. $\frac{x-2}{m}=\frac{y+1}{4}=\frac{z-5}{-3}$ to'g'ri chiziq va $3x-2y+Cz+1=0$ tekislik o'zaro perpendikulyar bo'lishi uchun m va C qanday qiymatlarni qabul qilishi kerak?J: $m=-6,\ C=\frac{3}{2}$.

17. $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+3}{\sqrt{2}}$ to’g’ri chiziq bilan $x+y+z\sqrt{2}-4=0$ tekislik orasidagi burchak topilsin.

18. $\frac{x-2}{m}=\frac{y+1}{4}=\frac{z-5}{-3}$ to’g’ri chiziq bilan $3x+2y+Cz+4=0$ tekislik m va C parametrlarning qanday qiymatida o’zaro perpendikulyar bo’ladi?

19. $\frac{x-3}{2}=\frac{y-5}{-3}=\frac{z+2}{-2}$ to’g’ri chiziq bilan $Ax+By+3z-5=0$ tekislik A va B parametrlarning qanday qiymatida o’zaro perpendikulyar bo’ladi?

20. $\frac{x+1}{3}=\frac{y-2}{n}=\frac{z+3}{-2}$ to’g’ri chiziq bilan $x-3y+6z+2=0$ tekislik n parametrning qanday qiymatida o’zaro perpendikulyar bo’ladi?

13-§. Ikkinchi tartibli sirtlar

$$Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Kz+L=0 \quad (1)$$

tenglamaga ikkinchi tartibli sirtlarning ***umumiy tenglamasi*** deyiladi. Bu yerda $A, B, C, D, E, F, G, H, K, L$ lar qandaydir berilgan sonlar va

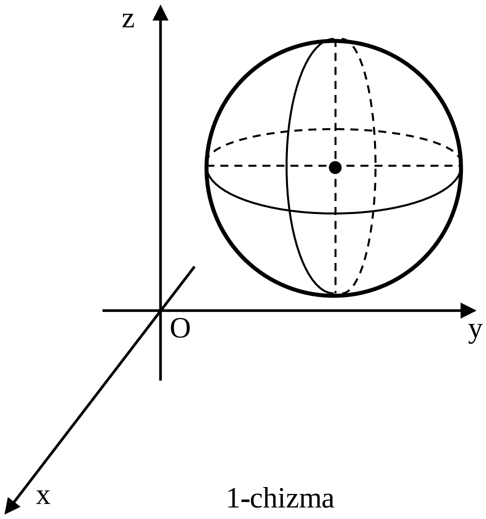
$$A^2+B^2+C^2+D^2+E^2+F^2\neq 0.$$

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2 \qquad (2)$$

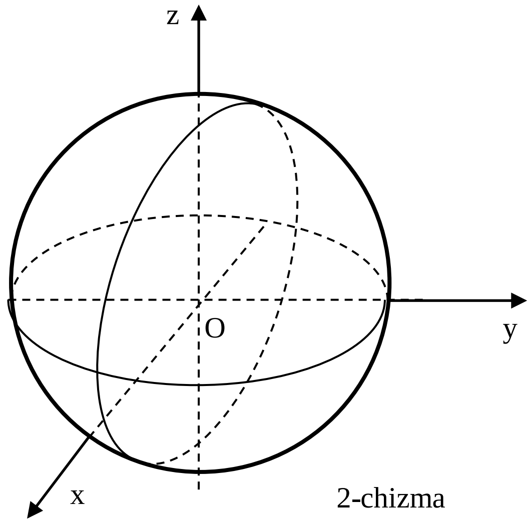
tenglama markazi $C(a;b;c)$ nuqtada va radiusi R ga teng bo’lgan sfera tenglamasidir(1- chizma).

$$x^2+y^2+z^2=R^2 \qquad (3)$$

tenglama esa markazi $O(0;0;0)$ nuqtada va radiusi R ga teng bo’lgan sfera tenglamasidir(2-chizma).



1-chizma



2-chizma

Sferani o’zaro perpendikulyar uchta yo’nalish bo’yicha tekis deformatsiyalash (cho’zish va siqish) natijasida hosil bo’lgan sirt ellipsoid deyiladi(3-chizma).

Ellipsoidning kanonik tenglamasi

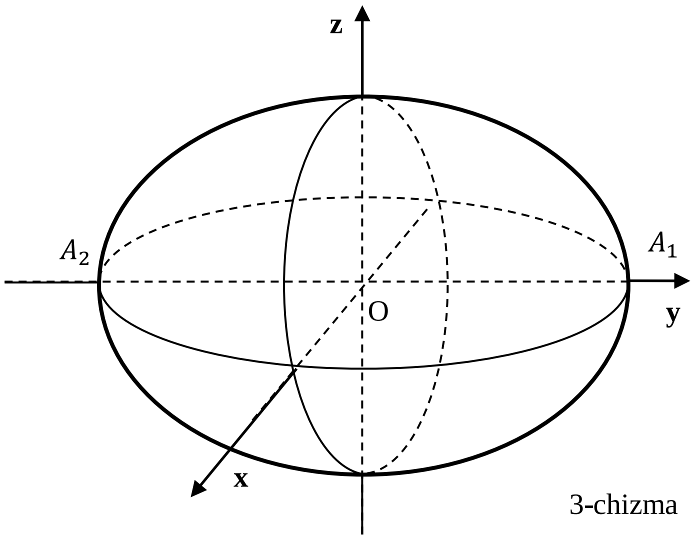
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \qquad (4)$$

ko’rinishda bo’ladi. a, b, c sonlar ellipsoidning yarim o’qlari deyiladi.

1°. Ellipsoid koordinata o’qlariga nisbatan simmetrikdir.

2°. Ellipsoid koordinata o’qlarini $A_1(a;0;0)$, $A_2(-a;0;0)$, $B_1(0;b;0), B_2(0;-b;0)$, $C_1(0;0;c)$ va $C_2(0;0;-c)$ nuqtalarda kesib o’tadi.

3°. Ellipsoidning $z=h$ tekislik bilan kesimi ellips bo’lib, uning tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1-\frac{h^2}{c^2}$ (5) bo’ladi.



3-chizma

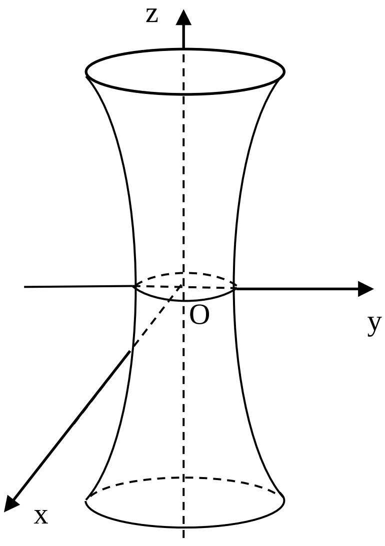
Quyidagi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \qquad (6)$$

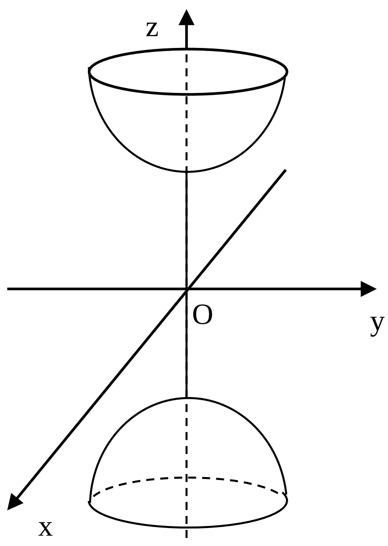
tenglama bilan aniqlangan sirt bir ***pallali giperboloid*** deb ataladi (4-chizma). Quyidagi

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=-1 \qquad (7)$$

tenglama bilan aniqlangan sirt ***ikki pallali giperboloid*** deb ataladi(5-chizma).



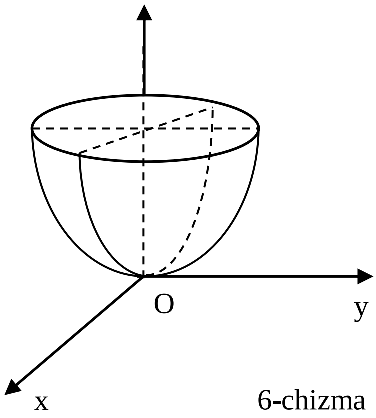
4-chizma



5-chizma

OXZ tekislikda $x^2=2pz,\ y=0$ (8) tenglama bilan berilgan parabolani Oz o'qi atrofida aylantirishdan hosil bo'lgan sirt ***paraboloid*** deb ataladi(6-chizma). U quyidagi tenglama bilan aniqlanadi:

$$x^2 + y^2 = 2pz \quad (9)$$



6-chizma

$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ (10) tenglama bilan aniqlangan sirt ***elliptik paraboloid*** deyiladi.

$2z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ (11) tenglama bilan aniqlangan sirt ***giperbolik paraboloid*** deyiladi.

$x^2 + y^2 = 2pz$ tenglama bilan berilgan aylanma paraboloid Oz o'qiga nisbatan simmetrik bo'ladi.

$2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ tenglama bilan berilgan elliptik paraboloidni $|z = h > 0|$ tekislik bilan kesish natijasida

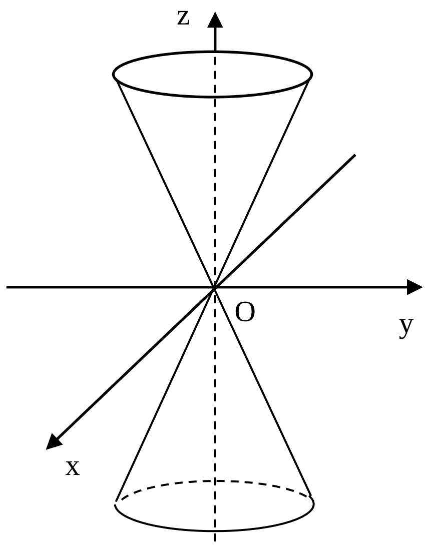
$$2h = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

ellips hosil bo'ladi.

$2z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ giperbolik paraboloidni $|z = h|$tekislik bilan kesilsa, kesimda $2h = \frac{x^2}{a^2} - \frac{y^2}{b^2}$giperbola hosil bo'ladi.

Quyidagi $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$ (12)

tenglama bilan aniqlangan sirtga ***konus*** deb ataladi(7-chizma).



1. Quyidagi tenglamalar bilan berilgan sferalarning markazi va radiusini toping.

   1) $x^2+y^2+z^2=4$; 2) $x^2+(y-2)^2+z^2=1$;

   3)$x^2+(y-2)^2+(z+2)^2=\frac{1}{4}$;4) $2x^2+2y^2+2(z-3)^2$=1;

   5) $(x+2)^2+(y-1)^2+(z-2)^2=1$.

2. Quyidagi sfera tenglamalarini kanonik ko'rinishga keltiring.

   1) $x^2+y^2+z^2-4x-6y-2z+13=0$;

   2) $x^2+y^2+z^2+2x-6y+8z+10=0$;

   3) $x^2+y^2+z^2-12x-6y+37=0$;

   4) $x^2+y^2+z^2-3y=0$;

   5) $4x^2+4y^2+4z^2-16x-4y-8z+17=0$;

   6) $x^2+y^2+z^2-4x+12y-2z+41=0$;

   7) $x^2+y^2+z^2-6\square+10=0$;

   8) $x^2+y^2+z^2-5x+\frac{4}{3}y-\frac{\sqrt{15}}{3}z=0$.

3. Markazi $C(-1;3;\sqrt{2})$ nuqtada va radiusi $r=5$ bo'lgan sferaning tenglamasi tuzilsin.

4. Markazi $C(2;0;-0{,}5)$ nuqtada va $4x-4y+2z+17=0$ tekislikka urunuvchi sfera tenglamasi yozilsin. J: $(x-2)^2+y^2++\left(z+\frac{1}{2}\right)^2=16$.

5. Quyidagi: 1) $2x-6y+3z-49=0$; 2) $4x-3y+101=0$;

3) $3x - 2y + z + 6 = 0$ tekisliklarning har biri $x^2 + y^2 + z^2 = 49$ sferaga nisbatan qanday joylashganini aniqlang.

<u>Ko'rsatma:</u> Sfera markazidan tekislikkacha masofani sfera radiusi bilan solishtiring. J: 1) urinadi; 2) kesmaydi; 3) kesadi.

6.1) $x^2 + y^2 + z^2 = 9$ sferaga $A(2;\ -1; 2)$ nuqtada urinuvchi;

2) $(x + 1)^2 + (y - 2)^2 + (z + 2)^2 = 49$ sferaga $A(5;\ 5; -4)$ nuqtada urinuvchi tekislik tenglamasini yozing.

<u>Ko'rsatma:</u> Sfera radiusi CA urinma tekislikka A nuqtada perpendikulyar.J: 1) $2x - y + 2z - 9 = 0$; 2) $6x + 3y \pm 2 - 53 = 0$.

7. Quyida berilgan tenglamalar qanday sirtni ifodalashini aniqlang.

1) $\frac{x^2}{4} + \frac{y^2}{16} + z^2 = 1$; 2) $x^2 + \frac{y^2}{4} + \frac{z^2}{16} = 1$;

3) $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 1$; 4) $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = -1$;

5) $\frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{16} = 0$; 6) $4(x^2 + y^2) - z^2 = 16$;

7) $y^2 - 16(x^2 + z^2) + 16 = 0$; 8) $4(y^2 - x^2) - z^2 + 16 = 0$

9) $\frac{x^2}{2} + \frac{y^2}{4} = z$; 10) $z = x^2 + y^2$.

8. $\frac{x^2}{9} + \frac{y^2}{1} - \frac{z^2}{4} = -1$ sirtga (-6; 2; 6) nuqtada urinuvchi tekislik tenglamasini tuzing.

<u>Ko'rsatma:</u> Urinma tekislik tenglamasi $\frac{xx_0}{a^2} + \frac{yy_0}{b^2} \pm \frac{zz_0}{c^2} = \pm 1$ dan iborat. J: $4x - 12y + 9z - 6 = 0$.

9) $\frac{x^2}{4} + \frac{y^2}{3} - z^2 = 0$ konusning (4; -6; 4) nuqtasiga urinuvchi urinma tekislik tenglamasini tuzing. J: $x - 2y - 4z = 0$.

<u>Ko'rsatma</u>:urinma tekislik tenglamasi $\frac{xx_0}{a^2} + \frac{yy_0}{b^2} \pm \frac{zz_0}{c^2} = 0$ bo'ladi.

IV BOB.MATEMATIK ANALIZGA KIRISH

1-§. O'zgaruvchi va o'zgarmas miqdorlar, funksiya tushunchasi

Faqat bitta sonli qiymat qabul qiladigan kattaliklar ***o'zgarmas miqdorlar*** deyiladi.

Turli sonli qiymatlar qabul qiladigan kattaliklar ***o'zgaruvchi miqdorlar*** deyiladi.

O'zgaruvchi miqdorning qabul qiladigan qiymatlari to'plami o'zgaruvchi miqdorning o'zgarish sohasi deyiladi.

Agar x o'zgaruvchi miqdor biror a sonidan b sonigacha $(a < b)$qiymatlarni qabulqilsa, u holda uni $a \leq x \leq b$ yoki $x\epsilon[a, b]$ kabi yoziladi va uni ***kesma*** deyiladi.

$a < x < b$ tengsizlikni qanoatlantiradigan $x$sonlar to'plami ***interval (oraliq)*** deb ataladi va uni (a,b) kabi yoziladi.

$a < x \leq b$ va $a \leq x < b$lar yarim yopiq oraliqlar deyiladi va (a,b] va [a,b) kabi yoziladi.

$-\infty < x < a$ va $a < x < +\infty$ lar cheksiz yarim intervallar (oraliqlar) deyiladi.

$-\infty < x < +\infty$ cheksiz interval (oraliq) deyiladi.

Agar x o'zgaruvchining biror sonli D to'plamga tegishli har bir qiymatiga ma'lum bir qonun qoida asosida y o'zgaruvchining biror E to'plamga tegishli yagona bir qiymati mos qo'yilgan bo'lsa, unda y o'zgaruvchi x o'zgaruvchining ***funksiyasi*** deyiladi.

yo'zgaruvchi x o'zgaruvchining funksiyasi ekanligi $y = f(x)$, $y = F(x)$, $y = \varphi(x)$, $y = g(x)$, $y = h(x)$ va hokazolardan biri bilan belgilanadi. Bu yerda x***erkli o'zgaruvchi*** yoki ***argument***, y esa ***erksiz o'zgaruvchi*** yoki ***funksiya*** deyiladi. D – funksiyaning ***aniqlanish sohasi***, E $-$***o'zgarish*** yoki ***qiymatlar sohasi*** deyiladi. Aniqlanish sohasi $D\{f\}$, qiymatlar sohasi $E\{f\}$ bilan belgilanadi.

XOY koodinata tekisligidagi $(x, y) = \big(x, f(x)\big)$, $x\epsilon D\{f\}$ koordinatali nuqtalarning geometrik o'rni $y = f(x)$ funksiyaning ***grafigi*** deyiladi.

Funksiya analitik, jadval, grafik va ta'rif usullarida beriladi.

Agar x argument bo'yicha bajariladigan matematik amallarni formulalar orqali berilsa, u holda funksiyani ***analitik usulda*** berilgan deyiladi.

Agar x va y o'zgaruvchilar orasidagi bog'lanish jadval ko'rinishida berilgan bo'lsa, u holda funksiyani ***jadval usulida*** berilgan deyiladi.

Agar x va y o'zgaruvchilar orasidagi bog'lanish tekislikdagi biror egri chiziq orqali berilsa, u holda funksiyani ***grafik usulda*** berilgan deyiladi.

Ta'rif usulida funksiya qiymatini aniqlash qonuni uni ta'riflash orqali beriladi. Masalan: Dirixle funksiyasi deb ataluvchi va [0,1] kesmada aniqlangan $D(x)$ funksiyani analitik, jadval yoki grafik ko'rinishlarda ifodalab bo'lmaydi. Bu funksiya qiymatlari ta'rif bo'yicha quyidagicha aniqlanadi:

$$D(x) = \begin{cases} 1, & \text{agar } x \text{ ratsional son bo'lsa} \\ 0, & \text{agar } x \text{ irratsional son bo'lsa} \end{cases}$$

Berilgan $y = f(x)$ funksiya biror $D \subset D\,(f)$ sohaga tegishli ixtiyoriy $x_1, x_2 \epsilon D$ va $x_1 < x_2$ nuqtalar uchun $f(x_1) < f(x_2) [f(x_1) \leq f(x_2)]$ shartni qanoatlantirsa, u holda funksiyani D sohada ***o'suvchi (kamaymovchi)*** deyiladi.

Berilgan $y = f(x)$ funksiya biror $D \subset D\,(f)$ sohaga tegishli ixtiyoriy $x_1, x_2 \epsilon D$ va $x_1 < x_2$ nuqtalar uchun $f(x_1) > f(x_2) [f(x_1) \geq f(x_2)]$ shartni qanoatlantirsa, u holda funksiyani shu sohada ***kamayuvchi (o'smovchi)*** funksiya deyiladi.

O'suvchi yoki kamaymovchi, kamayuvchi yoki o'smovchi funksiyalarni ***monoton funksiylar*** deyiladi.

Aniqlanish sohasi $D\{f\}$ nol nuqtaga nisbatan simmetrik bo'lgan sohadagi ixtiyoriy x uchun $f(-x) = f(x)$ $[f(-x) = -f(x)]$ shart bajarilsa, u holda funksiyani ***juft (toq)*** deyiladi.

Agar $y = f(x)$ funksiya uchun yuqoridagi shartlar bajarilmasa, u holda funksiyani ***juft ham, toq ham***emas deyiladi.

Agar $f(x)$ va $g(x)$ juft funksiyalar bo'lsa ularning umumiy aniqlanish sohasi D da $f(x) \pm g(x)$, $f(x) \cdot g(x)$ va $g(x) \neq 0$ bo'lganda $\frac{f(x)}{g(x)}$ funksiyalar ham juft bo'ladi.

Agar $f(x)$ va $g(x)$ toq funksiyalar bo'lsa, u holda $f(x) \pm g(x)$ toq $f(x) \cdot g(x)$ va $\frac{f(x)}{g(x)}$ funksiyalar juft bo'ladi.

Agar $f(x)$ juft va $g(x)$ toq funksiya bo'lsa, u holda $f(x) \cdot g(x)$ va $\frac{f(x)}{g(x)}$ toq funksiyalar bo'ladi.

Agar $y = f(x)$ funksiya uchun shunday $T > 0$ son mavjud bo'lsaki, $\forall x \epsilon D\{f\}$ uchun $x \pm T \epsilon D\{f\}$ bo'lganda $f(x \pm T) = f(x)$ shart bajarilsa, u holda funksiyani ***davriy funksiya*** deyiladi. Bu shartni qanoatlantiruvchi eng kichik musbat T soni ***funksiyani davri*** deyiladi.

Berilgan $y = f(x)$ funksiya uchun shunday $M > 0$ soni topilsaki, ixtiyoriy $x \epsilon D$ uchun $\left|f(x)\right| \le M$ shart bajarilsa, u holda funksiyani *D*sohada chegaralangan funksiya deyiladi. Aks holda $y = f(x)$ funksiyani chegaralanmagan deyiladi.

Agar $y = f(x)$ funksiya biror *D* sohaning har bir *x* nuqtasida o'zgarmas *C* songa teng bo'lsa, u holda funksiyani *D* sohada ***o'zgarmas funksiya*** deyiladi.

$y = f(u)$ bo'lib, o'z vaqtida $u = \varphi(x)$ bo'lsa, u holda $y = f[\varphi(x)]$ bo'lib, bu funksiyani ***murakkab funksiya*** yoki ***funksiyaning funksiyasi*** deyiladi.

$y = f(u)$ bo'lib, $u = \varphi(x)$ va $x = \psi(t)$ bo'lsa, u holda $f\{\varphi[\psi(t)]\}$ bo'lib, bu funksiya ham murakkab funksiya bo'ladi.

Aniqlanish sohasi $D\{f\}$ va qiymatlar sohasi $E\{f\}$ bo'lgan $y = f(x)$ funksiya uchun har bir $y \epsilon E\{f\}$ soniga $f(x) = y$ shartni qanoatlantiradigan yagona $x \epsilon D\{f\}$ sonini mos qo'yadigan $x = \varphi(y)$ funksiya mavjud bo'lsa, u berilgan f funksiyaga ***teskari funksiya*** deyiladi.

f funksiyaga teskari funksiya f^{-1} kabi belgilanadi ($f^{-1} = \frac{1}{f}$ degan ma'noni bildirmaydi).

Odatda argument *x*, funksiya y orqali belgilangani uchun, $y = f(x)$ funksiyaga teskari $x = \varphi(y)$ funksiya $y = \varphi(x)$ kabi yoziladi.

Quyidagi funksiyalar ***asosiy elementar funksiyalar*** deyiladi:

1)$y = x^{\alpha}, \alpha \epsilon R$ darajali funksiya;

2)$y = a^x,\ a > 0,\ a \neq 1$ ko'rsatkichli funksiya;

3) $y = \log_a x\,,\ a > 0,\ a \neq 1$ logarifmik funksiya;

4) $y = \sin x,\quad y = \cos x,\ y = tg\ x,\ y = ctg\ x,\quad y = \sec x, y = cosec\ x$ lar trigonometrik funksiyalar;

5) $y = \arcsin x,\ y = \arccos x,\quad y = arctgx,\quad y = arcctgx,\quad y = \mathrm{arcsec}\, x,$ $y = arccosec\ x$lar teskari trigonometrik funksiyalar.

Asosiy elementar funksiyalardan chekli sondagi arifmetik amallar yordamida tuzilgan murakkab funksiyalarga ***elementar funksiyalar*** deb ataladi.

1. Matematik mayatnikning kichik tebranishlar davri $T = -\pi\sqrt{\frac{l}{g}}$ formula bilan hisoblanadi, bu yerda $l -$ mayatnikning uzunligi, g –erkin tushish tezlanishi. Bu formulaga kiruvchi miqdorlardan qaysilari absolyut o'zgarmas miqdor, qaysilari parametr va qaysilari o'zgaruvchi miqdor ?

2. Izotermik jarayonda Boyl – Mariyot qonuni $PV = C$ formula bilan ifodalanishi ma'lum, bu yerda P-gazning bosimi, V–u egallab turgan hajm. Bu formuladagi o'zgarmas va o'zgaruvchi miqdorlarni ko'rsating.

3. Bo'shliqda erkin tushayotgan jismning bosib o'tgan S yo'li $S = \frac{gt^2}{2}$ formula bo'yicha hisoblanadi. Bu formulaga kiruvchi miqdorlarning qaysilari absolyut o'zgarmas miqdor, parametr yoki o'zgaruvchi bo'ladi?

4. Kesilgan konusning hajmi

$$V = \frac{\pi n}{3}\ (R^2 + Rr + r^2)$$

formula bo'yicha hisoblanadi. Bu formuladagi miqdorlardan qaysilari absolyut o'zgarmas miqdor, qaysilari o'zgaruvchi miqdor va qaysilari parametrbo'ladi ?

5. 1) $|x| < 4;$ 2) $x^2 \leq 9;$ 3) $|x - 4| < 2;$

4) $-1 < x - 3 \leq 2;$ 5) $x^2 > 9;$ 6) $(x - 2)^2 \leq 4.$

tengsizliklarni qanoatlantiruvchi x ning o'zgarish oraliqlari yasalsin.

6. O'zgaruvchilarning $[-1; 3];\ (0; 5);\ [-2; 3]$ o'zgarish oraliqlari tengsizliklar orqali yozilsin va yasalsin.

J: $-1 \leq x \leq 3;\ \ 0 < x < 5;\ \ -2 \leq x \leq 3.$

7. $x = 1 - \frac{1}{t}$ o’zgaruvchining o’zgarish oralig’i aniqlansin, bundagi t birdan kichik bo’lmagan har qanday qiymatni qabul qiladi. J: $0 \leq x < 1$.

8. $x = 2 + \frac{1}{t}$ o’zgaruvchining ixtiyoriy $t \geq 1$ qiymatlar qabul qilgandagi o’zgarish oralig’i aniqlansin. J: $2 < x \leq 3$.

9. $f(x) = 3x^2 - 2x - 1$ funksiya berilgan. $f(2)$, $f(-2)$, $f(1)$, $f(0)$, $f(a+2)$ va $f(-x)$ lar topilsin.

J: $f(2) = 7; f(-2) = 15;\ f(1) = 0; f(0) = -1; f(a+2) == 3a^2 + +10a + 7;\ \ f(-x) = 3x^2 + 2x - 1$.

10. $f(z) = 2z^3 - z^2 + z - 1$ funksiya berilgan. $f\left(\frac{1}{2}\right)$, $f(2)$, $f(-1)$, $f\left(\frac{a}{2}\right)$, va $f\left(\frac{a-1}{a+1}\right)$ lar topilsin.

J: $f\left(\frac{1}{2}\right) = -\frac{1}{2}, f(2) = 13;\ \ f(-1) = -5; f\left(\frac{a}{2}\right) = \frac{a^3}{2} - \frac{a^2}{4} + \frac{a}{4} - \frac{1}{2}; f\left(\frac{a-1}{a+1}\right) = \frac{a^3 - 7a^2 + 3a - 5}{(a+1)^3}$.

11. $f(x) = \frac{4x^2 - 7x + 2}{3x^2 + 5}$ funksiya berilgan. $f(a), f\left(\frac{1}{a^2}\right), f(2)$ va $f(0)$ lar topilsin.

J: $f(a) = \frac{4a^2 - 7a + 2}{3a^2 + 5}; f\left(\frac{1}{a^2}\right) = \frac{4 - 7a^2 + 2a^4}{5a^4 + 3}; f(2) = \frac{4}{17};\ \ f(0) = \frac{2}{5}$.

12. $f(x) = 5^{\frac{1}{x-1}}$ funksiya berilgan. $f(2)$ va $f(-3)$ lar topilsin.

J: $f(2) = \frac{1}{\sqrt{5}};\ \ f(-3) = \frac{1}{\sqrt[3]{5^4}}$.

13. $f(x) = \frac{5x+1}{2-x}$ funksiya berilgan. $f(3x)$, $f(x^3)$, $3f(x)$ va $[f(x)]^3$ lar topilsin.

J: $f(3x) = \frac{15x+1}{2-3x}$; $f(x^3) = \frac{5x^3+1}{2-x^4}$; $3f(x) = \frac{15x+3}{2-x}$;

$[f(x)]^3 = \frac{125x^3 + 75x^2 + 15x + 1}{8 - 12x + 6x^2 - x^3}$.

14. $f(x) = 4x - x^2$ bo’lsa, $f(a+1) - f(a-1)$ ni toping.

15. $f(x)=\begin{cases}2+x, & x>0\\ 5, & x=0\\ 2^x, & x<0\end{cases}$ funksiya berilgan. $f(-2),\ f(0),\ f(1)$ va $f(3)$ lar topilsin.

J: $f(-2)=\frac{1}{4};\ f(0)=5;\ f(1)=3;\ f(3)=5.$

16. $f(x)=\begin{cases}x^2, & -1\le x<0\\ -2x+1, & 0\le x<\frac{1}{2}\\ cos\pi x, & \frac{1}{2}\le x<1\end{cases}$

funksiya berilgan. $f\left(-\frac{1}{3}\right);\ f(0);\ f\left(\frac{1}{2}\right);\ f\left(\frac{4}{5}\right)$ lar topilsin.

J: $f\left(-\frac{1}{3}\right)=\frac{1}{9};\ f(0)=1;\ f\left(\frac{1}{2}\right)=0;\ f\left(\frac{4}{5}\right)=\cos\frac{4\pi}{5}.$

17. $f(x)=\frac{x+1}{x^3-1}$ funksiya berilgan. $f(-1),\ f(a+1),\ f(a)+1$ lar topilsin.

J: $f(-1)=0,\quad f(a+1)=\frac{a+2}{a^3+3a^2+3a},\quad f(a)+1=\frac{a^3+a}{a^3-1}.$

18. $f(x)=\begin{cases}3^{-x}-1, & -1\le x<0\\ \mathrm{tg}\frac{x}{2}, & 0\le x<\pi\\ \frac{x}{x^2-2}, & \pi\le x\le 6\end{cases}$

funksiya berilgan. $f(-1),\ f\left(\frac{\pi}{2}\right),\ f\left(\frac{2\pi}{3}\right),\ f(4),\ f(6)$ lar topilsin.

J: $f(-1)=2,\ f\left(\frac{\pi}{2}\right)=1,\ f\left(\frac{2\pi}{3}\right)=\sqrt{3},\ f(4)=\frac{2}{7},\ f(6)=\frac{3}{17}.$

19. $f(x)=\begin{cases}2x^3+1, & x\le 2\\ \frac{1}{x-1}, & 2<x\le 3\\ 2x-5, & x>3\end{cases}$

funksiya berilgan. $f(\sqrt{2}),\ f(\sqrt{8}),\ f(\sqrt{\log_2 1024})$ lar topilsin.

J: $f(\sqrt{2})=4\sqrt{2}+1,\ f(\sqrt{8})=\frac{1}{\sqrt{8}-1},\ f(\sqrt{\log_2 1024})=15.$

20. Agar $f(-4)=6$ va $f(4)=4$ bo'lsa, $f(x)$ chiziqli funksiyani yozing.

21. Agar $f(-2)=10$ va $f(1)=-5$ bo'lsa, $f(x)$ chiziqli funksiyani yozing.

22. Agar $f(-3)=3$ va $f(6)=0$ bo'lsa, $f(x)$ chiziqli funksiyani yozing.

23. Agar $f(0)=15;\ f(2)=30;\ f(4)=90$ bo'lsa, $f(x)=a+bc^x$ ($c>0$) funksiyani yozing.

24. Agar $f(0) = 5,\ f(-1) = 10,\ f(1) = 6$ bo'lsa, $f(x) = ax^2 + bx + c$ kvadrat funksiyani yozing.

J: $f(x) = 3x^2 - 2x + 5.$

25. Agar $f(1) = 3,\ f(-1) = 1,\ f(0) = 1$ bo'lsa, $f(\square) = ax^2 + bx + c$ kvadrat funksiyani yozing.

J: $f(x) = x^2 + x + 1.$

26. Quyidagi funksiyalarni aniqlanish sohalari topilsin.

1) $f(x) = \frac{1}{3x-2}$; 2) $f(x) = \sqrt{4 - x^2}$; 3) $f(x) = \frac{x}{x^2-1}$;

4) $f(x) = \sqrt{x^2 - 9}$; 5) $f(x) = \frac{1}{3x-2} + \frac{1}{2x-3}$;

6) $f(x) = \frac{1}{\sqrt{x+1}-\sqrt{x-1}}$; 7) $f(x) = \sqrt{x^2 + 4x - 5}$;

8) $f(x) = \frac{2x}{x^2+3x-10}$; 9) $f(x) = \frac{\lg(x-1)}{x^2+x-2}$;

10) $f(x) = \sqrt{x+2} - \sqrt{1-x}$; 11) $f(x) = \sqrt{x^2 - 1} + \frac{1}{\sqrt{4-x}}$;

12) $f(x) = \sqrt{x-4} - \sqrt{8 - x^2}$; 13) $f(x) = \frac{1}{\lg(2-x-x^2)}$;

14) $f(x) = \arcsin\frac{x-1}{3} + \arccos\frac{2x-1}{2}$.

J: 1) $\left(-\infty; \frac{2}{3}\right) \cup \left(\frac{2}{3}; +\infty\right)$; 2) $[-2; 2]$; 3) $f(x) = (-\infty; -1) \cup \cup (-1; 1) \cup (1; +\infty)$; 4) $(-\infty; -3] \cup [3; +\infty)$; 5) $\left(-\infty; \frac{2}{3}\right) \cup \left(\frac{2}{3}; \frac{3}{2}\right) \cup\cup \left(\frac{3}{2}; +\infty\right)$; 6) $[1; +\infty)$; 7) $(-\infty; -5] \cup [1; +\infty)$; 8) $(-\infty; -5) \cup\cup (-5; 2) \cup (2; +\infty)$; 9) $(1; +\infty)$; 10) $[-2; 1]$; 11) $(-\infty; -1) \cup \cup (1; 4)$; 12) $\emptyset$; 13) $\left(-2; \frac{-1-\sqrt{5}}{2}\right) \cup \left(\frac{-1+\sqrt{5}}{2}; 1\right)$; 14) $\left[-\frac{1}{2}; \frac{3}{2}\right]$.

27. Quyidagi funksiyalardan qaysilari juft, qaysilari toq, qaysilari juft ham emas, toq ham emas ?

1) $f(x) = \frac{x^2}{sin2x}$(toq); 2) $f(x) = 4 - 2x^4 + sin^2 x$ (juft);

3) $f(x) = x^3 + 2x - 1$ (juft ham toq ham emas);

4) $f(x) = \frac{1+a^{kx}}{1-a^{kx}}$ (toq);5) $f(x) = \frac{2^x+2^{-x}}{2^x-2^{-x}}$ (toq);

6) $f(x) = ln\frac{1-x}{1+x}$ (toq); 7) $f(x) = \frac{3^x+3^{-x}}{2}$ (toq);

8) $f(x) = \log_2 \frac{1+sinx}{1-sinx}$ (toq); 9) $f(x) = \sqrt{x^4 - |x|}\log_2 x^2$ (juft).

10) $f(x) = \sqrt{x^2(x-2)}$ (juft ham toq ham emas);

11) $f(x) = arcsin\frac{2x}{1+x^2}$ (toq); 12) $f(x) = \frac{1+sinx}{sinx}$ (juft ham, toq ham emas).

28. Quyidagi funksiyalarni chegaralangan yoki chegaralanmaganligi aniqlansin.

1) $y = 5cos3x + 2sin3x$ (chegaralangan);

2) $y = \frac{x^2+x+6}{x^2+x+1}$(chegaralangan);

3) $y = xsinx$(chegaralanmagan);

4) $y = 2^{\cos^3 x} + 3sinx$ (chegaralangan);

5) $y = 2sinx + cosx$ (chegaralangan);

6) $y = \frac{x^2+x+1}{x^2+1}$ (chegaralangan);

7) $y = \frac{x^3+1}{x^2+x}$ (chegaralanmagan).

29. Quyidagi funksiyalarning davrlari topilsin.

1) $f(x) = tg2x$ $\left(\frac{\pi}{2}\right)$;2) $f(x) = ctg\frac{x}{2}$ (2π);

3) $f(x) = sin2\pi x$ (1);4) $f(x) = sin^4x + cos^4x$ $\left(\frac{\pi}{2}\right)$;

5) $f(x) = |cosx|$ (π);6) $f(x) = sinx + cos2x + tg3x(2\pi)$;

7) $f(x) = 2sin3x + 3sin2x(2\pi)$; 8) $f(x) = sinx + cos2x(2\pi)$;

9) $f(x) = |sinx| + |cosx|(\pi)$; 10)$f(x) = tg\frac{2x}{3} + ctg\frac{3x}{2}(6\pi)$;

11)$f(x) = sin\frac{x}{2} + cos\frac{3x}{2} + +tg2x$ (4π);

12) $f(x) = sin\frac{3x}{2} - cos\frac{x}{3}$ (8π).

30. Quyidagi funksiyalarning grafiklari yasalsin.

1)$y = x^2$; 2) $y = x^3$; 3) $y = |x|$; 4) $y = |x-2|$;

5) $y = \frac{1}{x-2}$; 6) $y = \frac{x}{x-2}$; 7) $f(x) = [x]$; 8) $f(x) = \{x\}$;

9) $y = \frac{1}{x+3}$;10) $f(x) = \begin{cases} x^2, & \text{agar } x \geq 0 \text{ bo'lsa,} \\ 2x+1, & \text{agar } x < 0\ bo'lsa \end{cases}$;

11) $y = \begin{cases} x^2, & \text{agar } x \leq 0 \text{ bo'lsa}, \\ x, & \text{agar } x \geq 0 \text{ bo'lsa} \end{cases}$; 12) $y = \begin{cases} 2, & \text{agar } x > 0 \; bo'lsa, \\ -2, & \text{agar } x < 0 \; bo'lsa \end{cases}$;

13) $y = \begin{cases} 2x+2, & \text{agar } 0 \leq x \leq 3 \text{ bo'lsa}, \\ 8, & \text{agar } 3 \leq x \leq 6 \text{ bo'lsa}, \\ x+2, & \text{agar } x \geq 6 \text{ bo'lsa}; \end{cases}$;

14) $y = \begin{cases} -2x-2, & \text{agar } x < -1 \; bo'lsa, \\ -\sqrt{1-x^2}, & \text{agar } -1 \leq x \leq 1 \text{ bo'lsa}, \\ 2x-2, & \text{agar } x > 1 \; bo'lsa; \end{cases}$;

15) $y = \begin{cases} 2, & \text{agar } x \leq -1 \text{ bo'lsa}, \\ x+3, & \text{agar } -1 \leq x \leq 0 \text{ bo'lsa}, \\ -x+3, & \text{agar } 0 \leq x \leq 1 \text{ bo'lsa}, \\ 2, & \text{agar } x \geq 1 \text{ bo'lsa}. \end{cases}$

31. Quyidagi funksiyalarga teskari funksiyalar yozilsin.

1) $y = 3x - 1$; 2) $y = \frac{x-1}{2} + 3$; 3) $y = lg\frac{x}{5}$ $(x > 0)$;

4) $y = \arcsin\frac{x}{3} (-3 \leq x \leq 3)$; 5) $y = 5\text{arctg}x(-\infty < x < +\infty)$;

6) $y = \frac{x-1}{2-3x}$; 7) $y = 5^{lgx}$; 8) $y = cos^2x - sin^2x$.

J: 1) $y = \frac{x+1}{3}$; 2) $y = 2x - 5$; 3) $y = 5 \cdot 10^x$; 4) $y = 3\sin x$;

5) $y = \text{tg}\frac{x}{2}$;6) $y = \frac{2x+1}{3x+1}$; 7) $y = 10^{\frac{lgx}{lg5}}(0 < x < +\infty)$;

8) $y = \frac{1}{2}\arccos x \; (-1 \leq x \leq 1)$.

32. Murakkab funksiyalarga misollar keltiring.

33. $y = \ln \sin x$ bo'lsa, oraliq o'zgaruvchini yozing (u=sin$x$).

34. $y = \text{tg}\sqrt{x}$ bo'lsa, oraliq o'zgaruvchini yozing (u=$\sqrt{x}$).

35. Quyidagi funksiyalarning o'zgarish sohasini toping.

1) $y = \sqrt{16 - x^2}$J: [0;4];2) $y = 3cosx - 1$ J: [-4;2];

3) $y = 3^{-x^2}$ J: (0;1].

2-§. Ketma-ketlik va uning limiti

Agar har bir $n\epsilon N$ natural songa biror qonun qoida asosida ma'lum bir $x_n \epsilon R$ haqiqiy son mos qo'yilgan bo'lsa, u holda $x_1, x_2, x_3, \ldots, x_n, \ldots$ ga ***sonli ketma-ketlik*** deb ataladi. x_n sonli ketma-ketlikning ***umumiy hadi*** deyiladi. Sonli ketma-ketlikni qisqacha $\{x_n\}$ kabi yoziladi.

Agar shunday M (yoki m) soni mavjud bo'lsaki, $\{x_n\}$ ketma-ketlikning barcha hadlari uchun $x_n \leq M$ (yoki $x_n \geq m$) shart bajarilsa, u holda bu ketma-ketlik ***yuqoridan(quyidan)chegaralangan*** deyiladi.

Ham quyidan, ham yuqoridan chegaralangan ketma-ketlik ***chegaralanganketma-ketlik*** deyiladi.

Ixtiyoriy $M > 0$ son uchun $\{x_n\}$ ketma-ketlikning kamida bitta hadi $|x_n| > M$ tengsizlikni qanoatlantirsa, bu ketma-ketlik ***chegaralanmagan*** deyiladi.

Agar har qanday $n \epsilon N$ natural son uchun $x_{n+1} > x_n$ $(x_{n+1} < x_n)$ tengsizlik bajarilsa, $\{x_n\}$ ketma-ketlik o'suvchi (kamayuvchi) deyiladi. Faqat o'suvchi yoki kamayuvchi ketma-ketlik ***monoton ketma-ketlik*** deyiladi.

Hamma hadlari bir xil a soniga teng bo'lgan ketma-ketlik ***o'zgarmas ketma-ketlik*** deyiladi.

Agar istalgan $\varepsilon > 0$ son uchun shunday $N = N(\varepsilon) > 0$ son mavjud bo'lsaki, barcha $n > N$ lar uchun $|x_n - a| < \varepsilon$ tengsizlik bajarilsa, o'zgarmas a son $\{x_n\}$ ketma-ketlikning ***limiti*** deyiladi va u quyidagicha yoziladi: $\lim\limits_{n\to\infty} x_n = a$.

Agar $\{x_n\}$ ketma-ketlik limitga ega bo'lsa, u ***yaqinlashuvchi,*** aks holda ***uzoqlashuvchi*** deyiladi.

Har qanday chegaralangan va monoton ketma-ketlik limitga ega.

Ixtiyoriy $M > 0$ son uchun bu songa bog'liq shunday N_M soni topilsaki, $\{x_n\}$ ketma-ketlikning$n > N_M$shartni qanoatlantiruvchi barcha hadlari uchun $|x_n| > M$ tengsizlik bajarilsa, u holda bu ketma-ketlik ***cheksiz limitga*** ega deyiladi va u $\lim x_n = \pm\infty$ kabi yoziladi.

Agar $\{x_n\}$ va $\{y_n\}$ ketma-ketliklarning ikkalasi ham yaqinlashuvchi va $\lim x_n = A$, $\lim y_n = B$ bo'lsa, u holda quyidagi tengliklar o'rinli bo'ladi:

$$\lim(x_n \pm y_n) = \lim x_n \pm \lim y_n = A \pm B;$$

$$\lim(x_n \cdot y_n) = \lim x_n \cdot \lim y_n = A \cdot B;$$

$$\lim \frac{(x_n)}{(y_n)} = \frac{\lim x_n}{\lim y_n} = \frac{A}{B} \ (\lim y_n = B \neq 0).$$

Ketma-ketliklarni limitini hisoblashda ***ajoyib limit*** deb ataluvchi quyidagi tenglik muhimdir:

$$\lim\left(1+\frac{1}{n}\right)^n = e.$$

bu yerdagi e soni e ≈2,718281… irratsional son bo'lib u matematikada juda ko'p qo'llaniladi. Masalan, asosi e bo'lgan logarifm ***natural logarifm*** deb ataladi va **ln** bilan belgilanadi.

1. Umumiy hadi bilan berilgan quyidagi ketma-ketliklarning dastlabki bir nechta hadlari yozilsin:

1) $x_n = n$;2) $x_n = \frac{n}{n+2}$; 3) $x_n = \frac{2n}{3n-2}$; 4) $x_n = \frac{1}{n}$;

5) $x_n = -2^n$; 6) $x_n = n!$; 7) $x_n = \frac{1}{2^n}$; 8) $x_n = \frac{n^2-1}{n^2+1}$;

9) $x_n = \frac{1}{(3n-1)(3n+1)}$; 10) $x_n = sin\frac{\frac{n\pi}{2}}{n}$; 11) $x_n = \frac{1+(-1)^n}{2}$.

J: 1) 1, 2, 3, 4, 5, … ; 2) $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \dots$; 3) $\frac{2}{1}, \frac{4}{4}, \frac{6}{7}, \frac{8}{10}, \dots$;

4) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \dots$; 5) $-2, -4, -8, -16, \dots$; 6) 1!, 2!, 3!, 4!, … ;

7) $\frac{1}{2}, \frac{1}{4}, \frac{1}{,8}, \frac{1}{16}, \dots$; 8) $0, \frac{3}{5}, \frac{8}{10}, \frac{15}{17}, \dots$; 9)$\frac{1}{2\cdot4}, \frac{1}{5\cdot7}, \frac{1}{8\cdot10}, \frac{1}{11\cdot13}, \dots$;

10) $1, 0, -\frac{1}{3}, 0, -\frac{1}{5}, \dots$; 11) 0, 1, 0, 1, … .

2. Dastlabki bir nechta hadlari bilan berilgan quyidagi ketma-ketliklarning umumiy hadi yozilsin:

1) $\frac{6}{7}, \frac{9}{10}, \frac{14}{15}, \frac{21}{22}, \frac{30}{31}, \dots$, J: $x_n = \frac{n^2+5}{n^2+6}$;

2) $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \frac{1}{15}, \dots$, J: $x_n = \frac{1}{3n}$;

3) $\frac{1}{3\cdot4}, \frac{1}{5\cdot6}, \frac{1}{7\cdot8}, \frac{1}{9\cdot10} \dots$, J: $x_n = \frac{1}{(2n+1)(2n+2)}$;

4) $\frac{1}{6}, \frac{4}{11}, \frac{7}{16}, \frac{10}{21}, \frac{13}{26}, \dots$, J: $x_n = \frac{3n-2}{5n+1}$;

5) $\frac{3}{5}, \frac{7}{8}, \frac{11}{11}, \frac{15}{14}, \frac{19}{17}, \dots$, J: $x_n = \frac{4n-1}{3n+2}$;

6) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}, \dots$, J: $x_n = \frac{1}{3^n}$;

7) $0, \frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \dots$, J: $x_n = \frac{n-1}{n+1}$;

8) $\frac{3}{5}, \frac{12}{17}, \frac{27}{37}, \frac{48}{65}, \frac{75}{101}, \dots,$ J: $x_n = \frac{3n^2}{4n^2+1}$;

9) $\frac{2}{3}, \frac{5}{8}, \frac{10}{13}, \frac{17}{18}, \frac{26}{23}, \dots,$ J: $x_n = \frac{n^2+1}{5n-2}$;

10) $1, \frac{1}{2}, 2, \frac{1}{3}, 3, \frac{1}{4}, 4, \frac{1}{5}, \dots.$ J: $x_n = \begin{cases} k, & agar \ n = 2k-1 \\ \frac{1}{k+1}, & agar \ n = 2k \end{cases}.$

3. Quyida berilgan ketma-ketliklardan qaysilari monoton, qaysilari o'zgarmas va qaysilari chegaralangan ketma-ketliklar bo'ladi?

1) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots,$ J: Monoton kamayuvchi va chegaralangan;

2) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \dots, (-1)^{n+1}\frac{1}{n}, \dots,$ J: Chegaralangan;

3) $-1, 1, -1, 1, \dots (-1)^n, \dots,$ J: Chegaralangan;

4) 3, 3, 3, 3, …, 3, … , J: O'zgarmas;

5) $-1, -4, -9, -16, \dots, -n^2, \dots,$ J: Monoton kamayuvchi;

6) 2, 4, 8, 16,…2^n, … , J: Monoton o'suvchi

7) $1, 2, \frac{1}{3}, 4, \frac{1}{5}, \dots, n^{(-1)^n}, \dots,$

8) $-1, 2, -3, 4, -5, 6, \dots, (-1)^n \cdot n, \dots.$

4.$a_n = \frac{n}{n+1}$sonli ketma-ketlikning quyi va yuqori chegaralarini ko'rsating. J: 0; 1.

5. Quyidagi ketma-ketliklardan qaysilari chegaralangan?

$\{n^2+3\}$; $\{(-1)^n \cdot n\}$; $\left\{(n)^{(-1)^n}\right\}$; $\left\{\frac{n^2-1}{n^2}\right\}$; $\left\{\frac{(-1)^n}{3}\right\}$. J: $\left\{\frac{n^2-1}{n^2}\right\}$.

6. $x_n = \frac{n}{2n+1}$ ketma−ketlikni o'suvchi ekanligini isbotlang.

7. Umumiy hadi $x_n = \frac{n}{4n-3}$ bo'lgan ketma-ketlikni monoton kamayuvchi ekanligini isbotlang.

8. Umumiy hadi $x_n = \frac{n-1}{n}$ ketma-ketlikni monoton o'suvchi ekanligini isbotlang.

9. Umumiy hadi $x_n = \frac{3n}{n+1}$ bo'lgan ketma-ketlikni chegaralangan va monotono'suvchi ekanligi isbotlansin.

10. Umumiy hadi $x_n = \frac{2^n+1}{2^n}$ bo’lgan ketma-ketlikni chegaralangan va monoton kamayuvchi ekanligini isbotlang.

11. Umumiy hadi $x_n = \frac{2n-1}{3n+1}$ bo’lgan ketma-ketlikni o’suvchi ekanligi isbotlansin.

12. Ketma-ketlikning limiti ta’rifidan foydalanib quyidagilarni isbot qiling:

1) Umumiy hadi $x_n = \frac{n}{n+1}$ bo’lgan ketma-ketlikning limiti 1 ga tengligini;

2) Umumiy hadi $x_n = \frac{4n}{2n+1}$ bo’lgan ketma-ketlikning limiti 2 ga tengligini;

3) Umumiy hadi $x_n = \frac{2n-1}{2n+1}$ bo’lgan ketma-ketlikning limiti 1 ga tengligini;

4) Umumiy hadi $x_n = \frac{3n^2+1}{5n^2-1}$ bo’lgan ketma-ketlikning limiti $\frac{3}{5}$ ga tengligini;

13. Umumiy hadi $x_n = \frac{n^2-2}{2n^2-9}$ bo’lgan ketma-ketlikning limiti 0 ga teng emasligini isbotlang.

14. $1, \frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{5}, \frac{3}{4}, \frac{1}{7}, \frac{4}{5}, \ldots$ ketma-ketlikning limitga ega emasligi isbotlansin.

15. Umumiy hadi bilan berilgan quyidagi ketma-ketliklarni cheksiz kichik ekanligini isbotlang.

$$1)\ x_n = \frac{1}{n^k}(k > 0); \quad 2)\ x_n = \frac{1-(-1)^n}{n}; \quad 3)\ x_n = \frac{1}{n} sin\left[(2n-1)\frac{\pi}{2}\right]$$

<u>Ko’rsatma:</u> $\lim\limits_{n\to\infty} x_n = 0$ ekanligi ko’rsatiladi.

16. Umumiy hadi bilan berilgan quyidagi ketma-ketliklarni n→∞ ga cheksiz katta ekanligi isbotlansin.

1) $x_n = 3^{\sqrt[3]{n}}$; 2) $x_n = 7^n$.

17. $\lim\limits_{x\to 2+0} \frac{3}{x-2}$ va $\lim\limits_{x\to 2-0} \frac{3}{x-2}$ lar topilsin va jadvallar bilan tushuntirilsin.J: $+\infty$ va $-\infty$.

18. $\lim\limits_{x\to 0+0} 2^x$ va $\lim\limits_{x\to 0-0} 2^{\frac{1}{x}}$lar topilsin va jadvallar bilan tushuntirilsin.J:+∞ va 0.

19. Quyidagi 1) $\frac{2}{\infty} = 0$; 2) $\frac{2}{0} = \pm\infty$; 3) $3^{\infty} = \infty$; 4) $3^{-\infty} = 0$; 5) $\lg 0 = -\infty$ va 6) $tg\, 90^{\circ} = \pm\infty$ “shartli” yozuvlarning aniq ma’nolari tushuntirilsin.

20. Quyidagi limitlarni hisoblang.

1) $\lim\limits_{n\to\infty}\frac{1+3n+2n^2}{1-n}$; 2) $\lim\limits_{n\to\infty}\frac{7n^2+2n-3}{5n^2-4n+1}$; 3) $\lim\limits_{n\to\infty}\frac{3n^3+n^2-n+1}{5n^3-4n+17}$;

4) $\lim\limits_{n\to\infty}\frac{5n^2-4n+2}{n^{3-4n+1}}$; 5) $\lim\limits_{n\to\infty}\frac{1^2+2^2+3^2+\cdots+n^2}{3n^3+n+1}$; 6) $\lim\limits_{n\to\infty}\frac{5n+7}{3-4n}$;

7) $\lim\limits_{n\to\infty}\frac{n^3+1}{n^2-1}$; 8) $\lim\limits_{n\to\infty}\frac{n+5}{n^2+n-1}$; 9) $\lim\limits_{n\to\infty}\left(\frac{2n^2+n-1}{5n^2-7n+12}\right)^2$;

10) $\lim\limits_{n\to\infty}\left(1+\frac{3}{n}\right)\left(2-\frac{4}{n}\right)^2\left(\frac{5}{n^2}-1\right)$; 11) $\lim\limits_{n\to\infty}3^{\frac{6n+2}{3n-4}}$;

12) $\lim\limits_{n\to\infty}\left(\frac{1}{2}\right)^{\frac{n}{n-1}}$;13) $\lim\limits_{n\to\infty}\left(\sqrt{n+2}-\sqrt{n}\right)$;

14) $\lim\limits_{n\to\infty}(\sqrt{2n+3}-\sqrt{n-1})$; 15) $\lim\limits_{n\to\infty}\sqrt{n}\left(\sqrt{n+1}-\sqrt{n}\right)$;

16) $\lim\limits_{n\to\infty}\frac{\sqrt{n^2+1}}{n+1}$; 17) $\lim\limits_{n\to\infty}\frac{\sqrt{n^2+4n}}{\sqrt{n^3-3n^2}}$; 18) $\lim\limits_{n\to\infty}\sqrt[3]{\frac{n^2-n+1}{8n^2+n+3}}$;

19) $\lim\limits_{n\to\infty}\left(\sqrt[3]{\frac{5n}{4n+3}}\right)^{-\frac{1}{2}}$;20) $\lim\limits_{n\to\infty}\frac{1-2+3-4+5-6+\cdots-2n}{\sqrt{n^2+1}+\sqrt{4n^2-1}}$;

21) $\lim\limits_{n\to\infty}\frac{\sqrt[3]{n^2+5n}}{3n+2}$;22) $\lim\limits_{n\to\infty}\left(\sqrt[3]{1-n^3}+n\right)$; 23) $\lim\limits_{n\to\infty}\frac{1}{n(n+1)}$;

24) $\lim\limits_{n\to\infty}(\sqrt{2n+3}-\sqrt{n-1})$; 25) $\lim\limits_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n+1}+\sqrt{n}}$;26) $\lim\limits_{n\to\infty}\frac{\sqrt{n^2+4n}}{\sqrt[3]{n^3-3n^2}}$.

J: 1) -2; 2) $\frac{7}{5}$; 3) $\frac{3}{5}$; 4) 0; 5) $\frac{1}{9}$; 6) $-\frac{5}{4}$; 7) ∞; 8) 0; 9) $\frac{4}{25}$; 10) -4; 11) 9; 12) $\frac{1}{2}$; 13) 0; 14) ∞; 15) $\frac{1}{2}$; 16) 1;17) 1; 18) $\frac{1}{2}$; 19) $\left(\frac{4}{5}\right)^{\frac{1}{6}}$; 20) $-\frac{1}{3}$; 21) 0; 22) 0; 23) 1; 24) 0; 25) $\frac{1}{2}$; 26) 1.

3-§. Funksiyaning limiti

Agar oldindan berilgan ixtiyoriy $\varepsilon > 0$ son uchun unga bog'liq shunday $\delta > 0$son topilsaki, $|x-a| < \delta$ shartni qanoatlantiruvchi har qanday $x\epsilon D(f)$ va biror A son uchun $|f(x)-A| < \varepsilon$tengsizlik bajarilsa, A soni $y = f(x)$**funksiyaning** $\boldsymbol{x \to a}$ **bo'lgandagi limiti**deyiladi.

Ta'rifdagi tasdiq

$$\lim_{x\to a} f(x) = A$$

ko'rinishda yoziladi.

Agar har qanday katta $N > 0$ son uchun shunday $\delta = \delta(N) > 0$ son mavjud bo'lsaki, $|x - a| < \delta$ shartniqanoatlantiruvchi $\forall x \epsilon D\{f\}$ uchun $|f(x)| > N$ tengsizlik bajarilsa, u holda y $= f(x)$ funksiya $x \to a$ bo'lganda cheksiz limitga ($+\infty$ yoki $-\infty$)ega deyiladi va u quyidagicha yoziladi:

$$\lim_{x \to a} f(x) = \pm\infty$$

Agar har qanday kichik $\varepsilon > 0$ son uchun shunday katta $M = M(\varepsilon)$ son mavjud bo'lsaki, $|x| > M$ shartni qanoatlantiruvchi barcha $x \epsilon D\{f\}$ va biror chekli A soni uchun $|f(x) - A| < \varepsilon$ tengsizlik bajarilsa, u holda $y = f(x)$ funksiya $x \to \pm\infty$ bo'lganda ***chekli limitga ega*** deyiladi va u quyidagicha yoziladi:

$$\lim_{x \to \pm\infty} f(x) = A$$

Agar har qanday katta $N > 0$ soni uchun shunday M$= M(N)$son mavjud bo'lsaki, $|x| > M$ shartni qanoatlantiruvchi barcha $x \epsilon D\{f\}$ uchun $|f(x)| > N$ tengsizlik o'rinli bo'lsa, u holda $y = f(x)$ funksiya $x \to \pm\infty$ bo'lganda cheksiz limitga ega deyiladi va $\lim_{x \to \pm\infty} f(x) = \pm\infty$ko'rinishida yoziladi.

Agar □ $\to a$bo'lganda$y = f(x)$ funksiya limitga ega bo'lsa, u holda bu limit yagona bo'ladi.

Agar $y = f(x)$funksiya argumenti x chekli a soniga faqat chap ($x < a$) yoki faqat o'ng ($x > a$) tomondan yaqinlashib borganda ($x \to a - 0$ yoki $x \to a$+0 kabi belgilanadi) funksiya limiti biror A_1 yoki A_2 songa teng bo'lsa, bu songa funksiyaning a nuqtadagi chap yoki o'ng limiti deyiladi.

Ular $\lim_{x \to a-0} f(x) = f(a - 0) = A_1$ yoki $\lim_{x \to a+0} f(x) = f(a + 0) = A_2$ ko'rinishda yoziladi.

Biror a nuqtada $y = f(x)$ funksiya $x \to a$ bo'lganda chekli A limitga ega bo'lishi uchun uning shu a nuqtadagi chap va o'ng limitlari o'zaro teng va$f(a - 0) = f(a + 0) = A$ shart bajarilishi zarur va yetarlidir.

Agar $\alpha(x)$ funksiya uchun$\lim_{x \to a} \alpha(x) = 0$ shart bajarilsa, u holda bu funksiya $x \to a$ bo'lganda ***cheksiz kichik funksiya*** deyiladi.

Agar $x \to a$ bo'lganda $\alpha(x)$ va $\beta(x)$ cheksiz kichik funksiyalar bo'lib, $f(x)$ esa ixtiyoriy chegaralangan funksiya bo'lsa, u holda $x \to a$ bo'lganda$\alpha(x) \pm \beta(x)\alpha(x) \cdot \beta(x)$, $f(x) \cdot \alpha(x)$, $c\alpha(x)$ ($c-$ o'zgarmas son) funksiyalar ham cheksiz kichik funksiyalar bo'ladi.

$x \to a$ bo'lganda $\alpha(x)$ va $\beta(x)$ cheksiz kichik miqdorlar va $\lim\limits_{x \to a} \frac{\alpha(x)}{\beta(x)} = A = 0$ bo'lsa, u holda $\alpha(x)$ funksiya $x \to a$ bo'lganda $\beta(x)$ ga nisbatan yuqori tartibli cheksiz kichik funksiya deyiladi va u $\alpha(x) = 0(\beta(x))$ kabi yoziladi. Agar $A \neq 0$ chekli son bo'lsa, u holda $\alpha(x)$ va$\beta(x)$ lar bir xil tartibli cheksiz kichik funksiyalar deyiladi.Agar $A = 1$ bo'lsa, u holda $\alpha(x)$ va $\beta(x)$ lar ekvivalent cheksiz kichik funksiyalar deyiladi va $\alpha(x) \sim \beta(x)$ kabi yoziladi. Agar $A = \pm\infty$ bo'lsa, u holda $\alpha(x), x \to a$ bo'lganda $\beta(x)$ ga nisbatan quyi tartibli cheksiz kichik funksiya deyiladi.

Agar $f(x)$ funksiya uchun

$$\lim_{x \to a} \alpha(x) = \pm\infty$$

bo'lsa , u holda bu $\alpha(x)$ funksiya $x \to a$ bo'lganda ***cheksiz katta funksiya*** deyiladi.

Agar $f(x)$ va $g(x)$ funksiyalar $x \to a$ bo'lganda cheksiz katta funksiyalar bo'lsa, u holda $x \to a$ da quyidagilar o'rinlidir:

1. $|f(x)|$+$|g(x)|$va $f(x) \cdot g(x)$ cheksiz katta bo'ladi.
2. Agar$\lim\limits_{x \to a} h(x) \neq 0$ bo'lsa , u holda$f(x) \cdot h(x)$va$\frac{f(x)}{h(x)}$ cheksiz katta bo'ladi.
3. Ixtiyoriy co'zgarmas son va chegaralangan$f(x)$ funksiya uchun $cf(x)$ va $\alpha(x) \cdot f(x)$funksiyalarcheksiz katta bo'ladi.

Agar $f(x)$ funksiya $x \to a$ bo'lganda cheksiz katta funksiya bo'lsa, u holda $\frac{1}{f(x)}$ cheksiz kichik funksiya bo'ladi va aksincha , $f(x)$ funksiya $x \to a$ bo'lganda cheksiz kichik funksiya bo'lsa, u holda $\frac{1}{f(x)}$ cheksiz katta funksiya bo'ladi.

$y = f(x)$ funksiya $x \to a$ bo'lganda chekli A limitga ega bo'lishi uchun u $f(x) = A + \alpha(x)$ ko'rinishda bo'lishi zarur va yetarlidir.

Limitlarni hisoblashda quyidagilar o'rinlidir:

Agar $x \to a$ bo'lganda $f(x)$va $\varphi(x)$ funksiyalar chekli A va B limitlarga ega bo'lsa, u holda:

$\lim\limits_{x\to a}[f(x) \pm \varphi(x)] = \lim\limits_{x\to a} f(x) \pm \lim\limits_{x\to a} \varphi(x) = A \pm B$;

$\lim\limits_{x\to a} Cf(x) = C\lim\limits_{x\to a} f(x) = CA$;

$\lim\limits_{x\to a} f(x) \cdot \varphi(x) = \lim\limits_{x\to a} f(x) \cdot \lim\limits_{x\to a} \varphi(x) = AB$ va agar $\lim\limits_{x\to a} \varphi(x) \neq 0$ bo'lsa, $\lim\limits_{x\to a} \frac{f(x)}{\varphi(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} \varphi(x)} = \frac{A}{B}$tengliklar o'rinlidir.

Agar$x = a$ nuqtaning biror atrofida $\varphi(x) \leq f(x) \leq g(x)$ qo'sh tengsizlik o'rinli bo'lib, $x \to a$ da $\varphi(x)$va $g(x)$ funksiyalarning chekli limitlari mavjud va

$$\lim_{x\to a} \varphi(x) = \lim_{x\to a} g(x) = A$$

shart o'rinli bo'lsa,u holda $x \to a$ da $f(x)$ funksiya uchun ham chekli limit mavjud, ya'ni$\lim\limits_{x\to a} f(x) = A$ bo'ladi.

Agar$x = a$ nuqtaning biror atrofida $y = f(x)$ funksiya o'suvchi (kamayuvchi) bo'lib yuqoridan (quyidan) biror M (m) soni bilan chegaralangan bo'lsa, u holda bu funksiya $x \to a$ limitga ega va uning uchun$\lim\limits_{x\to a} f(x) \leq M$ ($\lim\limits_{x\to a} f(x) \geq m$)tengsizlik o'rinli bo'ladi.

Turli funksiyalarning limitini hisoblashda muhim limitlar deb ataluvchi quyidagi tengliklardan foydalaniladi:

$\lim\limits_{x\to 0} \frac{\sin x}{x} = 1$; $\lim\limits_{n\to\pm\infty} \left(1 + \frac{1}{x}\right)^x = \lim\limits_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$ =2,718281...;

$\lim\limits_{x\to 0} \frac{\ln(1+\alpha x)}{x} = \alpha$;$\lim\limits_{x\to 0} \frac{a^x-1}{x}$ $\ln a$;$\lim\limits_{x\to 0} \frac{(1+x)^\alpha-1}{x} = \alpha$.

Funksiya limitining " $\varepsilon - \delta$ " tilidagi ta'rifidan foydalanib quyidagilar isbotlansin.

1. $\lim\limits_{x\to 1}(3x - 8) = -5$; 2. $\lim\limits_{x\to\infty} \frac{5x+1}{3x+9} = \frac{5}{3}$;

3. $\lim\limits_{x\to 1} \frac{1}{(1-x)^2} = \infty$; 4. $\lim\limits_{x\to\infty} \log_a x = \infty$ $(a > 1)$;

5. $\lim\limits_{x\to\infty} arctgx = \frac{\pi}{2}$; 6. $\lim\limits_{x\to\frac{\pi}{2}} \sin x = \frac{1}{2}$;

7. $\lim_{x\to 1}(3x-2)=1$; 8. $\lim_{x\to 1}\frac{x-1}{\sqrt{x}+1}=2$;

9. $\lim_{x\to 0}\sin x=0$; 10. $\lim_{x\to 0}\cos x=1$; .

2. $y=\begin{cases}3x-1, & x<1\\ 2x+1, & x\geq 1\end{cases}$ funksiyaning $x=1$ nuqtadagi chap va o’ng limitlarini toping. J: chap limit 2, o’ng limit 3.

3. $y=\begin{cases}3x^2+2x-1, & x\leq 0\\ 2x^2+1, & x>0\end{cases}$ funksiyaning $x=0$ nuqtadagi chap va o’ng limitini toping. J: chap limit -1, o’ng limit 1.

4. $f(x)=\begin{cases}n-sin\,x, & x<0\\ n-e^{nx}, & x\geq 0\end{cases}$ funksiyaning $x=0$nuqtadagi o’ng limitini toping.

5.Quyidagi funksiyalarni ko’rsatilgan nuqtalardagi bir tomonlama limitlarini toping.

1) $f(x)=\begin{cases}-2x+3 & agar\ x\leq 1\ \ bo’lsa\\ 3x-5 & agar\ x>1\ \ bo’lsa\end{cases}$ni $x=1$ nuqtadagi;

J: chap limit 1, o’ng limit -2.

2) $f(x)=\frac{x^2-1}{|x-1|}$ni$x=1$ nuqtadagi;

3) $f(x)=\frac{\sqrt{1-cos\,2x}}{x}$ni $x=0$ nuqtadagi;

J: chap limit $-\sqrt{2}$; o’ng limit $\sqrt{2}$.

4) $f(x)=\frac{5}{(x-2)^3}$ni$x=2$ nuqtadagi;

5) $f(x)=\begin{cases}x+1, & agar\ 0\leq x<1\ bo’lsa\\ 3x+2, & agar\ 1<x<3\ bo’lsa\end{cases}nix=1$ nuqtadagi;

J: chap limit 2, o’ng limit 5.

6.Quyidagi limitlar hisoblansin.

1) $\lim_{x\to 3}(x^2-7x+4)$; 2) $\lim_{x\to 2}(2x^3-7x^2+4x+2)$;

3)$\lim_{x\to 4}(\frac{1}{2}x^3-x+2)$;4) $\lim_{x\to 3}\frac{x^2+x+2}{x^2+2x+8}$; 5) $\lim_{x\to -1}\frac{x^2-x+1}{2x^3-x^2+x+2}$;

6) $\lim_{x\to 2}\frac{x^3-8}{x-2}$;7)$\lim_{x\to 3}\frac{x^2+x-12}{2x^2-9x+9}$; 8)$\lim_{x\to 1}\frac{x^3-x^2-x+1}{x^3-3x+2}$;

9)$\lim_{x\to 0}\frac{5x^3-6x^2}{4x^5+2x^3+x^2}$;10) $\lim_{x\to -3}\frac{x^3+5x^2+3x-9}{x^3-3x^2-45x-81}$; 11) $\lim_{x\to -1}\frac{1+\sqrt[7]{x}}{1+\sqrt[5]{x}}$;

12) $\lim\limits_{x\to1}\frac{1-\sqrt[4]{x}}{1-\sqrt[6]{x}}$; 13) $\lim\limits_{x\to2}\left(\frac{1}{x-2}-\frac{1}{x^2-4}\right)$; 14) $\lim\limits_{x\to1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$;

15) $\lim\limits_{x\to0}\frac{\sqrt{1+x}-1}{x}$; 16) $\lim\limits_{x\to2}\frac{\sqrt{x^2+5}-3}{x-2}$; 17) $\lim\limits_{x\to3}\frac{\sqrt{x+6}-3}{x-3}$;

18) $\lim\limits_{x\to-2}\frac{\sqrt{6+x}-2}{x+2}$; 19) $\lim\limits_{x\to2}\frac{\sqrt{x^2+21}-5}{x-2}$; 20) $\lim\limits_{x\to1}\frac{\sqrt{5-x^2}-2}{1-x}$;

21) $\lim\limits_{x\to1}\frac{\sqrt{x+8}-\sqrt{8x+1}}{\sqrt{5-x}-\sqrt{7x-3}}$; 22) $\lim\limits_{x\to\infty}(\sqrt{x-2}-\sqrt{x})$;

23) $\lim\limits_{x\to\infty}x(\sqrt{x^2+1}-x)$; 24) $\lim\limits_{x\to\infty}\frac{\sqrt{x^2+4}}{x+2}$;

25) $\lim\limits_{x\to\infty}(\sqrt{x^2+x+1}-\sqrt{x^2-x+1})$; 26) $\lim\limits_{x\to\infty}(\sqrt{x^2+2}-x)$.

J: 1) -8; 2) -2; 3) 30; 4) $\frac{14}{23}$; 5) - $\frac{3}{2}$; 6) 12; 7) $\frac{14}{3}$; 8) $\frac{2}{3}$; 9) -6; 10) $\frac{1}{3}$; 11) $\frac{5}{7}$; 12) $\frac{3}{2}$; 13) ∞; 14) -1; 15) $\frac{1}{2}$; 16) $\frac{2}{3}$; 17) $\frac{1}{6}$; 18) $\frac{1}{4}$; 19) $\frac{2}{5}$; 20) $\frac{1}{2}$; 21) $\frac{7}{12}$; 22) 0; 23) $\frac{1}{2}$; 24) 1; 25) 1; 26) 0.

$\lim\limits_{x\to0}\frac{\sin x}{x}$ limitni qo'llashga doir misollar.

7. Quyidagi limitlar hisoblansin.

1) $\lim\limits_{x\to0}\frac{\sin 2x}{x}$; 2) $\lim\limits_{x\to0}\frac{\sin 17x}{8x}$; 3) $\lim\limits_{x\to0}\frac{\sin 6x}{\sin 2x}$;

4) $\lim\limits_{x\to0}\frac{\sin 3x}{\sin 5x}$; 5) $\lim\limits_{x\to0}\frac{1-\cos 6x}{4x^2}$; 6) $\lim\limits_{x\to0}\frac{\mathrm{tg}\,2x}{x}$;

7) $\lim\limits_{x\to0}\frac{1-\cos 8x}{2x^2}$; 8) $\lim\limits_{x\to0}\frac{1-\cos x}{x^2}$; 9) $\lim\limits_{x\to0}\frac{3\mathrm{tg}\,x}{x}$;

10) $\lim\limits_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\cos^2 x}$; 11) $\lim\limits_{x\to0}\frac{\mathrm{tg}\,x-\sin x}{x^3}$; 12) $\lim\limits_{x\to0}\frac{1-\cos 2x}{x\sin x}$;

13) $\lim\limits_{x\to0}\frac{\sin 3x}{\sqrt{x+2}-\sqrt{2}}$; 14) $\lim\limits_{x\to\frac{1}{2}}\frac{\arcsin(1-2x)}{4x^2-1}$; 15) $\lim\limits_{x\to0}\frac{\sin 4x}{\sqrt{x+1}-1}$;

16) $\lim\limits_{x\to0}\frac{\sqrt{1-\cos 2x}}{x}$; 17) $\lim\limits_{x\to0}\frac{2x\sin x}{\sec x-1}$; 18) $\lim\limits_{x\to0}\frac{1-\cos mx}{x^2}$.

J: 1) 2; 2) $\frac{17}{8}$; 3) 3; 4) $\frac{3}{5}$; 5) $\frac{9}{2}$; 6) 2; 7) 16; 8) $\frac{1}{2}$; 9) 3; 10) $\frac{1}{2}$; 11) $\frac{1}{2}$; 12) 2; 13) $6\sqrt{2}$; 14) $-\frac{1}{2}$; 15) 8; 16) $-\sqrt{2}$; 17) 4; 18) $\frac{m^2}{2}$.

$\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x=\lim\limits_{\alpha\to0}(1+\alpha)^{\frac{1}{\alpha}}=e$ limitga doir misollar.

8. Quyidagi limitlar hisoblansin.

1) $\lim\limits_{x\to\infty}\left(1+\frac{5}{x}\right)^{x}$; 2) $\lim\limits_{x\to\infty}\left(1-\frac{1}{3x}\right)^{x}$; 3) $\lim\limits_{x\to\infty}\left(\frac{x}{x+1}\right)^{x}$;

4) $\lim\limits_{x\to 0}(1+2x)^{\frac{1}{x}}$; 5)$\lim\limits_{x\to 0}(1-4x)^{\frac{1-x}{2}}$; 6) $\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+1}\right)^{2x}$;

7) $\lim\limits_{x\to 0}(\cos x)^{stg^2x}$ ($\sin x=\alpha$ deb oling); 8) $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$;

9)$\lim\limits_{x\to\infty}\left(\frac{x-3}{x}\right)^{\frac{x}{2}}$;10)$\lim\limits_{x\to\infty}\left(\frac{3x-2}{3x+1}\right)^{2x}$; 11)$\lim\limits_{x\to 0}(1+3tg^2x)^{ctg^2x}$;

12)$\lim\limits_{x\to 0}(1+5tg^2x)^{3ctg^2x}$; 13)$\lim\limits_{x\to\infty}\left(\frac{2x+3}{5x+7}\right)^{\frac{2x-1}{x+2}}$;14)$\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+4}\right)^{x}$;

15) $\lim\limits_{x\to\infty}\left(\frac{x+7}{x+5}\right)^{2x+4}$;16)$\lim\limits_{x\to 0}(\cos x)^{ctg^2x}$; 17)$\lim\limits_{x\to\infty}\left(\frac{3x-4}{3x+2}\right)^{\frac{x+1}{3}}$.

Javoblar: 1)$e^{\mathbf{5}}$; 2) $e^{-\frac{1}{3}}$; 3)$e^{-\mathbf{1}}$; 4)$e^{\mathbf{2}}$; 5)$e^{-\mathbf{4}}$; 6) $e^{-\mathbf{2}}$; 7)$\frac{\mathbf{1}}{\sqrt{e}}$; 8)$e^{\mathbf{6}}$; 9) $\frac{\mathbf{1}}{e\sqrt{e}}$; 10) $\frac{\mathbf{1}}{e^{\mathbf{2}}}$; 11) $e^{\mathbf{3}}$; 12) $e^{\mathbf{15}}$; 13) $\frac{\mathbf{4}}{\mathbf{15}}$; 14)$e^{\mathbf{2}}\sqrt{e}$; 15)$e^{\mathbf{4}}$; 16)$e^{-\frac{1}{2}}$; 17) $e^{-\frac{2}{3}}$.

4-§. Funksiyaning uzluksizligi

Agar $y=f(x)$ funksiya $x=a$ nuqtaning biror atrofida aniqlangan va

$$\lim_{x\to a}f(x)=f(a)$$

bo'lsa, u holda funksiyani $x=a$ nuqtada uzluksiz deyiladi. Bu ta'rif quyidagi 4 ta uzluksizlik shartini o'z ichiga oladi:

1)$f(x)$ funksiya a nuqtaning qandaydir atrofida aniqlangan bo'lishi kerak;

2)Chekli $\lim\limits_{x\to a-0}f(x)$va $\lim\limits_{x\to a+0}f(x)$limitlar mavjud bo'lishi kerak;

3) Bu chap va o'ng limitlar bir hil bo'lishi kerak;

4)Bu limitlar $f(a)$ ga teng bo'lishi kerak;

Agar funksiya$[a,b]$ kesmaning har bir ichki nuqtasida uzluksiz bo'lib, uning chegaralarida esa $\lim\limits_{x\to a+0}f(x)=f(a)$ va $\lim\limits_{x\to b-0}f(x)=f(b)$ bo'lsa, u holda funksiyani shu kesmada uzluksiz deyiladi.

Agar x nuqta $x_{\mathbf{0}}$ nuqta atrofidan olingan bo'lsa, $x-x_{\mathbf{0}}$ ayirma argument orttirmasi deyiladi va Δx bilan belgilanadi. Bu holda $f(x)-f(x_0)$ ayirma funksiya orttirmasi deyiladi va u Δf yoki Δyorqali belgilanadi.

Agar $x \to x_0$ bo'lsa, u holda $\Delta x \to 0$ bo'ladi. Bundan $x = x_0 + \Delta x$ ni yozish mumkun. Bundan foydalanib uzluksizlik shartini

$$\lim_{\Delta x \to 0} f(x_0 + \Delta x) = f(x_0)$$

ko'rinishida yozish mumkun. Bundan $\Delta f = f(x) - f(x_0) = f(x_0 + \Delta x) - f(x_0)$ ekanligidan foydalanib,

$$\lim_{\Delta x \to 0} \Delta y = 0$$

ni yozish mumkun. Demak, $f(x)$ funksiya uzluksiz bo'lishi uchun argumentning "kichik" Δx orttirmasiga funksiyaning "kichik" Δy orttirmasi mos kelishi kerak ekan.

Barcha asosiy elementar funksiyalar o'zlarining aniqlanish sohasidagihar bir x_0 nuqtada uzluksizdir.

Agar $f(x)$ va $g(x)$ funksiyalar x_0 nuqtada uzluksiz bo'lsa, u holda $f(x) \pm g(x)$, $f(x) \cdot g(x)$ va $\frac{f(x)}{g(x)}$ $(g(x) \neq 0)$ lar ham bu nuqtada uzluksiz bo'ladi.

Agar $y = \varphi(x)$ funksiya x_0 nuqtada uzluksiz, $y = f(u)$ funksiya $U_0 = \varphi(x_0)$ nuqtada uzluksiz bo'lsa, u holda $y = f[\varphi(x)]$ murakkab funksiya ham x_0 nuqtada uzluksiz bo'ladi.

$y = f(x)$ funksiya biror $x = a$ nuqtada aniqlangan bo'lib, bu nuqtada uning o'ng (chap) limiti mavjud va

$$\lim_{x \to a+0} f(x) = f(a+0) = f(a) \ \left(\lim_{x \to a-0} f(x) = f(a-0) = f(a)\right)$$

tenglik o'rinli bo'lsa, u holda $f(x)$ funksiya a nuqtada o'ngdan (chapdan) uzluksiz deyiladi.

$y = f(x)$ funksiya $x = a$ nuqtada uzluksiz bo'lishi uchun bu nuqtada u ham chapdan, ham o'ngdan uzluksiz bo'lishi zarur va yetarlidir.

$[a, b]$ kesmada uzluksiz $y = f(x)$ funksiya shu kesmada o'zining eng katta M va eng kichik m qiymatiga erishadi, ya'ni bu kesmada kamida bittadan x_1 va x_2 nuqtalar topiladiki, ularda $f(x_1) = M$ va $f(x_2) = m$ tengliklar o'rinli bo'ladi.

Agar$y = f(x)$ funksiya $[a, b]$ kesmada uzluksiz va uning chegaralarida turli ishorali qiymatlarni qabul qilsa, ya'ni $f(a) \cdot f(b)$<0 bo'lsa, u holda kamida bitta shunday $c\epsilon(a, b)$ nuqta mavjud bo'ladiki, unda $f(c) = 0$ bo'ladi.

Berilgan $y = f(x)$ funksiya uchun ixtiyoriy $\varepsilon > 0$ soni bo'yicha shunday $\delta = \delta(\varepsilon) > 0$ soni topilsaki, biror $D \subset D\{f\}$ sohadagi $|x_1 - x_2| < \delta$ shartni qanoatlantiruvchi ixtiyoriy x_1 va x_2 nuqtalar uchun $|f(x_1) - f(x_2)| < \varepsilon$ tengsizlik bajarilsa, u holda y = f(x) funksiya D sohada ***tekis uzluksiz*** deyiladi.

Agar funksiya$a$ nuqtadan chapda va o'ngda aniqlangan bo'lsa, ammo a nuqtada uzluksizlikning 4 ta shartidan aqalli bittasi bajarilmasa, u holda $f(x)$ funksiya $x = a$ bo'lganda uzilishga ega bo'ladi. Uzilishlar ikki turga ajraladi.

x_0 nuqtada $y = f(x)$ funksiya aniqlanmagan, biroq shu nuqtadagi bir tomonlama limitlar mavjud va o'zaro teng, ya'ni $f(x_0 - 0)$= =$f(x_0 + 0)$ bo'lsa, x_0 nuqta funksiyaning ***yo'qotiladigan uzilish nuqtasi*** deyiladi.

Agar $x = a$ nuqta y $= f(x)$ funksiyaning uzilish nuqtasi bo'lib, bu nuqtada funksiyaning chap $f(a - 0)$ va o'ng $f(a + 0)$ limitlari mavjud hamda chekli sonlardan iborat bo'lsa, u holda$x = a$nuqta funksiyaning ***1-tur uzilish nuqtasi*** deyiladi. Bunda$\Delta = f(a + 0) - f(a - 0)$ soni funksiyaning$a$ uzilish nuqtasidagi ***sakrashi*** deyiladi.

Agar $y = f(x)$ funksiyaning $x = a$ uzilish nuqtasida uning chap va o'ng limitlaridan kamida bittasi cheksiz yoki mavjud bo'lmasa, u holda$x = a$nuqta funksiyaning ***2-tur uzilish nuqtasi*** deyiladi.

1. $f(x) = 3x^3 - 4x + 5$ funksiyaning (- ∞; + ∞) oraliqdagi har qanday nuqtada uzluksizligi isbotlansin.

2. $f(x) = 3x^4 + 5x^3 + 2x^2 + 3x + 4$ funksiya x ning har qanday qiymatida uzluksiz bo'lishi isbotlansin.

3. $f(x) = \begin{cases} x + 1, & x \leq 1 \\ x^2 - 1, & x > 1 \end{cases}$ funksiyani $x = 1$ nuqtada chapdan uzluksiz, o'ngdan esa uzluksiz emasligini ko'rsating.

4. $f(x) = \begin{cases} 2x+1, & x \geq 3 \\ 2x-1, & x<3 \end{cases}$ funksiyani $x = 3$ nuqtada o’ngdan uzluksiz, chapdan esa uzluksiz emasligini ko’rsating.

5. $\text{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ funksiyani$x = 0$ nuqtada o’ngdan ham, chapdan ham uzluksiz emasligini ko’rsating.

6. $y = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ funksiya o’ngdan ham, chapdan ham uzluksiz emasligini ko’rsating.

7. $f(x) = \frac{3x^2+x+5}{x^2-6x+8}$ funksiya x ning qanday qiymatlarida uzluksiz bo’lishi aniqlansin. J: $(-\infty;2) \cup (2;4) \cup (4;+\infty)$.

8. Funksiya uzluksizligining orttirmalar tilidagi ta’rifidan foydalanib quyidagi funksiyalarni uzluksizlikka tekshiring.

1) $f(x)$ =$5x^2 - 6x + 2$ funksiyani ixtiyoriy x nuqtada;

2) $f(x) = \frac{1+x^2}{1-x^2}$funksiyani $x = 3$ nuqtada;

3) $f(x) = \sin x$ funksiyani ixtiyoriy x nuqtada;

4) $f(x) = \cos x$ funksiyani ixtiyoriy x nuqtada;

9. Quyidagi funksiyalarni uzluksizlikka tekshiring.

1) $f(x) = \begin{cases} \frac{\sin x}{x}, & agar\ x \neq 0\ bo’lsa, \\ 1, & agar\ x = 0\ bo’lsa, \end{cases}$;

2) $f(x) = \sin\frac{1}{x}$;

3) $f(x) = \begin{cases} 4 \cdot 3^x, & agar\ x < 0\ bo’lsa, \\ 2a + x, & agar\ x \geq 0\ bo’lsa, \end{cases}$

4) $f(x) = \frac{x^3+1}{x+1}$

J: 1) x ning barcha qiymatlarida uzluksiz; 2) $x \neq 0$ da uzluksiz va $x = 0$ da 2- tur uzilishga ega; 3) funksiya $x = 0$ nuqtada uzilishga ega; 4) funksiya$x = -1$ da uzilishga ega.

10. Quyidagi funksiyalarni uzilish nuqtalari mavjud bo’lsa, ular topilsin va har bir uzilish nuqtasidagi funksiyaning sakrashi topilsin.

1) $f_1(x) = \frac{1}{x^2-4}$;   2) $f_2(x) = \frac{3x-5}{x^2+2x+10}$;   3) $f_3(x) = arcctg\,\frac{1}{x}$;

4) $f_4(x) = \frac{|x-3|}{x-3}$;   5) $f_5(x) = lg(x^2+3x)$.

11. Quyidagi funksiyalarni uzilish nuqtalari mavjud bo'lsa, ular topilsin va har bir uzilish nuqtasidagi funksiyaning sakrashi topilib funksiya grafigi yasalsin.

1) $f(x) = \begin{cases} -\frac{1}{2}x^2, & \text{agar } x \leq 2 \text{ bo'lsa}, \\ x, & \text{agar } x > 2\ bo'lsa. \end{cases}$;

2) $\varphi(x) = \begin{cases} 2\sqrt{x}, & \text{agar } 0 \leq x \leq 1 \text{ bo'lsa}, \\ 4-2x, & \text{agar } 1 < x < 2\ bo'lsa, \\ 2x-7, & \text{agar } 2{,}5 \leq x < +\infty. \end{cases}$;

3) $f(x) = \begin{cases} 2x+5, & \text{agar } -\infty < x < -1 \text{ bo'lsa}, \\ \frac{1}{x}, & \text{agar } -1 \leq x < +\infty \text{ bo'lsa}. \end{cases}$

12. Quyidagi funksiyalarning uzluksizlik va aniqlanish sohalari ustma-ust tushishi yoki tushmasligi aniqlansin.

1) $y = x^3 - 2x$; 2) $y = \sqrt{x}$;3) $y = \frac{1}{x^2-9;}$; 4) $y = \cos 2x$.

13. $\sin x - x + 1 = 0$ tenglama yechimga ega yoki yechimga ega emasligi aniqlansin.J: Yechimga ega.

14. $x^5 - 18x + 2 = 0$ tenglama $[-1;1]$ kesmada yechimga egami?

J: Yechimga ega.

15. $[-2;2]$ kesmada $f(x) = \begin{cases} x^2+2, \text{agar } & -2 \leq x < 0 \text{ bo'lsa}, \\ -(x^2+2), \text{ agar } & 0 \leq x \leq 2 \text{ bo'lsa}, \end{cases}$

funksiya berilgan. Bu kesmada $f(x) = 0$ bo'ladigan nuqta mavjudmi?.

J: Yo'q, $x = 0$ nuqtada uzilishga ega.

16. $f(x) = \frac{x^3}{4} - \sin \pi x + 3$ funksiya $[-2;2]$ kesma ichida $2\frac{1}{3}$ ga teng qiymatni qabul qiladimi?J: Xa.

17. $f(x) = \begin{cases} 2^x+1, & \text{agar } -1 \leq x < 0 \text{ bo'lsa}, \\ 2^x, & \text{agar } x = 0 \text{ bo'lsa}, \\ 2^x-1, & \text{agar } 0 < x \leq 1 \text{ bo'lsa}, \end{cases}$ funksiya $[-1;1]$ kesmada aniqlangan va chegaralangan. Funksiya bu kesmada eng katta qiymatga ham, eng kichik qiymatga ham ega bo'lmasligini ko'rsating.

18. $y = \frac{x}{x+2}$ funksiyaning uzilish nuqtasi ko’rsatilsin, $\lim_{x\to -2-0} y$, $\lim_{x\to -2+0} y$, $\lim_{x\to \pm\infty} y$, lar topilsin va $x = -6, -4, -3, -1, 0, 2$ nuqtalar bo’yicha grafigi yasalsin.

J:$x = -2$ da 2-tur uzilish $\lim_{x\to -2-0} y = +\infty$, $\lim_{x\to -2+0} y = -\infty$, $\lim_{x\to \pm\infty} y = 1$.

19. $y = \begin{cases} 2, & \mathrm{x} = 0 \text{ va } \mathrm{x} = \pm 2 \text{ bo’lsa}, \\ 4 - \mathrm{x}^2, & 0 < |\,\mathrm{x}| < 2 \ bo’lsa, \\ 4, & |\mathrm{x}| > 2 \ bo’lsa, \end{cases}$ funksiyaning grafigi yasalsin va uzilish nuqtalari ko’rsatilsin. Uzilish nuqtalarida uzluksizlik shartlaridan qaysilari bajariladi va qaysilari bajarilmaydi.

J: $x = 0$ bo’lganda uzluksizlikning faqat to’rtinchi sharti bajarilmaydi. $x = \pm 2$ bo’lganda uchinchi va to’rtinchi shartlar bajarilmaydi.

20. Quyidagi funksiyalarning uzilish nuqtalari va ularning turlari aniqlansin.

1) $\mathrm{y} = \frac{2}{x-3}$. J:$x = 3$ ikkinchi tur uzilish nuqta;

2) $\mathrm{y} = \frac{x+1}{x(x-1)(x^2-9)}$. J:$x = 0; 1; -3; 3$lar ikkinchi tur uzilish nuqtalari;

3) $\mathrm{y} = \begin{cases} \mathrm{x}^2, \text{ agar } \mathrm{x} \geq 0 \text{ bo’lsa}, \\ -1, \text{ agar } \ \mathrm{x} < 0 \ bo’lsa, \end{cases}$ J:$x = 0$ birinchi tur uzilish nuqta;

4) $\mathrm{y} = \frac{\sin 3x}{x}$. J:$x = 0$ yo’qotiladigan uzilish nuqta;

5)$y = \sin \frac{1}{x}$. J: $x = 0$ ikkinchi tur uzilish nuqta;

6) $\mathrm{y} = \frac{x^3-27}{x-3}$. J:$x = 3$ yo’qotiladigan uzilish nuqta.

V BOB. HOSILA VA DIFFERENSIAL

1-§. Argument va funksiya orttirmasi. Hosila va uni hisoblash

x nuqta tayinlangan x_0 nuqtaning biror atrofida yotuvchi ixtiyoriy nuqta bo'lsa, u holda $x - x_0$ ayirma argumentning x_0 nuqtadagi orttirmasi deyiladi va Δx bilan belgilanadi. Demak, $\Delta x = x - x_0$yoki $x = x_0 + \Delta x$.

$f(x) - f(x_0) = f(x_0 + \Delta x) - f(x_0)$ayirmaga funksiya orttirmasi deyiladi va uni Δyyoki Δf bilan belgilanadi. Demak, $\Delta y = f(x_0 + \Delta x) - f(x_0)$

$y =$ f(x) funksiyaning x_0 nuqtadagi orttirmasi Δy ning argument orttirmasi Δx ga nisbatini Δx nolga intilgandagi limiti mavjud bo'lsa, uni $y = f(x)$ funksiyaning x_0 nuqtadagi ***hosilasi*** deyiladi.

$y = f(x)$ funksiyaning hosilasi $y', f'(x_0)$, $\frac{dy}{dx}$, $\frac{df}{dx}$ lardan biri bilan belgilanadi.

$y = f(x)$ funksiyaning hosilasi uning o'zgarish tezligini ifodalaydi va bu hosilaning ***mehanik ma'nosi*** deyiladi.

$y = f(x)$ funksiyaning hosilasi uning grafigini $M_0(x_0, y_0) == M_0(x_0, f(x_0))$ nuqtasiga o'tkazilgan urinmaning burchak koeffitsientini ifodalaydi va uni hosilaning**geometrik ma'nosi** deyiladi.

$y = f(x)$ funksiyaning hosilasini topish quyidagi ketma-ketlikda amalga oshiriladi.

1. $y = f(x)$ funksiyaning x argumentiga Δx orttirma beriladi. Bunday$y$ funksiya Δy orttirma oladi.

2. Funksiya orttirmasi $\Delta y = f(x + \Delta x) - f(x)$ topiladi.

3. Funksiya orttirmasiΔy ning argument orttirmasiΔx ga nisbati$\frac{\Delta y}{\Delta x}$ topiladi.

4. Funksiya orttirmasi Δy ni argument orttirmasi Δx ga nisbatining argument orttirmasi nolga intilgandagi limiti topiladi.

Bu ketma –ketlikka hosilani hisoblash algoritmi deyiladi.

Agar $y = f(x)$ funksiya x nuqtada chekli $f'(x)$ hosilaga ega bo'lsa, □ shu nuqtada ***differensiallanuvchi*** deyiladi.

$y = f(x)$ funksiyaning hosilasini topish amali ***differensiallash amali*** deyiladi.

Agar $y = f(x)$ funksiya x nuqtada differensiallanuvchi bo'lsa, y shu nuqtada uzluksiz bo'ladi.

$y = f(x)$ funksiya (a, b)oraliqning har bir *x*nuqtasida differensiallanuvchi bo'lsa, u holda uni shu oraliqda differensiallanuvchi deyiladi.

$y = f(x)$ funksiyaning hosilasini differensiallash qoidalari deb ataluvchi quyidagi qoidalar yordamida hisoblanadi:

I.$(C)' = 0$ (C –o'zgarmasson).II.$(Cf)' = C(f)'$.

III.$(f \pm g)' = f' \pm g'$. IV. $(f \cdot g)' = f' \cdot g + g' \cdot f$.

V. $(\frac{f}{g})' = \frac{f' \cdot g - g' \cdot f}{g^2}$ $(g \neq 0)$.

Berilgan $y = f(x)$ funksiya x nuqtaning biror atrofida qat'iy monoton va uzluksiz bo'lsin. Bundan tashqari $y = f(x\ )$ funksiya bu x nuqtada differensiallanuvchi va $y' = f'(x) \neq 0$ bo'lsin. Bu shartlarda $x = f^{-1}(y)$ teskari funksiya mavjud va differensiallanuvchi bo'lib, uning hosilasi uchun

$$\{\mathrm{f}^{-1}(y)\}' = \frac{1}{\mathrm{f}'(\mathrm{x})} \qquad \text{yoki} \qquad x'_y = \frac{1}{y'_x}$$

formula o'rinli bo'ladi.

Berilgan $\mathrm{y} = \mathrm{f}(u)$ murakkab funksiyada tashqi $\mathrm{f}(u)$ va ichki $u(x)$ funksiyalar argumentlari bo'yicha differensiallanuvchi bo'lsin. Bu holda $\mathrm{f}(u)$ murakkab funksiya x bo'yicha differensiallanuvchi bo'lib, uning hosilasi$f'_x(u) = f'_u(u) \cdot u'_x$ formuladan topiladi.

Agar $\mathrm{y} = □(x) > 0,\ v = v(x)$ esa ixtiyoriy funksiya bo'lsa, u holda $\mathrm{y} = u(x)^{v(x)} = u^v$ ko'rinishdagi murakkab funksiya ***darajali-ko'rsatkichli*** funksiya deyiladi.

Agary $= u(x)$ va $v = v(x)$ funksiyalar differensiallanuvchi bo'lsa, u holda y $= u^v$ darajali ko'rsatkichlifunksiya ham differensiallanuvchi bo'ladi va uning hosilasi quyidagicha topiladi:

$y = u^v$, $\ln y = v\ln u$, $\frac{1}{y} \cdot y' = v' ln\, u + \frac{vu'}{u}$, $y' = y\left(v' \ln u + \frac{v}{u}u'\right) ==$ $u^v \left(v' lnu + \frac{v}{u}u'\right) = u^{v\cdot} lnu \cdot v' + vu^{v-1} \cdot u'$.

Demak, $y = u^{v\cdot} lnu \cdot v' + vu^{v-1} \cdot u'$.

Barcha elementar funksiyalar o'zlarining aniqlanish sohasida differensiallanuvchi bo'ladi. Ularni hosilalarini quyidagi jadvalda keltiramiz.

1	$(x^\alpha)' = \alpha x^{\alpha-1}, \quad \alpha = (-\infty, +\infty)$	2	$(u^\alpha)' = \alpha u^{\alpha-1} \cdot u', \quad u = u(x)$
3	$(x)' = 1, (x^2)' = 2x, \ (x^3)' = 3x^2,$ $\left(\frac{1}{x}\right)' = -\frac{1}{x^2},$ $\left(\sqrt{x}\right)' = \frac{1}{2\sqrt{x}}$	4	$(u^2)' = 2uu', \ (u^3)' = 3u^2u',$ $\left(\frac{1}{u}\right)' = -\frac{1}{u^2} \cdot u', \ (\sqrt{u})' = \frac{u'}{2\sqrt{u}}$
5	$(a^x)' = a^x ln\, a, a > 0, a \neq 1$	6	$(a^u)' = a^u \cdot ln\, a \cdot u'$
7	$(e^x)' = e^x, \ (10^x)' = 10^x ln10$	8	$(e^u)' = e^u \cdot u'$
9	$(\log_a x)' = \frac{1}{x\, ln\, a} = \frac{\log_a e}{x}$	10	$(\log_a u)' = \frac{u'}{u\, ln\, a} = \frac{u' \log_a e}{u}$
11	$(\ln x)' = \frac{1}{x}, \ (lgx)' = \frac{1}{xln10} = \frac{lg\, e}{x}$	12	$(\ln u)' = \frac{1}{u} \cdot u'$
13	$(\sin x)' = \cos x, \ (\cos x)' = -\sin x$	14	$(\sin u)' = \cos u \cdot u', \ (\cos u)' =$ $= -\sin u \cdot u'$
15	$(\mathrm{tg}x)' = \frac{1}{cos^2 x}, \ (\mathrm{ctg}x)' = -\frac{1}{sin^2 x}$	16	$(\mathrm{tg}u)' = \frac{u'}{cos^2 u}, \ (\mathrm{ctg}u)' = -\frac{u'}{sin^2 u}$
17	$(\arcsin x)' = -(\arccos x)' = \frac{1}{\sqrt{1-x^2}}$	18	$(\arcsin u)' = -(\arccos u)' =$ $= \frac{u'}{\sqrt{1-u^2}}$
19	$(\mathrm{arctg}\, x)' = -(\mathrm{arcctg}\, x)' = \frac{1}{1+x^2}$	20	$(\mathrm{arctg}\, u)' = -(\mathrm{arcctg}\, u)' = \frac{u'}{1+u^2}$

$\frac{e^x-e^{-x}}{2}$, $\frac{e^x+e^{-x}}{2}$ifodalar va ularning nisbatlari mos ravishda giperbolik sinus, giperbolikkosinus,giperbolik tangens va giperbolikkotangenslar deyiladi va ular mos ravishda shx, chx, thx va $cthx$ lar bilan belgilanadi. Demak,

shx= $\frac{e^x-e^{-x}}{2}$,chx= $\frac{e^x+e^{-x}}{2}$,thx= $\frac{e^x-e^{-x}}{e^x+e^{-x}}$, cthx= $\frac{e^x+e^{-x}}{e^x-e^{-x}}$.

Giperbolik funksiyalar uchun quyidagilar o'rinlidir:

1) $ch^2x - sh^2x = 1$;2)$ch^2x + sh^2x = ch2x$; 3) $ch2x = 2shx \cdot chx$;

4) $sh0 = 0$; 5) $ch0 = 1$; 6) $(shx)' = chx$; 7) (chx)$' = shx$;

8) (thx)$' = \frac{1}{ch^2x}$; 9) (cthx)$' = \frac{1}{sh^2x}$.

1.Agar $x = 2$, $\Delta x = 0{,}1$ bo'lsa, $y = x^2$ funksiyaning orttirmasi topilsin.J: 0,41.

2.Agar $x = 3$, $\Delta x = 0{,}01$ bo'lsa, $y = \frac{1}{2}x^2$ funksiyaning orttirmasi topilsin.J: 0,03005.

3.Agar $x = 1$, $\Delta x = 0{,}1$ bo'lsa, $y = 3x^3 + x - 1$ funksiyaning orttirmasi topilsin.J: 1,093.

4.Agar $x = 5$, $\Delta x = 0{,}1$ bo'lsa, $y = x^3 - 7x^2 + 8$ funksiyaning orttirmasi topilsin.J: 0,05.

5.Agar $x = 2$, $\Delta x = -0{,}02$ bo'lsa, $y = 3x^2 + 5x - 4$ funksiyaning orttirmasin topilsin.J: -0,3388.

6. To'g'ri chiziq bo'ylab $S = t^3$ qonuniyat bo'yicha harakatlanayotgan moddiy nuqtaning t= 2 sekunddagi tezligi topilsin.

7. S= $\frac{gt^2}{2}$ qonuniyat bo'yicha tekis tezlanuvchan harakat qilayotgan moddiy nuqtaning ihtiyoriy t paytdagi va t= 3 sekund paytdagi tezligi topilsin. J: 29,4$\frac{M}{C}$.

8. Nuqta S= $\frac{1}{2}t^2 - 3t + 2$ qonuniyat bo'yicha to'g'ri chiziq bo'ylab harakat qilmoqda. Vaqtning $t = 3$ sekunddagi tezligi topilsin.J: 0.

9. Hosilaning ta'rifidan foydalanib, quyidagi funksiyalarning hosilalari topilsin.

1) $y = x^3$; 2) $y = x^4$; 3) $y = \sqrt{x}$; 4) $y = \sin x$;

5) $y = \frac{1}{x}$;6) $y = \frac{1}{\sqrt{x}}$;   7) $y = \frac{1}{x^2}$; 8) $y = tgx$; 9) $y = \frac{1}{x^3}$;

10) $y = 4x^2 - 2$;11) $y = \sqrt{2x + 1}$;12)$y = \frac{1}{3x+2}$;

13) $y = \sqrt{1 + x^2}$; 14) $y = \sin 2x$.

10.Hosilani hisoblash qoidalari vahosilalar jadvalidanfoydalanibquyidagi funksiyalarning hosilalari topilsin.

1) $y = \frac{x^3}{3} - 2x^2 + 4x - 5$;   2) $y = -x^3 + 9x^2 - x + 2$;

3) $y = \frac{x^5}{5} - \frac{2x^3}{3} + x$;   4) $y = \left(1 - \frac{x^2}{2}\right)^2$;

5) $y = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3} + 10$;   6) $y = x + \frac{1}{x^2} - \frac{1}{5x^5} + \sqrt{x}$;

7) $y = 6\sqrt[3]{x} - 4\sqrt[4]{x}$;   8) $y = (\sqrt{a} - \sqrt{x})^2$;

9) $y = \frac{8}{\sqrt[4]{x}} - \frac{6}{\sqrt[3]{x}} + 7$;   10) $y = \frac{3}{\sqrt[3]{x}} - \frac{2}{\sqrt{x}}$;

11) $y = x - 3\sin x + 4\cos x$;   12) $y = x - tgx + 14$;

13) $y = x^2 \cos x + x \sin x$;   14) $y = \sqrt{x}\cos x$;

15) $y = \frac{x}{1-4x}$;   16) $y = \frac{x^2}{x^2+1}$;

17) $y = \frac{2x^2+x+1}{x^2-x+1}$;   18) $y = \frac{\sqrt{x}}{\sqrt{x}+1}$;

19) $y = \frac{x^2-1}{x^2+1}$;   20) $y = \frac{4\sqrt{x}}{\sqrt{x}+1} + 7$;

21) $y = \frac{\cos x}{1-\sin x}$;   22) $y = \frac{\cos x}{1+2\sin x}$;

23) $y = \frac{\sqrt{x}}{x\sqrt[3]{x}}$;   24) $y = \frac{1-x^5}{1+x^5}$;

25) $f(x) = \frac{x^3}{3} - x^2 + x$ funksiya berilgan : $f'(0)$, $f'(1)$ va $f'(-1)$ lar hisoblansin.

26) $f(x) = x^2 - \frac{1}{2x^2}$funksiya berilgan:$f'(2), -f'(-2)$ hisoblansin.

27) $f(x) = \frac{(\sqrt{x}-1)^2}{x}$ funksiya berilgan: $0{,}01 \cdot f'(0{,}01)$ hisoblansin.

28) $f(x) = \frac{x}{2x-1}$funksiya berilgan:$f'(0), f'(2)$ va $f'(-2)$ hisoblansin.

29) $f(x) = \frac{\cos x}{1-\sin x}$ funksiya berilgan: $f'\left(\frac{\pi}{6}\right)$ hisoblansin.

Javoblar: 1) $y' = x^2 - 4x + 4$; 2) $y' = -3x^2 + 18x - 1$;

3) $y' = x^4 - 2x^2 + 1$; 4) $y' = x^3 - 2x$; 5) $y' = -\frac{1}{x^2} - \frac{2}{x^3} - \frac{3}{x^4}$;

6) $y' = 1 - \frac{2}{x^3} + \frac{1}{x^6} + \frac{1}{2\sqrt{x}}$; 7) $y' = \frac{2}{\sqrt[3]{x^2}} - \frac{1}{\sqrt[4]{x^3}}$; 8) $y' = 1 - \sqrt{\frac{a}{x}}$;

9)$y' = \frac{2}{x}\left(\frac{1}{\sqrt[3]{x}} - \frac{1}{\sqrt[4]{x}}\right)$;10)$y' = \frac{1}{x}\left(\frac{1}{\sqrt{x}} - \frac{1}{\sqrt[3]{x}}\right)$;

11) $y' = 1 - 3\cos x - 4\sin x$; 12) $y' = 1 - \frac{1}{cos^2 x}$;

13)$y' = 3\,\mathrm{x}\cos x - x^2 \sin x + \sin x$; 14) $y' = \frac{\cos x - 2x\sin x}{2\sqrt{x}}$;

15) $y' = \frac{1}{(1-4x)^2}$;16) $y' = \frac{2x}{(x^2+1)^2}$; 17) $y' = \frac{-3x^2+2x+2}{(x^2-x+1)^2}$;

18) $y' = \frac{1}{2\sqrt{x}(\sqrt{x}+1)^2}$; 19) $y' = \frac{4x}{(x^2+1)^2}$; 20) $y' = \frac{2}{\sqrt{x}(\sqrt{x}+1)^2}$;

21) $y' = \frac{1}{1-\sin x}$;22) $y' = -\frac{2+\sin x}{(1+2\sin x)^2}$; 23) $y' = -\frac{5}{6x\sqrt[6]{x^5}}$;

24) $y' = \frac{10x^4}{(1+x^5)^2}$;25) $1, 0, 4$;26) $8, 25$; 27) -90;

28)-1; $-\frac{1}{9}$; $-\frac{1}{25}$; 29) 2.

11. Quyidagi funksiyalarning hosilalari topilsin.

1) $y = xlnx$; 2) $y = \frac{1+lnx}{x}$; 3) $y = lg5x$; 4) $y = lnx - \frac{2}{x} - \frac{1}{2x^3}$; 5) $y = x^3 lnx$; 6) $y = x^2 + 3^x$; 7) $y = x^2 \cdot e^x$; 8) $y = \frac{1+e^x}{1-e^x}$; 9) $y = e^x(\sin x + \cos x)$.

Javoblar:1) $l$nx+1;2) $-\frac{lnx}{x^2}$; 3) $\frac{1}{xln10}$; 4) $\frac{1}{x} + \frac{2}{x^2} + \frac{3}{2x^4}$;5) $x^2(3lnx + 1)$;

6) 2x+$3^x ln3$; 7) $xe^x(x+2)$; 8) $\frac{2e^x}{(1-e^x)^2}$; 9) $2e^x \cos x$.

12. Quyidagi berilgan murakkab funksiyalarning hosilalari topilsin.

1) $y = \sin 6x + \cos 6x$; 2) $y = \cos(a - bx)$; 3) $y = \sin\frac{x}{2} + \cos\frac{x}{2}$;

4) $y = (1-5x)^4$; 5) $y = 3\sqrt{(4+3x)^2}$; 6) $y = \frac{1}{(1-x^2)^5}$;

7) $y = \sqrt{1-x^2}$; 8) $y = \sqrt{\cos 4x}$; 9) $y = \sqrt{2x - \sin 2x}$;

10) $y = \sin^4 x + \cos^4 x$;11) $y = \sin^3 x + \cos^3 x$;

12) $y = \mathrm{tg}^3 x - 3tgx + 3x$;13) $y = \sqrt[4]{1 + cos^2 x}$;

14) $y=\sqrt{1+\sin 2x}-\sqrt{1-\sin 2x}$; 15) $y=\sin\sqrt{x}$;

16) $y=\frac{1}{(1+cos4x)^5}$; 17) $y=ctg^3\frac{x}{3}$; 18) $y=\frac{sin^2x}{\cos x}$;19) $y=\frac{1+\sin 2x}{1-\sin 2x}$;

20) $y=\sqrt{\frac{x}{2}-sin\frac{x}{2}}$; 21) $y=cos^2(\frac{\pi}{4}-\frac{x}{2})$;

22) $y=\sqrt{x+2\sqrt{x}}$funksiya berilgan. $y'(1)$ topilsin.

23) $y=\sqrt{1+cos^2x^2}$funksiya berilgan. $y'(\frac{\sqrt{\pi}}{2})$ topilsin.

Javoblar:1) $6(\cos 6x-\sin 6x)$; 2) $-bsin(a-bx)$;

3)$\frac{1}{2}(\cos\frac{x}{2}-sin\frac{x}{2})$;4) $-20(1-5x)^3$; 5) $\frac{2}{\sqrt[3]{4+3x}}$;6) $\frac{10x}{(1-x^2)^6}$; 7) $-\frac{x}{\sqrt{1-x^2}}$; 8) -2tg4x$\sqrt{\cos 4x}$; 9) $\frac{2sin^2x}{\sqrt{2x-sin2x}}$; 10) –sin4x;11) $\frac{3}{\sqrt{2}}sin2xsin(x-\frac{\pi}{4})$; 12) $3tg^4x$;

13) $\frac{-sin2x}{4\sqrt[4]{(1+cosx)^3}}$; 14) $\pm\left(\sqrt{1+\sin 2x}-\sqrt{1-\sin 2x}\right)$ “+” ishora $\cos 2x>0$ bo’lganda, $"-"$ ishora $\cos 2x<0$ da, $\cos 2x=0$ bo’lganda y' hosila mavjud emas; 15)$\frac{\cos\sqrt{x}}{2\sqrt{x}}$;

16) $\frac{20sin4x}{(1+cos4x)^6}$; 17) $\frac{ctg^2\frac{x}{3}}{sin^2\frac{x}{3}}$; 18) sin x(1+$sec^2x$);

19) $\frac{4cos2x}{(1-sin2x)^2}$; 20) $\frac{sin^2\frac{x}{4}}{2\sqrt{\frac{x}{2}-sin\frac{x}{2}}}$; 21) $\frac{1}{2}cosx$; 22) $\frac{1}{\sqrt{3}}$; 23) $-\sqrt{\frac{\pi}{6}}$.

13.Quyidagi funksiyalarning hosilalari topilsin:

1) $y=ln(x^2+2x)$; 2)$y=ln(1+cosx)$; 3)$y=ln\frac{x^2}{1-x^2}$;

4) $y=ln\frac{a^2+x^2}{a^2-x^2}$; 5) $y=lntg(\frac{\pi}{4}+\frac{x}{2})$; 6) $y=ln\sqrt{\frac{1+2x}{1-2x}}$;

7) $y=ln(x+\sqrt{a^2+x^2})$; 8) $y=ln\frac{cosx}{sin^2x}$ + lntg$\frac{x}{2}$; 9) $y=a^{sinx}$;10) $y=x^2e^{-2x}$; 11) $y=\sqrt{x}\cdot e^{\sqrt{x}}$; 12) $y=e^{-x}(sinx+cosx)$;13) $y=ln\sqrt{\frac{sin2x}{1-sin2x}}$; 14) $y=ln\sqrt{\frac{e^{4x}}{e^{4x}+1}}$; 15) $y=x^{\frac{1}{x}}$;16) $y=\sqrt{1-x^2}+arcsinx$;

17) $y=$ arcsin$\sqrt{1-4x}$;

18) $y = \arccos(1-2x)$; 19) $y = \mathrm{arcctg}\frac{1+x}{1-x}$; 20) $y = \mathrm{arctg}\sqrt{6x-1}$; 21) $y = \arccos\frac{1}{\sqrt{x}}$;

22) $f(x) = \arcsin\frac{x-1}{x}$ funksiya berilgan: f′(5) topilsin.

Javoblar: 1) $\frac{2(x+1)}{x(x+2)}$; 2) $-tg\frac{x}{2}$; 3) $\frac{2}{x(1-x^2)}$; 4) $\frac{4a^2x}{a^4-x^4}$; 5) $\frac{1}{\cos x}$;6) $\frac{2}{1-4x^2}$; 7) $\frac{1}{\sqrt{a^2+x^2}}$; 8) $-\frac{2ctg^2x}{\sin x}$; 9) $a^{sinx}cosx\ ln\ a$; 10) 2x(1-x)e^{-2x};11) $\frac{1}{2}e^{\sqrt{x}}(1+\frac{1}{\sqrt{x}})$; 12) $-2e^{-x}sin\ x$; 13) $\frac{ctg2x}{1-sin2x}$; 14) $\frac{2}{e^{4x}+1}$; 15) $x^{\frac{1}{x}}\cdot\frac{1-lnx}{x^2}$;16) $\frac{1-x}{\sqrt{1-x^2}}$;17) $-\frac{1}{\sqrt{x-4x^2}}$; 18) $\frac{1}{\sqrt{x-x^2}}$; 19) $-\frac{1}{1+x^2}$; 20) $\frac{1}{2x\sqrt{6x-1}}$; 21) $\frac{1}{2x\sqrt{x-1}}$; 22) $\frac{1}{15}$.

14. Quyidagi funksiyalarning hosilalari topilsin.

1) $y = sh^2x$; 2) y $= x - thx$; 3) y $= 2\sqrt{Chx-1}$;

4) y $= thx + cthx$;5) y $= x - cthx$; 6) y $= \frac{sh5x}{ch\frac{x}{3}}$;

7) y $= cth(tgx) - th(ctgx)$; 8) y $= sh^2x + ch^3x^2$;

9) y $= sh\frac{x}{2} + ch\frac{x}{2}$; 10) y $= \sqrt{1+sh^24x}$;

11) y $= e^{ax}(chbx + shbx)$; 12) y $= lntg\frac{x}{2} + lnsh\frac{x}{2}$.

Javoblar: 1) sh2x; 2) th^2x; 3) $\sqrt{chx+1}$; 4) $-\frac{4}{sh^22x}$; 5) cth^2x;6) $5ch5x\cdot ch\frac{x}{3} + \frac{1}{3}sh5x\cdot sh\frac{x}{3}$; 7) $-\frac{sec^2x}{sh^2(tgx)} + \frac{cosec^2x}{ch^2(ctgx)}$; 8) $3x(xsh2x^3 + +chx^2\cdot sh2x^2)$; 9) $\frac{1}{2}(ch\frac{x}{2} + sh\frac{x}{2})$; 10) $4sh4x$; 11) $(a+b)e^{(a+b)x}$; 12) $\frac{1}{sinx} + 2cth\frac{x}{2}$.

15. Quyidagi funksiyalarning hosilalari topilsin.

1) y $= x^x$ (x> 0); 2) $y = (sinx)^{cosx}$; 3) $y = (cosx)^{sinx}$;

4) $y = x^{sinx}$; 5) $y = x^{cosx}$; 6) $y = (cosx)^{sin2x}$;

7) y $= \sqrt[x]{x}$; 8) y $= x^{x^x}$.

Javoblar:1)$x^x(lnx+1)$; 2) $(sinx)^{cosx}\cdot(-sinx\cdot lncosx + \frac{cos^2x}{sinx})$;

3) $(cosx)^{sinx} \cdot (cos\, xln\, cosx - tgx\, sinx)$; 4) $x^{sinx} \cdot (cos\, xln\, x + +\frac{sinx}{x})$;5) $-x^{cosx} \cdot (-sinxlnx + \frac{cosx}{x})$; 6) $2cos2xlncosx - 2sin^2x$; 7) $\sqrt[x]{x} \cdot ln\frac{e}{x}$; 8) $x^{x^x} \cdot x^x\left(\frac{1}{x} + lnx + ln^2x\right)$.

2-§.Oshkormas funksiya va uning hosilasi. Parametrik shaklda berilgan funksiya va uning hosilasi

x o'zgaruvchining y funksiyasi oshkormas shaklda $F(x,y) = 0$ tenlama bilan berilgan bo'lsa, u holda y' hosilani topish uchun $F(x,y) = 0$ tenglikni ikkala qismini x bo'yicha differensiallab so'ngra y' ga nisbatan hosil bo'lgan chiziqli tenglamadan hosilani topish kerak. Ikkinchi va undan yuqoriroq tartibli hosilalar ham shu tartibda topiladi.

Agar y funksiyaning x argumentga bog'liqligi

$$\begin{cases} x = \varphi(t) \\ y = f(t) \end{cases}$$

tenglama bilan berilgan bo'lsa, u holda funksiyani ***parametrik*** shaklda berilgan deyiladi. Bu funksiyaning hosilasi

$$y'_x = \frac{y'_t}{x'_t}$$

formuladan topiladi.

1. Oshkormas shaklda berilgan quyidagi funksiyalarning hosilalari topilsin:
1) $x^2 + y^2 = 64$; 2) $y^2 = 2px$; 3) $x^2 + xy + y^2 = 6$;
4) $x^2 + y^2 - xy = 0$;5) $x^3 + y^3 - 3axy = 0$;
6) $5x^2 + 3xy - 2y^2 + 2 = 0$.

J: 1) $-\frac{x}{y}$; 2) $\frac{p}{y}$; 3) $-\frac{2x+y}{x+2y}$; 4) $\frac{2x-y}{x-2y}$; 5) $\frac{x^2-ay}{ax-y^2}$; 6) $\frac{10x+3y}{4y-3x}$.

2. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$funksiya berilgan.$y'(a)$topilsin. J: $-\sqrt[3]{\frac{y}{x}}$.

3.$e^y + xy = e$funksiya berilgan.y'_xni 0,1nuqtadagi qiymati topilsin:J: $-\frac{1}{e}$

4. $x^2 \sin y - \cos y + \cos 2y = 0$ funksiya berilgan $x = \frac{\pi}{2}$ bo’lganda y' hisoblansin.J:± 2.

5. Parametrik shaklda berilgan quyidagi funksiyalarning hosilalari topilsin:

1) $\begin{cases} x = a\cos t \\ y = b\sin t \end{cases}$; 2) $\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$; 3) $\begin{cases} x = \frac{1-t}{1+t}, \\ y = \frac{2t}{1+t} \end{cases}$;

4) $\begin{cases} x = \frac{a\sin t}{1+b\cos t}, \\ y = \frac{c\cos t}{1+b\cos t} \end{cases}$; 5) $\begin{cases} x = t^2, \\ y = t^3 \end{cases}$; 6) $\begin{cases} x = \frac{3at}{1+t^3} \\ y = \frac{3at^2}{1+t^3} \end{cases}$.

J: 1) $-\frac{b}{a} ctg\, t$; 2) $ctg\, \frac{t}{2}$; 3) -1; 4) $-\frac{c\sin t}{a(b+\cos t)}$; 5) $\frac{3}{2}t$; 6) $\frac{2t-t^4}{1-2t^3}$.

6. $\begin{cases} x = k\sin t + \sin kt \\ y = k\cos t + \cos kt \end{cases}$ funksiya berilgan y'_x ni $t = 0$ da hisoblang.J: 0.

7. $\begin{cases} x = \sin 2t \\ y = sin^2 t \end{cases}$ funksiya berilgan y'_x ni $t = \frac{\pi}{8}$ da hisoblang.J: $\frac{1}{2}$.

8. $\begin{cases} x = \frac{2}{3}\sqrt{2t^3} \\ y = \frac{1}{2}t^2 \end{cases}$ funksiya berilgan y'_x ni $t = \frac{\pi}{8}$ da hisoblang.

J: 0,833.

3-§. Yuqori tartibli hosilalar

$y = f(x)$ funksiyaning ikkinchi tartibli hosilasi deb uning birinchi tartibli hosilasidan olingan hosilaga, ya’ni $(y')'$ ga aytiladi.

Ikkinchi tartibli hosila quyidagilardan biri bilan belgilanadi:

$$y'',\ f''(x),\ \frac{d^2y}{dx^2}.$$

Uchinchi tartibli hosila deb uning ikkinchi tartibli hosilasidan olingan hosilaga, ya’ni $(y'')'$ ga aytiladi.

Uchinchi tartibli hosila quyidagilardan biri bilan belgilanadi:

$$y''', f'''(x), \frac{d^3y}{dx^3}.$$

$y = f(x)$ funksiyaning n -tartibli hosilasi deb uning $(n-1)$ -tartibli hosilasidan olingan hosilaga aytiladi va u quyidagilardan biri bilan belgilanadi:

$$y^{(n)}, \quad f^{(n)}(x), \quad \frac{d^ny}{dx^n}.$$

$F(x,y)$ ko'rinishidagi oshkormas funksiyani ikkinchi tartibli hosilasini topish uchun birinchi tartibli hosila y' ni x bo'yicha differensiallaymiz.

$\begin{cases} x = \varphi(t) \\ y = f(t) \end{cases}$ tenglamalar bilan berilgan funksiyaning ikkinchi tartibli hosilasini topish uchun y'_x hosiladan, ya'ni$\frac{y_t'}{x_t'}$dan yana t bo'yicha hosila olamiz:

$$y''_{xx} = \left(\frac{y_t'}{x_t'}\right)' \frac{1}{x_t'} = \frac{y''_{tt}x'_t - x''_{tt}y'_t}{(x'_t)^3}.$$

1. Quyidagi funksiyalarning ikkinchi tartibli hosilalari topilsin:

1) $y = sin^2x$; 2) $y = cos^2x$;3) $y = tg\ x$; 4) $y = \sqrt{1+x^2}$.

J: 1) $2\cos 2x$; 2) $-4\sin 2x$; 3) $2tg\ x\ sec^2x$; 4) $\frac{1}{(1+x^2)^{\frac{3}{2}}}$.

2. Quyidagi funksiyalarning uchinchi tartibli hosilalari topilsin:

1) $y = cos^2x$; 2) $y = \frac{1}{x^2}$; 3) $y = x\sin x$; 4) $y = x\ln x$;

5) $y = arctg\frac{x}{a}$; 6) $y = \sqrt{x}$; 7) $y = \frac{1}{\sqrt{x}}$; 8) $y = xe^{-x}$

J: 1) $4\sin 2x$; 2) $-\frac{24}{x^5}$; 3) $-(x\cos x + 3\sin x)$; 4) $-\frac{1}{x^2}$;

5) $\frac{2a(3x^2-a^2)}{(x^2+a^2)}$; 6) $\frac{3\sqrt{x}}{8x^3}$; 7) $-\frac{1\cdot 3\cdot 5}{2^3x^3\sqrt{x}}$; 8) $e^{-x}(3-x)$.

3. Quyidagi funksiyalarning$n$- tartibli hosilalari topilsin:

1) $y = \frac{1}{x}$; 2) $y = \cos x$; 3) $y = \sin x$;

4) $y = \ln x$; 5) $y = e^x$; 6) $y = a^x$.

J: 1) $(-1)^n\frac{n!}{x^{n+1}}$; 2) $\cos(x + \frac{n\pi}{2})$; 3) $\sin\left(x + \frac{n\pi}{2}\right)$;

4) $\frac{(-1)^{n-1}(n-1)!}{x^n}$;5) e^x; 6) $a^x ln^n a$.

4. Leybnits formulasidan foydalanib quyidagi funksiyalarning ikkinchi tartibli hosilalari topilsin:

1) $y = e^x\cos x$; 2) $y = a^x x^3$; 3) $y = x^2\sin x$.

J: 1) $-2e^x\sin x$;2) $xa^x(x^2ln^2a + 6xln\ a + 6)$; 3) $3\sin x + +4x\cos x - x^2\sin x$.

5. Leybnits formulasidan foydalanib quyidagi funksiyalarning uchinchi tartibli hosilalari topilsin.

1) $y=e^{-x}\cos x$; 2) $y=x^2\ln x$; 3) $y=x\cos x$; 4) $y=x^2\sin\frac{x}{a}$.

J: 1) $2e^{-x}(\sin x+\cos x)$; 2) $\frac{2}{x}$; 3) $x\sin x-3\cos x$.

6. $f(x)=\arcsin\frac{1}{x}$ funksiya berilgan. $f(2)$, $f'(2)$ va $f''(2)$ lar topilsin:J: $\frac{\pi}{6}$; $-\frac{\sqrt{3}}{6}$; $\frac{7\sqrt{3}}{36}$.

7. $y=e^x\cos x$ funksiya $y'''+4y=0$ni qanoatlantirishi isbotlansin.

8. $y=xe^{-\frac{1}{x}}$ funksiya $x^3y''-xy'+y=0$ ni qanoatlantirishi ko'rsatilsin.

9. Oshkormas holda berilgan quyidagi funksiyalarning ikkinchi tartibli hosilalari topilsin:

1) $x^2+y^2=a^2$; 2) $ax+by-xy=c$; 3) $x^my^n=1$;

4) $arctg\, y=x+y$; 5) $x^2+xy+y^2=a^2$; 6) $x^3+\square^3-3axy$=0.

J: 1) $-\frac{a^2}{y^3}$; 2) $\frac{2(y-a)}{(x-b)^2}$; 3) $\frac{m(m+n)y}{n^2x^2}$; 4) $-\frac{2(1+y^2)}{y^5}$; 5) $-\frac{6a^2}{(x+2y)^3}$; 6) $\frac{2a^3xy}{(ax-y^2)^3}$.

10. Parametrik shaklda berilgan quyidagi funksiyalarning ikkinchi tartibli hosilalari topilsin:

1) $\begin{cases}x=a\cos t\\ y=a\sin t\end{cases}$; 2) $\begin{cases}x=t^2\\ y=\frac{t^3}{3}-t\end{cases}$; 3) $\begin{cases}x=a(t-\sin t)\\ y=a(1-\cos t)\end{cases}$;

4) $\begin{cases}x=2\cos t\\ y=\sin t\end{cases}$; 5) $\begin{cases}x=t^2\\ y=t+t^3\end{cases}$; 6) $\begin{cases}x=e^{2t}\\ y=e^{3t}\end{cases}$; 7) $\begin{cases}x=arc\sin t\\ y=\sqrt{1-t^2}\end{cases}$.

J: 1) $-\frac{1}{asin^3t}$; 2) $\frac{t^2+1}{4t^3}$; 3) $-\frac{1}{4asin^3\frac{t}{2}}$; 4) $-\frac{1}{4sin^3t}$; 5) $\frac{3t^2-1}{4t^3}$; 6) $\frac{3}{4e^t}$; 7) $-\sqrt{1-t^2}$.

4-§. Hosilaning geometrik ma'nosi

$y=f(x)$ egri chiziqning $(x_0;y_0)$ nuqtasida o'tkazilgan urinmaning burchak koeffitsienti $f(x)$ funksiya hosilasining (x_0,y_0) nuqtadagi qiymatiga teng, ya'ni

$$k=tg\varphi=f'(x_0)=y'|_{x=x_0} \quad (1)$$

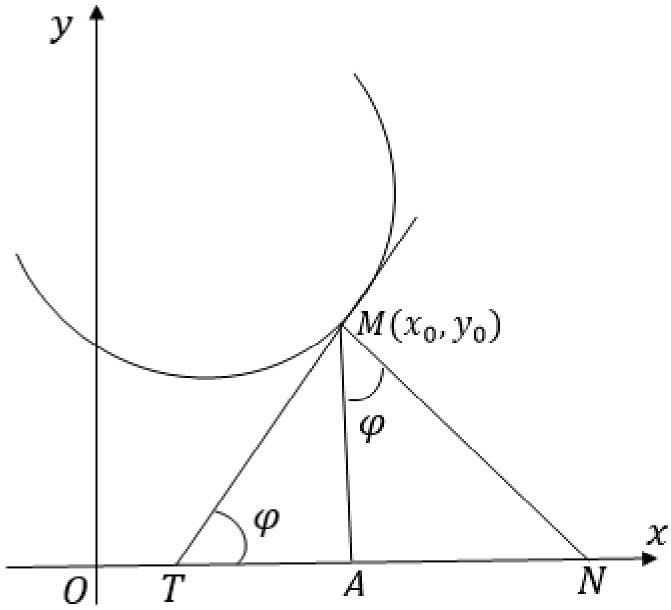
Bu k son bahzan chiziqning $(x_0;y_0)$ nuqtadagi og'maligi xam deyiladi. Egri chiziqning $(x_0;y_0)$ nuqtasida o'tkazilgan urinmaning (1-chizma) tenglamasi:

$$y-y_0=k\,(x-x_0) \quad (2)$$

normalning tenglamasi

$$y-y_0=-\frac{1}{k}\,(x-x_0) \quad (3)$$

$TA = y_0 ctg\,\varphi$,$AN = y_0 tg\,\varphi$ kesmalar mos ravishda ***urinma osti*** va ***normal osti*** deyiladi, MT va MN kesmalarning uzunliklari esa – ***urinma va normal uzunliklari*** deyiladi.



1-chizma

1. $y = 2x^2 - 2$parabolaning absissalari mos ravishda $x_1 = 1,\ x_2 = 2$ va$x_3 = 0$ bo'gan nuqtalariga o'tkazilgan urinmalarning burchak koeffitsentlari topilsin. ($k_1 = 4$; $k_2 = -8$; $k_3 = 0$).

2. $y = x^2$ parabolaning absissasi $x = -1$ bo'lgan nuqtasiga o'tkazilgan urinmaning burchak koeffitsienti topilsin. (-2).

3.$y = x^2 + 5x - 2$ egri chiziqning (1;4) nuqtasiga o'tkazilgan urinma og'ish burchagining tangensini toping. ($tg\ a = 7$).

4. $y = x^2$ parabolaning A(1;1), B(-1;1) va D(0;0) nuqtalariga o'tkazilgan urinmalarning tenglamalari yozilsin. ($y_1 = 2x - 1$; $y_2 = -2x - 1$; $y_3 = 0$)

5.Quyidagi egri chiziqlarga ko'rsatilgan nuqtalarda o'tkazilgan urinmalarning tenglamalari tuzilsin.

1) $y = x^2 + 1$ ga absissasi$x = -1$ bo'lgan nuqtada; (J: $y = -2x$);

2) $y = x - x^3$ga 0(0;0) nuqtada; (J:$y = x$);

3)$y = \sin x$ga$A_1(\frac{\pi}{2}; 1)$nuqtada; (J:$y = 1$);

4) $y = x^2 - 3x + 4$ga$A(3;4)$nuqtada; (J:$y = 3x - 5$);

5)$y = \frac{x^3}{3}$ga$x = -1$ nuqtada; $\left(\text{J: } y = x + \frac{2}{3}\right)$;

6)$y^2 = x^3$ga$x_1 = 0$,$x_2 = 1$ nuqtada; (J:y=0,$y = \pm\frac{1}{2}(3x - 1)$);

7) $y = \sin x$ga$x = \pi$ nuqtada; (J:$y = \pi - x$).

6. Quyidagi egri chiziqlarga o'tkazilgan urinma va normal tenglamalari tuzilsin:

1) $y = x^3 - 3x + 2$ egri chiziqqa $A(2;4)$ nuqtada;

J: $9x - y - 14 = 0$va$x + 9y - 38 = 0$;

2) $y = x^4 + 3x^2 - 16$ egri chiziqqa uning $y = 3x^2$ parabola bilan kesishgan nuqtalarida;

J:$y = -44x - 76, y = 44x - 76$;$y = \frac{1}{44}(x + 2) + 12$,

$y = -\frac{1}{44}(x - 2) + 12$.

3) $y = x^2 - 4x$ parabolaga absissasi$x = 1$ bo'lgan nuqtada

J: $2x + y + 1 = 0$ va $x - 2y - 7 = 0$

7. $y = x^3 - 3x + 5$ egri chiziqda shunday nuqta topingki, u nuqtada o'tkazilgan urinma: a) $y = -2x$ to'g'ri chiziqqa parallel; b) $y = -\frac{x}{9}$ to'g'ri chiziqqa perpendikulyar bo'lsin.

J: a) $M_1\left(-\frac{1}{\sqrt{3}}; 5 + \frac{8\sqrt{3}}{9}\right)$, $M_2\left(\frac{1}{\sqrt{3}}; 5 - \frac{8\sqrt{3}}{9}\right)$; b) $M_1(-2;3)$, $M_2(2;7)$.

8. $y = (x + 1)\sqrt[3]{3 - x}$ egri chiziqqa absissasi $x_0 = -1$ bo'lgan nuqtada o'tkazilgan urinma va normal tenglamasini tuzing.

J: $y = \sqrt[3]{4}(x + 1)$ va $y = -\frac{1}{\sqrt[3]{4}}(x + 1)$.

9. $y = \frac{8}{4+x^2}$ egri chiziqqa absissasi $x_0 = 2$ bo'lgan nuqtada o'tkazilgan urinma va normal tenglamasi tuzilsin.

J:$y = -\frac{x}{2} + 2$va$y = 2x - 3$.

10. Quyidagi chiziqlarning kesishish burchaklari topilsin:

1)$y = 4 - x$ to'g'ri chiziq bilan $y = -\frac{x^2}{2}$parabolani;

2)$y=\sin x$ sinusoida bilan$y=\cos x$kosinusoidani;

3)$x^2+4y^2=4$ ellips bilan $4y=4-5x^2$ parabolani;

4)$y=8-x^2$va$y=x^2$ parabolalarni;

5)$2y=x^2$va$2y=8-x^2$ parabolalarni.

J: 1) $\varphi_1=45°$ va$\varphi_2\approx 18{,}5°$; 2)$\varphi=arctg\ 2\sqrt{2}$; 3) $\varphi\approx 92°$; 4)$\varphi_1=arctg\ \left(-\frac{8}{15}\right)va\varphi_2=arctg\ \frac{8}{15}$; 5) $\varphi=arctg\frac{4}{3}$.

11. $\begin{cases} x=t^2+3t-8 \\ y=2t^2-2t-5 \end{cases}$ egri chiziqning $M(2;-1)$ nuqtasiga o'tkazilgan urinmaning burchak koeffitsienti topilsin. J: $k=\frac{6}{7}$

12. $\begin{cases} x=t-1 \\ y=t^3-12t+1 \end{cases}$ egri chiziqning qaysi nuqtalariga o'tkazilgan urinmlar: 1) OX o'qiga; 2) $9x+y+3=0$ to'g'ri chiziqqa parallel bo'ladi? J: 1) (1;-15), (-3;17); 2) (0;-10), (-2;12).

13. $\begin{cases} x=2\sqrt{3}\cos t \\ y=2\sin t \end{cases}$ ellipsning $t=\frac{\pi}{6}$ bo'lgan nuqtasiga o'tkazilgan urinma va normal tenglamasi topilsin.J:$x+y=4$; $x-y=2$.

14.$\begin{cases} x=acos^3t \\ y=asin^3t \end{cases}$astroidaning$t=\frac{\pi}{4}$ bo'lgan nuqtasiga o'tkazilgan urinma va normal tenglamasi topilsin. J: $\sqrt{2}(x+y)=a$; $y=x$.

15. $\begin{cases} x=a(t-\sin t) \\ y=a(1-\cos t) \end{cases}$ sikloidaga $t=\frac{3\pi}{2}$ nuqtada o'tkazilgan urinma tenglamasi yozilsin.

16. $4x^3-3xy^2+6x^2-5xy-8y^2+9x+14=0$ egri chiziqqa$M(-2;3)$ nuqtada o'tkazilgan urinma va normal tenglamasi topilsin.

J: $y=-\frac{9}{2}(x+2)+3$va$y=\frac{2}{9}(x+2)+3$.

17. $x^5+y^5-2xy=0$ egri chiziqqa $M(1;1)$ nuqtada o'tkazilgan urinma va normal tenglamasi topilsin.J:$x+y-2=0$va$y=x$.

5-§. Hosilaning fizik tatbiqlari

Nuqta OX o'q bo'yicha xarakat qilib, vaqtning t paytida $S=f(t)$ koordinataga ega bo'lsin, u holda vaqtning t paytidagi tezlik:

$V = \lim_{\Delta t \to 0} \frac{\Delta S}{\Delta T} = \frac{dS}{dt}$ va tezlanish:$a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt} = \frac{d^2x}{dt^2}$ bo'ladi.

1. Jism$S = t^2 - t + 3$ qonun bo'yicha to'g'ri chiziqli xarakat qiladi. Harakat boshlangandan 2 sek. keyingi tezlik topilsin. J:$v = 3$.
2. Jism $S = 3t^2 - 2t + 4$ qonun bo'yicha xarakatlanmoqda beshinchi sekund oxirida jismning xarakat tezligi qancha?J: 28.
3. Nuqta$S = 2t^3 + t^2 - 4$ qonuniyat bo'yicha to'g'ri chiziqli xarakat qiladi. Nuqtaning $t = 4$ sek. dagi tezligi topilsin.J: 104.
4. $S = 6t - t^2$ qonun bo'yicha xarakatlanayotgan nuqtaning tezligi qachon nolga teng bo'ladi? J: $t = 3$.
5. $S = t^3 + t^2 - 27t$ va $S = t^2 + 1$ qonuniyat bo'yicha xarakatlanayotgan jismlarning tezliklari qachon teng bo'ladi? J: $t = 3\ s$.
6. Massasi 8 kg bo'lgan jism $\square = 2t^2 + 3t - 1$ qonun bo'yicha to'g'ri chiziqli xarakat qiladi. Jismning xarakat boshlangandan so'ng uchinchi sekund o'tgandagi kinetik energiyasi topilsin. J: 900 Dj.
7. Material nuqta $S = 2t^3 - 6t^2 + 4t$ qonun bo'yicha xarakat qiladi. Nuqtaning 3-sekund oxiridagi tezlanishni toping. J: 24 $\frac{m}{c^2}$.
8. $S = t^3 + 2t^2$qonun bo'yicha xarakatlanayotgan material nuqtaning 3 sekund oxiridagi tezlanishi topilsin. J: 22 $\frac{m}{c^2}$.
9. Jism $x = \frac{t^3}{3} - 2t^2 + 3t$ qonunga asosan to'g'ri chiziq bo'yicha harakat qiladi. Harakat tezligi va tezlanishi aniqlansin.J:$v = t^2 - 4t + +3$; $a = 2t - 4$.
10. Qandaydir kimyoviy reaksiya natijasida hosil qilinadigan jism miqdori x bilan t vaqt orasidagi bog'lanish $x = A(1 - e^{-kt})$ tenglama bilan ifodalanadi. Reaktsiya tezligi topilsin.J:$\frac{dx}{dt} = Ake^{-kt}$.

6-§. Aniqmasliklar va Lopital qoidalari

Agar $x \to a$ (a - chekli yoki cheksiz son) bo'lganda $f(x)$ va $g(x)$ funksiyalar cheksiz kichik funksiyalar, ya'ni

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0$$

bo’lsa, ularning $\frac{f(x)}{g(x)}$nisbati$x \to a$ bo’lganda$\frac{0}{0}$ko’rinishdagi ***aniqmaslik*** deyiladi.

Agar $x \to a$ (a -chekli yoki cheksiz son) bo’lganda $f(x)$ va $g(x)$ funksiyalar cheksiz katta funksiyalar bo’lsa, ya’ni

$$\lim_{x \to a} f(x) = \pm\infty, \quad \lim_{x \to a} g(x) = \pm\infty$$

bo’lsa, ularning$\frac{f(x)}{g(x)}$ nisbati $x \to a$bo’lganda$\frac{\infty}{\infty}$ko’rinishdagi ***aniqmaslik***deyiladi.

$\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko’rinishdagi $\frac{f(x)}{g(x)}$ aniqmaslikning $x \to a$ dagi limitini topish aniqmaslikni ochish deyiladi.

Lopitalning 1-qoidasi: $f(x)$ va $g(x)$ funksiyalar $x = a$ nuqta atrofida aniqlangan, differensiallanuvchi va $g'(\square) = 0$ bo’lsin. Bundan tashqari $f(x)$ va $g(x)$ funksiyalar $x \to a$ shartda cheksiz kichik miqdorlar bo’lsin, ya’ni

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0$$

bo’lsin. Bu holda, agar

$$\lim_{x \to a} \frac{f'(x)}{g'(x)}$$

mavjud bolsa (chekli yoki cheksiz), u holda

$$\lim_{x \to a} \frac{f(x)}{g(x)}$$

ham mavjud bo’ladi va quyidagi tenglik o’rinli bo’ladi:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

Lopitalning II–qoidasi: $f(x)$ va $g(x)$ funksiyalar $x = a$ nuqta atrofida aniqlangan, differensiallanuvchi va $g'(x) \neq 0$ bo’lsin. Bundan tashqari $f(x)$ va$g(x)$ funksiyalar$x \to a$ da cheksiz katta miqdorlar bo’lsin, ya’ni

$$\lim_{x \to a} f(x) = \infty, \quad \lim_{x \to a} g(x) = \infty$$

bo’lsin. Bu holda, agar

$$\lim_{x \to a} \frac{f'(x)}{g'(x)}$$

mavjud bo’lsa (chekli yoki cheksiz), u holda

$$\lim_{x\to a}\frac{f(x)}{g(x)}$$

ham mavjud bo'ladi va quyidagi tenglik o'rinli bo'ladi:

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Agar $\lim\limits_{x\to a} f(x)=0$, $\lim\limits_{x\to a} g(x)=\infty$ bo'lsa, $f(x)\cdot g(x)$ ko'paytma $x\to a$ bo'lganda $0\cdot\infty$ ko'rinishdagi aniqmaslik deyiladi.

Agar $\lim\limits_{x\to a} f(x)=1$, $\lim\limits_{x\to a} g(x)=\infty$ bo'lsa, $f(x)^{g(x)}(f(x)>0)$ ifoda $x\to a$ bo'lganda 1^{∞} ko'rinishdagi aniqmaslik deyiladi.

Agar $f(x)$ va $g(x)$ funksiyalar uchun $\lim\limits_{x\to a} f(x)=0$ va $\lim\limits_{x\to a} g(x)$ =0 yoki $\lim\limits_{x\to a} f(x)=\infty$ va $\lim\limits_{x\to a} g(x)=\infty$ bo'lsa, u holda $f(x)^{g(x)}(f(x)>0)$ ifoda $x\to a$ da 0^0 yoki ∞^0 ko'rinishdagi aniqmaslik deyiladi.

Agar $f(x)$ va $g(x)$ funksiyalar uchun $\lim\limits_{x\to a} f(x)=\infty$, $\lim\limits_{x\to a} g(x)=\infty$ bo'lsa, unda $f(x)-g(x)$ ayirma $x\to a$ da $\infty-\infty$ ko'rinishdagi aniqmaslik deyiladi.

$0\cdot\infty$, 1^{∞}, 0^0, ∞^{∞}, $\infty-\infty$ ko'rinishdagi aniqmasliklar ham Lopital qoidalariga keltirish orqali ochiladi (Fransua Lopital 1661-1704 yillarda yashagan fransuz matematigi).

Aniqmasliklarni ochish uchun Lopital qoidasini bir necha marta qo'llash mumkin, ya'ni

$$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}=\lim_{x\to a}\frac{f''(x)}{g''(x)}=\lim_{x\to a}\frac{f'''(x)}{g'''(x)}$$

1.Quyidagi limitlar topilsin:

1) $\lim\limits_{x\to 0}\frac{x-\sin x}{x^3}$; 2) $\lim\limits_{x\to 0}\frac{e^x-1}{\sin 2x}$; 3) $\lim\limits_{x\to 0} x^2\ln x$; 4) $\lim\limits_{x\to 1}\frac{x^3-7x^2+4x+2}{x^{3-5x+4}}$;

5) $\lim\limits_{x\to}\frac{tg\, x-\sin x}{x-\sin x}$; 6)$\lim\limits_{x\to 0}\frac{e^{5x}-1}{\sin 3x}$; 7) $\lim\limits_{x\to a}\frac{x-a}{x^n-a^n}$; 8) $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$;

9) $\lim\limits_{x\to a}\frac{\ln x}{ctg\, x}$; 10) $\lim\limits_{x\to\infty}\frac{\ln x}{x}$; 11) $\lim\limits_{x\to\pi}(\text{p}-x)tg\frac{x}{2}$; 12) $\lim\limits_{x\to\frac{\pi}{4}}\frac{tg\, x}{tg\, 3x}$

13) $\lim\limits_{x\to 0}\frac{x-arctg\, x}{x^3}$;14) $\lim\limits_{x\to\frac{\pi}{6}}\frac{1-2\sin x}{\cos 3x}$; 15) $\lim\limits_{x\to\frac{\pi}{4}}\frac{1-tg\, x}{\cos 2x}$;

16) $\lim_{x\to 0}(1-e^{2x})ctg\,x$;17) $\lim_{x\to 1}\frac{\ln x}{1-x^3}$; 18) $\lim_{x\to\infty} x^n e^{-x}$;

19) $\lim_{x\to\infty}\left(1+\frac{3}{x}\right)^x$; 20) $\lim_{x\to 0}(\sin x)^x$.

J: 1) $\frac{1}{6}$; 2) $\frac{1}{2}$; 3) 0; 4) $\frac{7}{2}$; 5) 3; 6) $\frac{5}{3}$; 7) $\frac{1}{na^{n-1}}$; 8) $\frac{1}{2}$; 9) 0; 10) 0; 11) 2; 12) 3; 13) $\frac{1}{3}$; 14) $\frac{1}{\sqrt{3}}$; 15) 1; 16) -2; 17) $-\frac{1}{3}$; 18) 0; 19) e^3; 20) 1.

7-§. Funksiyaning o'sish va kamayishi

Agar $y=f(x)$ funksiya biror (a,b) oraliqda aniqlangan va bu oraliqqa tegishli ixtiyoriy ikkita $x_1<x_2$ nuqtalarda $f(x_1)<f(x_2)\big(f(x_1)>f(x_2)\big)$ tengsizlik bajarilsa, u holda y shu oraliqda o'suvchi (kamayuvchi) deyiladi.

Funksiyaning o'sish va kamayish oraliqlarini birgalikda monotonlik oraliqlari deyiladi.

Differensiallanuvchi $y=f(x)$ funksiya biror (a,b) oraliqda kamayuvchi (o'smovchi) bo'lsa, u holda bu oraliqda uning hosilasi $f'(x)\geq 0(f(x)\leq 0)$ shartni qanoatlantiradi.

1. Quyidagi funksiyalarning monotonlik oraliqlarini toping:

1) $f(x)=2x^2-\ln x$; 2) $f(x)=2x^3-9x^2-24x+7$;

3) $f(x)=4x^3-21x^2+18x+20$; 4) $f(x)=x^3+3x^2+3x$;

5) $f(x)=\sqrt{(x^2-9)^3}$; 6) $f(x)=\cos x-x$;

7) $f(x)=x^2e^{-x}$; 8) $f(x)=e^x+5x$;

9) $f(x)=\ln(1-x^2)$; 10) $f(x)=x\big(1+2\sqrt{x}\big)$;

11) $f(x)=x-2sinx,\ 0\leq x\leq 2\pi$;

12) $f(x)=x^5-5x^4+5x^3+1$.

J: 1) $\left(0;\frac{1}{2}\right)$ da kamayadi, $\left(\frac{1}{2};\infty\right)$ da o'sadi; 2) $(-\infty;-1)\cup(4;+\infty)$ da o'sadi, $(-1;4)$ da kamayadi; 3) $\left(-\infty;\frac{1}{2}\right)\cup(3;+\infty)$ da o'sadi, $\left(\frac{1}{2};3\right)$ da kamayadi; 4) $(-\infty;+\infty)$ da o'sadi; 5) $(-\infty;-3]$ da kamayadi, $[-3;+\infty)$ da o'sadi; 6) $(-\infty;+\infty)$ da kamayadi; 7) $(-\infty;0)\cup(2;+\infty)$ da kamayadi, $(0;2)$ da o'sadi; 8) $(-\infty;+\infty)$ da o'sadi; 9) $(-1;0)$ da o'sadi, $(0;1)$ da kamayadi; 10)

$[0;+\infty)$ da o'sadi; 11) $\left(\frac{\pi}{3};\frac{5\pi}{3}\right)$ da o'sadi, $\left[0;\frac{\pi}{3}\right)\cup\left(\frac{5\pi}{3};2\pi\right]$ da kamayadi; 12) $(-\infty;0)\cup\cup(0;1)\cup(3;+\infty)$ da o'sadi, $(1;3)$ da kamayadi.

8-§. Funksiyaning ekstremumlari

Berilgan $y=f(x)$ funksiya x_0 nuqta va uning biror atrofida aniqlangan bo'lib, u bu atrofdagi ixtiyoriy x nuqtada $f(x_0)\geq f(x_1)[f(x_0)\leq f(x_1)]$ shartni qanoatlantirsa, y shu x_0 nuqtada lokal ***maksimumga(minimumga)*** ega deyiladi.

Funksiyaning lokal maksimum va minimum nuqtalari birgalikda uning ***lokal ekstremumlari*** deyiladi.

Ferma teoremasi: Agar $y=f(x)$ funksiya x_0 nuqtada differentsallanuvchi va lokal ekstremumga ega bo'lsa, u holda bu nuqtada funksiyaning hosilasi $f'(x_0)=0$ shartni qanoatlantiradi.

Funksiyaning hosilasini nolga aylantiradigan yoki mavjud qilmaydigan nuqtalar shu funksiyaning ***kritik nuqtalari*** deyiladi.

Lokal ekstremumning birinchi yetarli sharti: Agar $y=f(x)$ funksiya x_0 kritik nuqtaning biron atrofida differensiallanuvchi bo'lib, bu kritik nuqtani chapdan o'nga qarab bosib o'tishda $f'(x)$ hosila o'z ishorasini musbatdan (manfiydan) manfiyga (musbatga) o'zgartirsa, u holda funksiya x_0 nuqtada maksimumga (minimumga) ega bo'ladi.

Agar $y=f(x)$ funksiya hosilasi x_0 kritik nuqtaning chap va o'ng atrofida ishorasini o'zgartirmasa, bu nuqtada funksiya ekstremumga ega bo'lmaydi.

Lokal ekstremumning ikkinchi yetarli sharti: Agar x_0 kritik nuqtada $f'(x_0)=0$, $f''(x_0)\neq 0$ va chekli bo'lsa, unda bu nuqtada $y=f(x)$ funksiya lokal ekstremumga ega bo'ladi. Jumladan, $f''(x_0)<0(f''(x_0)>0)$ bo'lsa, $f(x_0)$ funksiyaning lokal maksimumi (lokal minimumi) bo'ladi.

1.Quyidagi funksiyalarning ekstremumlari topilsin:

1) $y=4x-x^2$; 2) $y=x^2+2x-3$; 3) $y=\frac{x^3}{3}+x^2$;

4) $y=x^3+6x^2+9x$; 5) $y=\frac{x^2}{x-2}$; 6) $y=x^3+\frac{x^4}{4}$;

7) $y=\frac{x^4}{4}-2x^2$; 8) $y=2x-3\sqrt[3]{x^2}$; 9) $y=\frac{(x-1)^2}{x^2+1}$;

10) $y=xe^{-\frac{x^2}{2}}$; 11) $y=x-2\ln x$; 12) $y=x^{\frac{2}{3}}(x-5)$;

13) $y=\sin 2x-x$ ni $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda;

14) $y=2x+ctg\,x$ ni $(0;\pi)$ oraliqda;

15) $y=x+arcctg\,2x$; 16) $y=2tg\,x-tg^2x$;

17) $y=\frac{3-x^2}{x+2}$; 18) $y=\frac{1+x+x^2}{1-x+x^2}$;

19) $y=\sqrt{1-\cos x}$; 20) $y=(1-x^2)(1-x^3)$.

J: 1) $y_{max}(2)=4$; 2) $y_{min}(-1)=-4$;3) $y_{max}(-2)=\frac{4}{3}$;
4) $y_{min}(-1)=-4$,$y_{max}(-3)=0$;5) $y_{max}(0)=0$,$x=2$ da $y=\pm\infty$, $y_{min}(4)=8$; 6) $y_{min}(-3)=-6{,}75$; 7) $y_{\min}(\pm 2)=-4$, $y_{max}(0)=0$;8) $y_{max}(0)=0$, $y_{min}(1)=-1$; 9) $y_{max}(-1)=2$, $y_{min}(1)=0$; 10) $y_{min}(-1)=-\frac{1}{\sqrt{e}}$,$y_{max}(1)\approx 0{,}6$;
11) $y_{min}(2)=(1-\ln 2)$;12) $y_{max}(2)=0$, $y_{min}(5)=-4{,}8$;
13) $y_{max}\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}-\frac{\pi}{6}$, $y_{min}\left(-\frac{\pi}{6}\right)=-0{,}34$; 14) $y_{min}\left(\frac{\pi}{4}\right)=\frac{\pi}{2}+1$, $y_{max}\left(\frac{3\pi}{4}\right)$=3,71; 15) $y_{min}\left(\frac{\pi}{4}\right)=2{,}57$, $y_{max}\left(\frac{3\pi}{4}\right)=3{,}7$;
17) $y_{min}(-3)=6$,$y_{max}(-1)=2$; 18) $y_{min}(-1)=\frac{1}{3}$,$y_{max}(1)=3$;
19) $y_{min}(2\pi n)=0$,$y_{max}[(2n+1)\pi]=\sqrt{2}$; 20) $y_{max}(0)=1$, $y_{min}(1)=0$.

2. Quyidagi funksiyalarni 2-tartibli hosila yordamida ekstremumga tekshirilsin.

1) $y=4x-x^2$; 2) $y=\frac{1}{3}x^3-\frac{5}{2}x^2+6x$; 3) $y=x^5$;

4) $y=x^4-8x^2$; 5) $y=x+\cos 2x$, $\left(0;\frac{\pi}{4}\right)$; 6) $y=x^2e^{-x}$;

7) $y=2x^3+6x^2-18x+120$; 8) $y=3x^4-4x^3$.

J: 1) $y_{max}(2)=4$; 2) $y_{max}(2)=\frac{4}{3}$, $y_{min}(3)=\frac{9}{2}$; 3) ekstremum mavjud emas; 4) $y_{min}(-2)=-16$, $y_{max}(0)=0$, $y_{min}(2)=-16$;5) $y_{max}\left(\frac{\pi}{12}\right)\approx 1{,}13$;6)

$y_{min}(0) = 0$, $y_{max}(2) = 4e^{-2}$;7) $y_{min}(1) = 110$, $y_{max}(-3) = 174$;8) $y_{min}(1) = -1$,$y_{max}(0) = 0$.

9-§. Funksiyaning kesmadagi eng katta va eng kichik qiymatlari

$[a, b]$ kesmada uzluksiz bo'lgan funksiyaning eng katta va eng kichik qiymatlarini topish uchun quyidagi ishlar bajariladi:

1. $f(x)$ funksiyaning $[a, b]$ kesma ichida yotuvchi barcha kritik nuqtalari va hosilani mavjud qilmaydigan nuqtalari topiladi.

2. Bu nuqtalarda funksiyaning qiymatlari topiladi.

3. $[a, b]$kesmaning chetki nuqtalaridagi funksiyaning qiymatlari topiladi.

4. Topilgan barcha qiymatlardan eng kattasi va eng kichigi ajratiladi.

Eslatma.Agar $f(x)$ funksiyaning kritik nuqtalari $[a, b]$ kesmaga tegishli bo'lmasa, u holda $f(a)$ va$f(b)$ lar topiladi.

1. Quyidagi funksiyalarning ko'rsatilgan kesmalardagi eng katta va eng kichik qiymatlari topilsin.

1)$y = \frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2 + 2$ ni$[-2; 4]$ kesmadagi;

2)$y = x^3 - 3x^2 + 3x + 2$ ni$[2; 5]$ kesmadagi;

3)$y = \frac{x^4}{4} - \frac{x^3}{3} - 7x^2 + 24x + 1$ ni$[-5; 2]$ kesmadagi;

4)$y = x^4 + 8x^3 + 16x^2$ ni$[-3; 1]$ kesmadagi;

5)$y = \frac{x-1}{x+1}$ ni$[0; 4]$ kesmadagi;

6)$y = arctg\frac{1-x}{1+x}$ ni$[0; 1]$ kesmadagi;

7)$y = \frac{1-x+x^2}{1+x-x^2}$ ni$[0; 1]$ kesmadagi;

8) $y = \sqrt[3]{x+1} - \sqrt[3]{x-1}$ ni$[0; 1]$ kesmadagi;

9)$y = x + 2\sqrt{x}$ ni$[0; 4]$ kesmadagi;

10) $y = x - 2\ln x$ ni $[1; e]$ kesmadagi;

11) $y = 2\sin x + \cos 2x$ ni $\left[0; \frac{\pi}{2}\right]$ kesmadagi;

J: 1) $f(-2) = \frac{16}{3}$ eng katta, $f(3) = -\frac{37}{4}$ eng kichik; 2) $f(2) = 4$ eng kichik,$f(5) = 67$ eng katta; 3) $y(2) = \frac{67}{3}$ eng katta,$y(-4) = -\frac{365}{3}$eng kichik; 4)

$y(0) = 0$ eng kichik, $y(1) = 25$ eng katta; 5) 0,6 eng katta, -1 eng kichik; 6) $\frac{\pi}{4}$ eng katta, 0 eng kichik; 7) 1 eng katta, 0,6 eng kichik; 8) 2 eng katta, $\sqrt[3]{2}$ eng kichik; 9) 8 eng katta, 0 eng kichik; 10) $f(2) = 2(1 - \ln 2)$ eng kichik, $f(1) =$ 1 eng katta; 11) $y\left(\frac{\pi}{6}\right) = \frac{3}{2}$ eng katta, $y\left(\frac{\pi}{2}\right) = 1$ eng kichik.

Eng katta va eng kichik qiymatlarni topishga olib keluvchi masalalar

1. 20 soni shunday ikkita qo'shiluvchiga ajratilsinki, ularning ko'paytmasi eng katta bo'lsin. J: 10 va 10.

2. a soni shunday ikkita qo'shiluvchiga ajratilsinki, ularning ko'paytmasi eng katta bo'lsin. J: $\frac{a}{2}$ va $\frac{a}{2}$.

3. Uzunligi 120 metrlik panjara bilan bir tomondan uy bilan chegaralangan eng katta yuzaga ega to'g'ri to'rtburchak shaklidagi maydon o'rab olinishi kerak. Bu maydonning o'lchovlari aniqlansin.

J: 30×60.

4. Asosi 60 sm va balandligi 20 sm bo'lgan uchburchakka eng katta yuzli to'g'ri to'rtburchak ichki chizilgan. To'g'ri to'rtburchak yuzi topilsin. J: $30sm^2$.

5. Perimetri $2\,p$ bo'lgan to'g'ri to'rtburchaklar ichidan yuzi eng katta bo'lganini toping. J: Tomoni $\frac{p}{2}$ bo'lgan kvadrat.

6. Jism $S(t) = -t^3 + 9t^2 + 24t$ qonun bo'yicha to'g'ri chiziqli xarakat qiladi. Vaqtning qanday paytida jism xarakatining tezligi eng katta bo'ladi va tezlikning miqdori qancha bo'ladi?

J: $t = 3,\ V(3) = 51\frac{s}{m}$.

7.Yuqoriga tik otilgan jismning xarakat qonuni $S(t) = 19{,}6t - 4{,}9t^2$ tenglama bilan berilgan. Vaqtning qanday paytida jism eng yuqori balandlikda bo'ladi va bu balandlik necha metr bo'ladi?

J: $t = 2, S(2) = 196$.

8. Berilgan S yuzga ega bo'lgan barcha to'g'ri to'rtburchaklar ichida eng kichik perimetrga ega bo'lganini toping. J: Kvadrat.

9. Berilgan V xajmga ega bo'lgan barcha silindrlar ichidan to'la sirti eng kichik bo'lganini toping. J: $R_S = \sqrt[3]{\frac{V}{2\pi}}; H = 2R$.

10.Tunnelning kesimi bir tomoni yarim aylanadan iborat to'g'ri to'rtburchak shakliga ega. Kesim perimetri 18m. Yarim aylana radiusi qanday bo'lsa, kesim yuzi eng katta bo'ladi?J: $\frac{18}{\pi+4} \approx 2{,}5$.

11.Tomoni 60sm bo'lgan kvadrat shaklidagi tunikaning to'rtala uchidan kattaligi bir xil kvadratlar kesib olinib, qolgan qismidan usti ochiq quti yasalgan. Qutining xajmi eng katta bo'lishi uchun kesib tashlangan kvadratning tomoni qanday bo'lishi kerak?J: 10 sm.

12.Tubi kvadrat shaklida, xajmi 32 m^3 ga teng ochiq xovuzning o'lchovlari qanday bo'lganda, uning devorlari bilan tagini qoplash uchun eng kam material sarflanadi?J: $4 \times 4 \times 2$.

13.Asosining radiusi 4 dm, balandligi 6 dm bo'lgan konusga xajmi eng katta bo'lgan silindr ichki chizilgan. O'sha silindrning xajmi topilsin.J: $V = \frac{128\pi}{9} dm^3$.

14. $A(0;3)$ va $B(4;5)$ nuqtalar berilgan. OX o'qida shunday P nuqta topilsinki, $S = AP + PB$ masofa eng kichik bo'lsin.J: $x = 1{,}5$.

10-§. Funksiya grafigining qavariqlik va botiqlik oraliqlari. Bukilish nuqtalar. Assimptotalar

$y = f(x)$ funksiyaning grafigi (a, b) oraliqning istalgan nuqtasida o'tkazilgan urinmadan pastda (yuqorida) yotsa, u holda funksiya grafigi shu oraliqda qavariq (botiq) deyiladi.

Funksiya grafigining qavariq qismini botiq qismidan ajratuvchi $M_0\big(x_0; f(x_0)\big)$ nuqta grafikning bukilish nuqtasi deyiladi.

<u>Funksiya grafigining qavariq yoki botiq bo'lishini yetarlilik shartlari:</u> Agar (a, b) oraliqda differensiallanuvchi $f(x)$ funksiyaning ikkinchi tartibli hosilasi manfiy (musbat), ya'ni $f''(x) < 0$ ($f''(x) > 0$) bo'lsa, u holda funksiya grafigi shu oraliqda qavariq (botiq) bo'ladi.

$f''(x) = 0$ yoki $f''(x)$ majud bo'lmaydigan nuqtalar ***ikkinchi tur kritik nuqtalar*** deyiladi.

Bukilish nuqtalari mavjud bo'lishining yetarlilik shartlari: Agar x_0 nuqta $y = (x)$ funksiya uchun ikkinchi tur kritik nuqta bo'lsa va $f''(x)$ ikkinchi tartibli hosila bu nuqtadan o'tishda ishorasini o'zgartirsa, u holda bu funksiya grafigining x_0 absissali nuqtasi ***bukilish nuqta*** bo'ladi.

Agar $y = f(x)$ funksiya grafigidagi nuqta shu grafik bo'ylab cheksiz uzoqlashganda undan biror to'g'ri chiziqqacha bo'lgan masofa nolga intilsa, u holda bu to'g'ri chiziq funksiya grafigining ***asimptotasi*** deb ataladi.

Agar$\lim\limits_{x\to a} f(x) = \infty$bo'lsa,$x = a$ to'g'ri chiziq$y = f(x)$ funksiya grafigining ***vertikal asimptotasi*** deyiladi.

Agar $k = \lim\limits_{x\to+\infty} \frac{f(x)}{x}$ va $b = \lim\limits_{x\to+\infty} [f(x) - kx]$ limitlar mavjud bo'lsa, u holda $y = kx + b$ to'g'ri chiziq $y = f(x)$ funksiyaning ***og'ma asimptotasi*** deyiladi.

Agar$k = 0$ bo'lsa, u holda ***gorizantal asimptotaga*** ega bo'lamiz.

1.Quyidagi funksiyalar grafiklarining bukilish nuqtalari topilsin:

1) $y = \frac{x^3}{6} - x^2$; 2) $y = e^{-x^2}$;3) $y = \frac{2x}{1+x^2}$; 4) $y = 2^{\frac{1}{x}}$;

J: 1) $\left(2; -\frac{8}{3}\right)$; 2) $\left(\pm\frac{1}{\sqrt{2}}; e^{-\frac{1}{2}}\right)$; 3) $\left(\pm\sqrt{3}; \pm\frac{\sqrt{3}}{2}\right)$va (0;0);
4) $\left(-\frac{\ln 2}{2}; e^{-\frac{2}{\ln 2}}\right)$.

2. Quyidagi funksiyalarning qavariqlik, botiqlik oraliqlari va bukilish nuqtalarini toping:

1) $y = x^5 + 5x - 6$; 2) $y = (x - 4)^5 + 4x + 4$;

3) $y = e^{-\frac{x^2}{2}}$; 4) $y = xe^x$;

5) $y = ln(1 + x^2)$; 6) $y = arctg\, x - x$.

J: 1) $(-\infty; 0)$ da qavariq, $(0; +\infty)$ da botiq, $M_0(0; 6)$ bukilish nuqta; 2) $(-\infty; 4)$ da botiq, $(4; +\infty)$ da qavariq, $M_0(4; 20)$ bukilish nuqta; 3) $(-\infty; -1)$ va $(1; +\infty)$ da botiq, $(-1; 1)$ da qavariq, $M_1(-1; e^{-\frac{1}{2}})$ va $M_2(1; e^{-\frac{1}{2}})$ bukilish nuqtalar; 4) $(-\infty;-2)$ da qavariq, $(-2;+\infty)$ da botiq, $(-2;-2e^{-2})$ bukilish nuqta; 5) $(-\infty;-1)$ va $(1;+\infty)$ da qavariq, $(-1;1)$ da botiq, $M_1(1;\ln 2)$ va $M_2(-1;\ln 2)$ bukilish nuqtalar; 6) $(-\infty;0)$ da qavariq, $(0;+\infty)$ da botiq, $0(0;0)$ bukilish nuqta;

3.Quyidagi funksiyalar grafiklarining asimptotalari topilsin.

1) $y=\sqrt[5]{\frac{x}{x-2}}$; 2) $y=3x+arctg\,5x$; 3) $y=\frac{\ln(x+1)}{x^2}+2x$;

4) $y=\frac{x^2}{\sqrt{x^2-1}}$; 5) $y=\frac{x^3}{4(x+1)^2}$; 6) $y=\frac{1}{x}$;

7) $y=\frac{x-2}{x+4}$; 8) $y=\frac{x}{1+x^2}$; 9) $y=\frac{2x^2+x+3}{x+6}$;

10) $y=\frac{2x^2+4x\sqrt{x}+2}{2x+4}$; 11) $y=\frac{x^2+2}{2x+3}$; 12) $y=\frac{5x}{x-3}$.

J: 1) $x=2$va$y=1$; 2) $x\to+\infty$da $y=3x+\frac{\pi}{2}$va$x\to-\infty$da$y=3x-\frac{\pi}{2}$; 3) $x=0,\ \ y=2x$va$x\to-1+0$da$x=-1$; 4) $x=\pm1,\ \ y=\pm x$; 5) $x=-1$, $y=\frac{1}{2}x+1$; 6) $x=0$;7) $x=-4$, $y=1$; 8) $y=0$; 9) $x=-6$, $y=2x-11$; 10) $x=-2$; 11) $x=-\frac{3}{2},\ y=\frac{1}{2}x-\frac{3}{4}$;12) $x=3,\ y=3x+3$.

4. Quyida berilgan funksiyalarni to'la tekshiring va grafigini yasang.

1) $y=3\left(\frac{x^4}{2}-x^2\right)$; 2) $y=-4x+x^3$;

3) $y=x^3-9x^2+24x-15$; 4) $y=x^5-\frac{5}{3}x^3$;

5) $y=x^4-8x^3+16x^2$; 6) $y=x^2+\frac{1}{3}x^3-\frac{x^4}{4}$;

7) $y=x^4-2x^2+3$; 8) $y=\frac{x-1}{x^2-2x}$;

9) $y=\frac{2x^2}{4x^2-1}$; 10) $y=\frac{3x}{1+x^2}$.

11-§. Funksiyaning differensiali

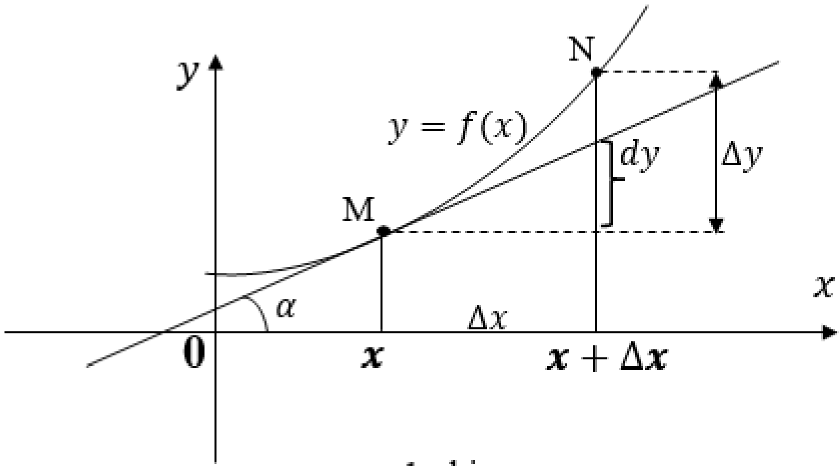
$y=f(x)$ funksiyaning differensiali deb, funksiya orttirmasini erkli o'zgaruvchi x ning orttirmasiga nisbatan chiziqli bo'lgan bosh qismiga aytiladi.

$y=f(x)$funksiyaning differensialidy yokidf bilan belgilanadi.

Demak, $dy=df=f'(x)dx$yoki$dy=y'dx$

Differensial geometrik jihatdan $y = f(x)$ funksiya grafigiga $M(x; y)$ nuqtadan o’tkazilgan urinma ordinatasining orttirmasiga teng (1-chizma).

Funksiyaning differensiali dy o’zining Δy orttirmasidan Δx ga nisbatan yuqori tartibli cheksiz kichik miqdorga farq qiladi.



1-chizma

Agar $u(x)$ va $v(x)$ funksiyalar differensiallanuvchi bo’lsa, u holda differensialning ta’rifi va differensiallash qoidalaridan bevosita differensialning asosiy xossalariga ega bo’lamiz:

1.$d(c) = 0$ (c - o’zgarmas son)

2. $d(cu) = cdu.$

3. $d(u \pm v) = du + dv.$

4. $d(u \cdot v) = udv + \square du.$

5. $d\left(\frac{u}{v}\right) = \frac{vdu - udv}{v^2},\ v \neq 0$

6. $df(u) = f'_u u' dx = f'(u)du.$

$y = f(x)$ funksiyaning ikkinchi tartibli differensiali deb birinchi tartibli differensialdan olingan differensialga aytiladi va u

$$d^2y = d(dy)$$

kabi yoziladi.

$y = f(x)$ funksiyaning n -tartibli differensiali deb $(n-1)-$ tartibli differensialdan olingan differensialga aytiladi, ya’ni

$$d^n y = d(d^{n-1}y)$$

Shunday qilib biz quyidagiga ega bo’lamiz:

$$d^2y = y''dx^2, d^3y = y'''dx^3, \ldots, d^ny = y^{(n)}dx^n.$$

Funksiyaning dy differensiali uning Δy orttirmasidagi $\Delta x = dx$ ga nisbatan yuqori tartibli cheksiz miqdorga farq qiladi, shu sababli $\Delta y \approx dy$ yoki

$$f(x + \Delta x) - f(x) = f'(x)\Delta x$$

deb yozish mumkin. Undan

$$f(x + \Delta x) \approx f(x) + f'(x)\Delta x$$

formulani hosil qilamiz. Bu fotmuladan taqribiy hisoblashlarda foydalanish mumkin.

1.Quyidagi funksiyalarning differensiallari topilsin:

1) $y = x^4$; 2) $y = x^3 - 3x^2 + 3x$; 3) $y = \sqrt{1 + x^2}$;

4) $S = \frac{gt^2}{2}$; 5) $r = 2\varphi - \sin 2\varphi$; 6) $y = \cos\frac{x}{2}$;

7) $y = \arcsin\frac{1}{x}$; 8) $y = \ln\sin 2x$; 9) $y = \frac{1-x^2}{1+x^2}$;

10) $y = \ln(x + \sqrt{1 + x^2})$; 11) $y = \ln tg\, 2x$; 12) $y = e^{\sin 2x}$

J: 1) $dy = 4x^3dx$; 2) $dy = (3x^2 - 6x + 3)dx$; 3) $dy = \frac{xdx}{\sqrt{1+x^2}}$;

4) $ds = gtdt$; 5) $dr = 2(1 - \cos 2\varphi)d\varphi$; 6) $dy = -\frac{1}{2}\sin\frac{x}{2}dx$;

7) $dy = -\frac{dx}{x\sqrt{x^2-1}}$; 8) $dy = 2ctg\, 2xdx$; 9) $dy = -\frac{4xdx}{(1+x^2)^2}$;

10) $dy = \frac{dx}{\sqrt{1+x^2}}$; 11) $dy = \frac{4dx}{\sin 4x}$; 12) $dy = 2e^{\sin 2x} \cdot \cos 2xdx$;

2.Quyidagi funksiyalarning differensiallari hisoblansin:

1) $x = 0$, $\Delta x = 0{,}1$ bo'lganda $y = ln(1 + e^{10x}) + arctg\, x$ funksiyaning differensiali topilsin. J: $dy = 0{,}25$.

2) $x = -10$ va $\Delta x = 0{,}1$ bo'lganda $y = x(1 + x)(1 - x)$ funksiyaning differensiali topilsin. J: -29,9.

3) $\varphi = -\frac{\pi}{4}$, $d\varphi = 0{,}2$ bo'lganda $r = \varphi + (\varphi^2 + 1)arcctg\, \varphi$ funksiyaning differensiali topilsin. J: -0,31.

3.Quyidagi funksiyalarning ko'rsatilgan tartibli differensiallari topilsin:

1) $y = 4x^5 - 7x^2 + 3, d^2y$ topilsin. J:$(80x^3 - 14)dx^2$.

2)$y = 4^{-x^2}$,d^2y topilsin. J:$d^2y = 2^{-2x^2+1}\ln 4(2x^3ln4 - 1)dx^2$.

3)$y = \sqrt{ln^2x - 4}$, d^2y topilsin. J: $d^2y = \frac{4lnx - 4 - ln^3x}{x^2\sqrt{(ln^2x-4)^3}}$.

4) $y = sin^2x, d^3y$ topilsin. J: $d^3y = -4\sin 2xdx^3$.

4. Quyidagi funksiyalarning taqribiy qiymatlarini verguldan keyingi ikki xonasigacha aniqlikda hisoblang:

1)$y = x^3 - 4x^2 + 5x + 3$ ni$x = 1{,}03$ da;

2)$y = \sqrt{1+x}$ni$x = 0{,}2$ da;

3)$y = \sqrt[3]{\frac{1-x}{1+x}}nix = 0{,}1$ da;

4)$y = \sqrt{x^2 - 7x + 10}$ni$x = 0{,}98$ da.

J: 1) 5,00; 2) 1,10; 3) 1,03; 4) 2,09.

5. Quyidagilarni taqribiy qiymatlari topilsin:

1) $\cos 31°$; 2) $\sqrt[5]{33}$; 3) $\sqrt[4]{17}$; 4) $arctg$ 0,98; 5) $\sin 29°$.

J: 1) 0,851; 2) 2,0125; 3) 2,031; 4) 0,7754; 5) 0,4848.

12-§. Teylor va Makloren formulalari

Agar $y = f(x)$ funksiya x_0 nuqtaning biror atrofida (n+1)–tartibgacha hosilalarga ega bo'lsa ($(n+1)$-tartibli hosila ham kiradi), u holda bu atrofning har qanday x nuqtasi uchun ***Teylor formulasi*** deb ataluvchi quyidagi formula o'rinlidir:

$$f(x)=f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + R_n(x)$$

bu yerda$R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x - x_0)^{n+1}$ ga Teylor formulasining Lagranj shaklidagi qoldiq hadi deyiladi. Bu yerdagi ξ nuqta x va x_0 nuqtalar orasida yotadi, ya'ni

$$\xi = x_0 + \theta(x - x_0) \text{ va} 0 < \theta < 1.$$

Agar Teylor formulasida $x_0 = 0$ deb olinsa, u holda ***Makloren formulasi*** deb ataluvchi quyidagi formulaga ega bo'lamiz:

$$f(x) = f(0) + \frac{f'(0)}{1!}(x) + \frac{f''(0)}{2!}(x)^2 + \cdots + \frac{f^{(n)}(0)}{n!}(x)^n + R_n(x)$$

bu yerda $R_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x)^{n+1}$ – qoldiq xad, ξ nuqta x va 0 nuqtalar orasida yotadi, ya'ni $\xi = \theta x$, $0 < \theta < 1$.

Quyida ko’p uchrab turadigan funksiyalarning Makloren qatorini keltiramiz:

1. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \frac{e^{\theta x}}{(n+1)!} x^{n+1};$

2. $\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots - (-1)^{n+1} \frac{x^{2n-1}}{(2n-1)!} + (-1)^n cos\theta x \frac{x^{2n+1}}{(2n+1)!};$

3. $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots - (-1)^n \frac{x^{2n}}{(2n)!} + (-1)^{n+1} cos\theta x \frac{x^{2n+1}}{(2n+1)};$

4. $\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n+1} \frac{x^n}{n} + R_n(x);$

5. $(1 + x)^m = 1 + mx + \frac{m(m-1)}{2!} x^2 + \frac{m(m-1)(m-2)}{3!} x^3 + \ldots + \frac{m(m-1)\ldots(m-n+1)}{n!} x^n +$
$+R_n(x).$

1. Teylor formulasidan foydalanib $P(x) = x^5 - 2x^4 + x^3 - x^2 + +2x - 1$ ko’phadni $x - 1$ ning darajalari bo’yicha yoying.

2. Teylor formulasidan foydalanib $f(x) = x^3 - 2x^2 + 3x + 5$ ko’phadni $x - 2$ ikkihadning darajalari bo’yicha yoying. J: $(x - 2)^3 + +4(x - 2)^2 +$ $+7(x - 2) + 11$

3. $x_0 = -1$ da $f(x) = e^x$ funksiya uchun uchinchi tartibli Teylor formulasini yozing. J: $e^x = \frac{1}{e} + \frac{1}{e} \cdot \frac{x+1}{1!} + \frac{1}{e} \cdot \frac{(x+1)^2}{2!} + \frac{1}{e} \cdot \frac{(x+1)^3}{3!} + R_3(x).$

4. e^x , $\sin x, \cos x$ va (1+ $x)^m$ larning Makloren formulasi bo’yicha yoyilmalaridan foydalanib quyidagi funksiyalarni Makloren formulasi bo’yicha yoyilmalari yozilsin:

1) $y = e^x;$ 2) $y = e^{\frac{x}{2}};$ 3) $y = e^{-x^2}$

4) $y = \sin 2x;$ 5) y=$\sin 3x;$ 6) $y = \sin \frac{x}{2};$

7) $y = \cos 2x$ 8) $y = \cos 3x$ 9) $y = \cos \frac{x}{3};$

10) $y = (1 + x)^{10};$ 11) $y = (1 + x)^5;$ 12) $y = (1 + x)^{2m};$

5. *e* sonini 0,0001 gacha aniqlikda hisoblang. J: 2,718.

6. $\sqrt[3]{29}$ning qiymatini 0,001 gacha aniqlikda hisoblang. J: 3,072.

7. $\cos 41°$va $\sqrt[3]{121}$ larning qiymatlarini 0,001 gacha aniqlikda hisoblang.

J: 1) 0,754; 2) 4,946.

VI BOB. ANIQMAS INTEGRAL

1-§. Boshlang'ich funksiya va aniqmas integral. Aniqmas integralni bevosita hisoblash

Biror chekli yoki cheksiz oraliqdagi har bir x nuqtada differensiallanuvchi va hosilasi

$$F'(x) = f(x) \quad (1)$$

shartni qanoatlantiruvchi $F(x)$ funksiya berilgan $f(x)$ funksiya uchun ***boshlang'ich funksiya*** deyiladi.

Agar $F(x)$ funksiya $f(x)$ funksiya uchun boshlang'ich funksiya bo'lsa, u holda ixtiyoriy c o'zgarmas son uchun $f(x) + c$ funksiya ham $f(x)$ funksiya uchun boshlang'ich funksiya bo'ladi. Chunki,

$$(F(x) + c)' = F'(x) + (c)' = f(x) + 0 = f(x)$$

Agar $f(x)$ funksiya biror (a, b) oraliqda $f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa u holda $F(x) + c$ funksiyalar to'plami $f(x)$ funksiyaning aniqmas integrali deyiladi.

$f(x)$ funksiyaning aniqmas integrali $\int f(x\,)dx$ kabi yoziladi. Demak, ta'rifga asosan,

$$\int f(x\,)dx = F(x) + c \quad (2)$$

bu yerda $\int$– aniqmas integral belgisi, $f(x) -$ aniqmas integral ostidagi funksiya $f(x)dx -$ aniqmas integral ostidagi ifoda, $x -$ integrallash o'zgaruvchisi deyiladi. Berilgan $f(x)$ funksiyaning $\int f(x)dx$ aniqmas integralini to'pish amali bu funksiyani integrallash deyiladi.

Aniqmas integral bir qator xossalarga ega :

1. $(\int f(x)dx)' = f(x)$. 2. $d(\int f(x)\,dx) = f(x)dx$.

3. $\int F'(x)dx = F(x) + c$. 4. $\int dF \quad (x) = F(x) + c$.

5. $\int kf(x)dx = k \int f(x)dx$.

6. $\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$.

7. $f(x)dx = F(x) + c$ bo’lsa, $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + c$.

Aniqmas integrallarni hisoblashda quyidagi jadvallardan foydalaniladi:

1. $\int x^{\alpha}\, dx = \frac{x^{\alpha+1}}{\alpha+1} + c \ \ (\alpha \neq -1)$. 2. $\int dx = x + c$.

3. $\int xdx = \frac{x^2}{2} + c$. 4. $\int \frac{dx}{x^2} = -\frac{1}{x} + c$.

5. $\int \frac{dx}{\sqrt{x}}$ =$2\sqrt{x} + c$. 6. $\int \frac{dx}{x} = ln\lfloor x\rfloor + c$.

7. $\int e^x\, dx = e^x + c$. 8. $\int a^x\, dx = \frac{a^x}{lna} + c$.

9. $\int \sin xdx = -\cos x + c$. 10. $\int \cos xdx = \sin x + c$.

11. $\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + c \quad \left(x \neq \frac{\pi}{2} + k\pi, k \in z\right)$.

12. $\int \frac{dx}{\sin^2 x} = -ctg\, x + c \quad (x \neq k\pi, k \in z)$.

13. $\int \operatorname{tg} xdx = -ln|\cos x| + c \left(x \neq \frac{\pi}{2} + k\pi, k \in z\right)$.

14. $\int ctgxdx = ln|sinx| + c(x \neq k\pi, k \in z)$.

15. $\int \frac{dx}{1+x^2} = \begin{cases} arctgx + c \\ -arcctgx + c. \end{cases}$ 16. $\int \frac{dx}{\sqrt{1-x^2}} = \begin{cases} \arcsin x + c \\ -\arccos x + c. \end{cases}$

17.$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} ln\left|\frac{x-a}{x+a}\right| + c$.18.$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = ln\left|x + \sqrt{x^2 \pm a^2}\right|$+c.

19. $\int \frac{dx}{a^2+x^2} = \frac{1}{a} arctg \frac{x}{a} + c$. 20. $\int \frac{dx}{\sqrt{a^2-x^2}} = arsin \frac{x}{a} + c$.

Bu yerda keltirilgan integrallarning to’g’riligini tenglikning o’ng tomonidan hosila olish orqali tekshiriladi.

Berilgan funksiyaning integralini integralning xossalari va jadvallari yordamida topilsa, u holda bunga ***bevosita hisoblash*** deyiladi.

Ba’zi aniqmas integrallarni hisoblashda differensial belgisi ostiga kiritish usulidan ham foydalanish mumkin.

Masalan, $dx = \frac{1}{k}d(kx+a)$ (a va $k-$ o’zgarmas sonlar), $cosxdx == d(sinx)$,$\frac{dx}{x}$=$d(lnx)$, $\frac{d□}{cos\ \ ^2x} = d(tgx)$,$\frac{dx}{1+x^2} = d(arctgx)$va hokazo.

1. Hosilasi $y' = 4x - 3$ bo’lgan va $x = 2$ da $y = 6$ qiymat qabul qiladigan funksiyani toping. J: $y = 2x^2 - 3x + 4$.

2. Hosilasi $y' = sinx + cosx$ bo'lgan va $x = \frac{\pi}{2}$ da $y = 4$ qiymat qabul qiladigan funksiyani toping. J: $y = sinx - cosx + 3$.

3. Agar $M(2; -4)$ nuqtadan o'tuvchi egri chiziqqa o'tkazilgan urinmaning burchak koeffitsienti uning har bir nuqtasida $2x - 6$ ga teng bo'lsa, shu egri chiziqning tenglamasini toping. J: $y = x^2 - 6x + 4$.

4. Moddiy nuqtaning harakat tezligi $v = 3t^2 + 2$. Agar bu nuqta $t = 2$ sekund vaqt ichida $40\ m$ yo'l bosib o'tgan bo'lsa, uning harakat qonunini toping. J: $s = t^3 + 2t + 28$.

5. To'g'ri chiziqli harakat qilayotgan nuqtaning tezligi $v = 2cost$ formula bilan berilgan. Agar bu nuqta $t = \frac{\pi}{6}$ sekund momentda sanoq boshidan $S = 6\ m$ masofada turgan bo'lsa, uning harakat qonunini toping. J: $S = 2\sin t + 5$.

6. Jism $v_0$boshlang'ich tezlik bilan yuqoriga tik otilgan. Bu jismning harakat qonununi toping. J: $S = v_0 t - \frac{gt2}{2}$.

7. Nuqta $a = 6t + 12$ tezlanish bilan to'g'ri chiziqli harakat qilyapdi. Vaqtning $t = 0$ momentida boshlang'ich tezlik $v_0 = 6\frac{m}{s}$, sanoq boshigacha bo'lgan masofa $s_0 = 8m$; 1) nuqtaning harakat tezligi va qonuniyatini toping; 2) $t = 2c$ momentdagi tezlanish, tezlik, va yo'lni toping. J:1) $s = t^3 + 6t^2 + 6t + 8$; 2)$a = 24\ \frac{m}{s}, v = 42\ \frac{m}{s}, s = 52\ m$.

8. Ushbu

1) $d(\quad) = 3x^2dx$; 2) $d(\quad) = x^4dx$; 3) $d(\quad) = -sinxdx$;

4) $d(\,) = -\frac{dx}{x^2}$; 5) $d(\,) = \frac{dx}{sin^2x}$; 6) $d(\quad) = \frac{dx}{\sqrt{1-x^2}}$.

tengliklardagi bo'sh joylar mulohazalar yordamida to'ldirilsin.

J: 1) x^3; 2) $\frac{x^5}{5}$; 3)$cosx$; 4) $\frac{1}{x}$; 5) $-ctgx$; 6) $arcsinx$.

9. Bevosita hisoblashga doir quyidagi integrallar hisoblansin:

1) $\int \left(x^2 + 2x + \frac{1}{x}\right) dx$; 2) $\int \left(3x^4 + 4x^3 + 5\sqrt{x} + \frac{4}{x^2} + 7\right) dx$;

3) $\int \frac{10x^8+3}{x^4} dx$; 4) $\int \frac{x-2}{x^3} dx$; 5) $\int (\sqrt{x} + \sqrt[3]{x}) dx$;

6) $\int\left(\frac{1}{\sqrt{x}}-\frac{1}{\sqrt[4]{x^3}}\right)dx$; 7) $\int\frac{(x^2+1)^2}{x^3}dx$; 8) $\int e^x\left(1-\frac{e^{-x}}{x^2}\right)dx$;

9) $\int\frac{cos2x}{cos^2x-sin^2x}dx$; 10) $\int\frac{dx}{sin^2x\cdot cos^2x}$; 11) $\int tg^2xdx$;

12) $\int ctg^2xdx$; 13) $\int\frac{3-2ctg^2x}{cos^2x}dx$; 14) $\int sin^2\frac{x}{2}dx$;

15) $\int cos^2\frac{x}{2}dx$; 16) $\int\left(\frac{2}{1+x^2}-\frac{3}{\sqrt{1-x^2}}\right)dx$; 17) $\int\frac{x^4dx}{1+x^2}$;

18) $\int\left(\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}\right)dx$;19) $\int\left(sin\frac{x}{2}-cos\frac{x}{2}\right)^2dx$; 20) $\int\frac{1-sin^3x}{sin^2x}dx$;

21) $\int\frac{x^2+5x-1}{\sqrt{x}}dx$; 22) $\int(x^2+5)^3dx$; 23) $\int\left(\frac{1}{\sqrt[3]{x^2}}-\frac{1}{x\sqrt{x}}\right)dx$;

24) $\int\frac{1+2x^2}{x^2(1+x^2)}dx$; 25) $\int a^x\left(1+\frac{a^{-x}}{x^5}\right)dx$.

J: 1) $\frac{x^3}{3}+x^2+lnx+c$; 2) $\frac{3x^5}{5}+x^4+\frac{10}{3}x\sqrt{x}-\frac{4}{x}+7x+c$;

3) $2x^5-\frac{1}{x^3}+c$; 4) $\frac{1-x}{x^2}+c$; 5) $x\left(\frac{2}{3}\sqrt{x}+\frac{3}{4}\sqrt[3]{x}\right)+c$;

6) $2\sqrt{x}-4\sqrt[4]{x}+c$; 7) $\frac{x^2}{2}+2lnx-\frac{1}{2x^2}+c$; 8) $e^x+\frac{1}{x}+c$;

9) $-ctgx-tg\,x+c$; 10) $tgx-ctgx+c$; 11) $tgx-x+c$;

12) $-ctgx-x+c$; 13) $3tgx+2ctgx+c$; 14) $\frac{x}{2}-\frac{sinx}{2}+c$;

15) $\frac{x}{2}+\frac{sinx}{2}+c$; 16) $2\,arctgx-3\,arcsinx+c$;

17) $\frac{x^3}{3}-x+arctgx+c$; 18) $lnx-\frac{1}{x}-\frac{1}{2x^2}+c$;

19) $x+sinx+c$; 20) $\cos x-ctgx+c$;

21) $\frac{2}{5}x^2\sqrt{x}+\frac{10}{3}x\sqrt{x}-2\sqrt{x}+c$; 22) $\frac{x^7}{7}+3x^5+25x^3+125x+c$;

23) $3\sqrt[3]{x}+\frac{2}{\sqrt{x}}+c$; 24) $-\frac{1}{x}+arctgx+c$; 25) $\frac{a^x}{lna}-\frac{1}{4x^4}+c$.

10. Quyidagi integrallarni differensial belgisi ostiga kiritish usulidan foydalanib hisoblang.

1) $\int\frac{dx}{\sqrt{3x-5}}$; 2) $\int\frac{3x^2-4x}{x^3-2x^2+4}dx$;

3) $\int\frac{x^2dx}{\sqrt[3]{1+x^3}}$; 4) $\int e^{-x^2}xdx$;

5) $\int\frac{e^{\sqrt{x}}}{\sqrt{x}}dx$; 6) $\int e^{x^3}x^2dx$;

7) $\int \frac{3x^2dx}{\sqrt[3]{1+x^3}}$; 8) $\int \frac{xdx}{\sqrt{1-x^2}}$;

9) $\int \frac{sinx\, dx}{\sqrt{1+2cos\, x}}$; 10) $\int \sqrt[3]{x^3-8}\cdot x^2dx$;

11)$\int \frac{x-arcsinx}{\sqrt{1-x^2}}dx$; 12) $\int \sqrt{x^2+1}xdx$.

J: 1) $\frac{2}{3}\sqrt{3x-5}+c$; 2) $ln|x^3-2x^2+4|+c$; 3) $\frac{1}{2}(\sqrt[3]{1+x})^2+c$;

4) $-\frac{1}{2}e^{-x^2}+c$; 5) $2e^{\sqrt{x}}+c$; 6) $\frac{1}{3}e^{x^3}+c$; 7) $\frac{3}{2}\sqrt[3]{(1+x^3)^2}+c$;

8) $-\sqrt{1-x^2}+c$; 9) $-\sqrt{1+2cosx}+c$; 10) $\frac{1}{4}\sqrt[3]{(1+x^3)^4}+c$;

11) $-\sqrt{1-x^2}-\frac{1}{2}\arcsin^2 x+c$; 12) $\frac{1}{3}\sqrt{(x^2+1)^3}+c$.

2-§. Aniqmas integralda o’zgaruvchini almashtirish. Bo’laklab integrallash

Aniqmas integralda o’zgaruvchini almashtirish quyidagicha amalga oshiriladi;

1) $x=\varphi(t)$, bunda $\varphi(t)-$ yangi o’zgaruvchi t ning differensiallanuvchi funksiyasi. Bu holda o’zgaruvchini almashtirish formulasi quyidagi ko’rinishda bo’ladi:

$$\int f(x)dx = \int f(\varphi(t))\varphi'(t)dt.$$

2) $\varphi(x)=t,$ bunda t –yangi o’zgaruvchi. Bu holda o’zgaruvchini almashtirish formulasi quyidagi ko’rinishga ega bo’ladi:

$$\int f(\varphi(x))\varphi'(x)dx = \int f(t)dt.$$

3) Har ikkala holda ham integrallashdan so’ng eski o’zgaruvchi x ga qaytish kerak bo’ladi.

Bo’laklab integrallash usuli:

$$\int udv = u\square - \int vdu.$$

formulaga asoslanadi. Bu yerda u va v lar x ning integrallanuvchi funksiyalari. Bu usuldan

$$\int P_n(x)e^{\alpha x}dx; \int P_n(x)cosaxdx, \int P_n(x)sinaxdx, \int P_n(x)arcsinxdx,$$

$$\int P_n(x)arccosxdx, \int P_n(x)arctgxdx, \int P_n(x)arcctg\square dx,$$

$$\int P_n(x)cosxdx, \int P_n(x)\, sinxdx, \int P_n(x)lnxdx,$$

$\int e^{\alpha x}cos\beta xdx, \int e^{\alpha x}sin\beta xdx$ va hokazo ko'rinishdagi integrallarni hisoblashda foydalaniladi.

1. Quyidagi integrallar hisoblansin:

O'zgaruvchini almashtirish (o'rniga qo'yish) usuli bilan hisoblanadigan integrallar.

1) $\int cos3xdx$; 2) $\int sin\frac{x}{2}dx$; 3) $\int e^{-3x}dx$;

4) $\int \frac{dx}{cos^2 5x}$; 5) $\int \left(e^{\frac{x}{2}} + e^{-\frac{x}{2}}\right)dx$; 6) $\int \sqrt{4x-1}dx$;

7) $\int \sqrt[3]{5-6x}\,dx$; 8) $\int \frac{dx}{\sqrt{3-2x}}$; 9) $\int \frac{2x-5}{x^2-5x+7}dx$;

10)$\int \frac{xdx}{x^2+1}$; 11) $\int \frac{e^{2x}dx}{1-3e^{2x}}$; 12)$\int \frac{sinxdx}{1+3cosx}$;

13) $\int sin^2xcosxdx$; 14) $\int cos^3xsinxdx$; 15) $\int \frac{sinx}{cos^3x}dx$;

16) $\int \frac{1-2cosx}{sin^2x}dx$; 17) $\int e^{cosx}sinxdx$; 18) $\int e^{x^3}x^2dx$;

19) $\int e^{-x^2}xdx$; 20) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$; 21) $\int \sqrt{x^2+1}\,xdx$;

22) $\int \sqrt[3]{x^3-8}\,x^2dx$; 23) $\int \frac{sinxdx}{\sqrt{1+2cosx}}$; 24) $\int \sqrt{1+4sinx}\,cosxdx$;

J: 1) $\frac{1}{3}sin3x + c$; 2)-2cos$\frac{x}{2} + c$; 3) $-\frac{1}{3}e^{-3x} + c$; 4) $\frac{1}{5}tg5x + c$;

5) $2(e^{\frac{x}{2}} - e^{-\frac{x}{2}}) + c$; 6) $\frac{1}{6}(4x-1)^{\frac{3}{2}} + c$; 7) $-\frac{1}{8}(5-6x)^{\frac{4}{3}} + c$;

8) $-\sqrt{3-2x} + c$; 9) $\ln(x^2-5x+7) + c$; 10) $\frac{1}{2}\ln(x^2+1) + c$;

11) $-\frac{1}{6}ln|1-3e^{2x}| + c$; 12) $-\frac{1}{3}ln|1+3cosx| + c$; 13) $\frac{sin^3x}{3}$+c;

14)$-\frac{cos^4x}{4} + c$; 15)$\frac{1}{2cos^2x} + c$; 16)$\frac{2-cosx}{sinx} + c$; 17) $-e^{-cosx} + c$;

18) $\frac{1}{3}e^{x^3}$+c;19) $-\frac{1}{2}e^{-x^2}$+c;20) $2e^{\sqrt{x}}$+c; 21) $\frac{1}{3}\sqrt{(x^2+1)^3}$+c;

22) $\frac{1}{4}\sqrt[3]{(x^3-8)^4}$+c;23)$-\sqrt{1+2cosx}$+c; 24) $\frac{1}{6}(1+4sinx)^{\frac{3}{2}}$+c.

Quyidagi integrallar bo'laklab integrallash formulasidan foydalanib hisoblansin:

1) $\int \ln x\, dx$; 2) $\int arcsinx\, dx$; 3) $\int arctgx\, dx$;

4) $\int xcosx\, dx$; 4) $\int xsinx\, dx$; 6) $\int arccosx\, dx$;

7) $\int xe^{-5x}dx$; 8) $\int arcctgx\, dx$; 9) $\int xe^{2x}\, dx$;

10) $\int x^2 \cos x\, dx$; 11) $\int xln(x-1)dx$; 12) $\int xarctgx\, dx$;

13) $\int (x^2+1)cosx\, dx$; 14) $\int \frac{x\, dx}{sin^2x}$; 15) $\int \frac{lnx\, dx}{x^2}$;

16) $\int \ln(x^2+1)\, dx$; 17) $\int \frac{x\, dx}{cos^2x}$; 18) $\int x^3e^{-x}dx$;

19) $\int e^x sinx\, dx$; 20) $\int e^x cosx\, dx$; 21) $\int x\, arcctgx\, dx$;

22) $\int \frac{arcsinx\, dx}{\sqrt{1+x}}$; 23) $\int arctg\sqrt{2x-1}dx$; 24) $\int \frac{arcsin\frac{x}{2}}{\sqrt{2-x}}dx$;

25) $\int (x^2+3x+5)cos2x\, dx$; 26) $\int (x^3+1)cosx\, dx$;

27) $\int (3x^3-17)e^{2x}dx$; 28) $\int x\, ln\left(1+\frac{1}{x}\right)dx$.

Javoblar: 1) $xln|x|-x+c$; 2) $x\, arcsinx+\sqrt{1-x^2}+c$;

3) $xarctgx-\frac{1}{2}\ln(1+x^2)+c$; 4) $x\, sinx+cosx+c$;

5) $-xcosx+sinx+c$; 6) $x\, arccosx-\sqrt{1-x^2}$+c;

7) $-\frac{x}{5}e^{-5x}-\frac{1}{25}e^{-5x}+c$; 8) $x\, arcctgx+\frac{1}{2}\ln(1+x^2)$+c;

9) $\frac{1}{2}e^{2x}\left(x-\frac{1}{2}\right)+c$; 10) $x^2sinx+2cosx-2sinx$ +c;

11) $\frac{x^2}{2}ln|x-1|-\frac{1}{2}\left(\frac{x^2}{2}+x+ln|x-1|\right)+c$;

12) $\frac{x^2+1}{2}arctgx-\frac{x}{2}+c$;13) $2x\, cosx$+$(x^2-1)sinx+c$;

14) $-xctgx+ln|sinx|+c$; 15) $-\frac{ln|x|+1}{x}+c$;

16) $x\ln(x^2+1)-2x+2arctgx+c$;17) $x\, tgx+ln|cosx|+c$;

18) $-e^{-x}(x^3+3x^2+6x+6)$+c;19) $\frac{1}{2}e^x(sinx-cosx)+c$;

20) $\frac{1}{2}e^x(sinx+cosx)+c$;21) $\frac{x^2}{2}arcctgx+\frac{1}{2}x-\frac{1}{2}arcctgx$+c;

22) $2\sqrt{1+x}arcsinx$+$4\sqrt{1-x}$+c;23) $xarctg\sqrt{2x-1}-\frac{\sqrt{2x-1}}{2}$+c;

24) $4\sqrt{2+x}-2\sqrt{2-x}\,arcsin\frac{x}{2}+c$;

25) $\left(\frac{x}{2}+\frac{3}{4}\right)cos2x+\left(\frac{1}{2}x^2+\frac{3}{2}x+\frac{9}{4}\right)sin2x+c$;

26) $(x^3-6x+1)sinx+(3x^2-6)cosx+c$;

27) $\left(\frac{3}{2}x^3-\frac{9}{4}x^2+\frac{9}{4}x-\frac{77}{8}\right)e^{2x}+c$;

28) $\frac{1}{2}(x^2-1)ln|x+1|-\frac{x^2}{2}lnx+\frac{x}{2}+c$.

3-§. Trigonometrik funksiyalarni integrallash

$\int R(sinx, cosx,)dx$ ko'rinishdagi integrallar $tg\frac{x}{2}=t$ almashtirish bilan integrallanadi. Bu yerda

$$sinx=\frac{2tgx\frac{x}{2}}{1+tg^2\frac{x}{2}}, \qquad cosx=\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}.$$

formulalardan foydalaniladi va t ga nisbatan ratsional funksiyani integrallashga keltiriladi. Ya'ni:

$$\int R(sinx, cosx)dx=\int R\left(\frac{2t}{1+t^2},\frac{1-t^2}{1+t^2}\right)\cdot\frac{2dt}{1+t^2}$$

$\int R(sinx)cosx\,dx,\ \int R(cosx)sinx\,dx$ ko'rinishdagi integrallar mos ravishda $sinx=t,\ cosx\,dx=dt$ va $cosx=t,\ sinx\,dx=-dt$ o'rniga qo'yishlar orqali ratsional funksiyalardan olingan integrallarga keltiriladi.

$\int sin^n x\cdot cos^m x\,dx$ ko'rinishdagi integrallar m va n ning qiymatlariga qarab turlicha integrallanadi.

1) Agar n musbat va toq bo'lsa, $cosx=t,\ sinx=-dt$ o'rniga qo'yish bilan integrallanadi;

2) Agar m musbat va toq bo'lsa, u holda $sinx=t,\ cosx\,dx=dt$ o'rniga qo'yish bilan integrallanadi;

3) Agar $m,n\geq 0$ va musbat sonlar bo'lsa, u holda $sin\alpha\cdot cos\alpha==\frac{1}{2}sin2\alpha$, $sin^2\alpha=\frac{1}{2}(1-cos2\alpha)$, $cos^2\alpha=\frac{1}{2}(1+cos2\alpha)$ formulalardan foydalanib integrallanadi;

4) Agar $m, n \leq 0$ va ulardan biri toq bo'lsa, u holda surat va maxrajni $sinx$ yoki $cosx$ ga qo'shimcha ko'paytirish usulidan foydalanib integrallanadi(m va n larning qaysinisini toq darajadaligiga qarab);

5) Agar $m + n < 0$ va juft bo'lsa, u holda $tgx = t$ yoki $ctgx = t$ o'rniga qo'yishdan foydalaniladi. Agar $m < o,\ n < o$ bo'lsa u holda sun'iy usuldan, ya'ni suratdagi 1 ni $(sin^2\alpha + cos^2\alpha)^k$ bilan almashtirilib integrallanadi. Bu yerda:

$$k = \frac{|m+n|}{2} - 1.$$

$\int tg^n x\, dx$ va $\int ctg^n x\, dx$ shakldagi integrallar ($n > 0$ −butun son) ni hisoblashda tg^2x yoki ctg^2x ko'paytuvchilarga ajratiladi va

$$tg^2x = \frac{1}{cos^2x} - 1 \qquad \text{va} \qquad ctg^2x = \frac{1}{sin^2x} - 1$$

formulalardan foydalaniladi. Bu integrallarni $tgx = t$ yoki $ctgx = t$ almashtirishlar bilan ham hisoblash mumkin:

$\int sec^n xdxva \int cosec^n xdx$ ko'rinishdagi integrallarni hisoblashda ikki holni qarash mumkin:

a) Agar n toq bo'lsa, u hoda $tg\frac{x}{2} = t$ almashtirishdan foydalaniladi;

b) Agar n juft bo'lsa, u holda tgx=t o'rniga qo'yishdan foydalaniladi. Ba'zi hollarda sec^2x yoki $cosec^2x$ ko'paytuvchi ajratilib $sec^2xdx == d(tgx)$ yoki $cosec^2x\, dx = d(ctgx)$ deb olinib qolgan darajalar $sec^2x = 1 + tg^2x$ yoki $cosec^2x = 1 + ctg^2x$ formulalar bo'yicha almashtiriladi.

c) $\int sin\alpha x \cdot cos\beta x\, dx, \int cos\alpha x \cdot cos\beta x\, dx, \int sin\alpha x \cdot sin\beta x\, dx$ ko'rinishidagi integrallar

$$sin\alpha \cdot cos\beta = \frac{1}{2}[sin(\alpha + \beta) + sin(\alpha - \beta)];$$

$$cos\alpha \cdot cos\beta = \frac{1}{2}[cos(\alpha + \beta) + \cos(\alpha - \beta)];$$

$$sin\alpha \cdot sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

formulalardan foydalanib hisoblanadi.

Trigonometrik funksiyalarni integrallashga doir masalalar

1. Quyidagi integrallar hisoblansin;

1)$\int sin^2 3x\, dx$; 2) $\int (1+2cosx)^2 dx$; 3) $\int (1-sin2x)^2 dx$;

4) $\int cos^4 x\, dx$; 5) $\int sin^4 x\, dx$; 6) $\int sin^2 x \cdot cos^2 x\, d\, x$;

7) $\int sin^4 x \cdot cos^4 x\, dx$; 8) $\int sin^2 x \cdot cos^4 x dx$; 9) $\int sin^5 x\, dx$;

10) $\int sin^2 x \cdot cos^3 x\, dx$; 11)$\int sin^3 x \cdot cos^2 x\, dx$; 12)$\int cos^7 x\, dx$;

13)$\int (1+2cosx)^3 dx$; 14)$\int \frac{cos^3 x}{sin^2 x} dx$; 15) $\int \frac{sin^3 x}{cos^2 x} dx$;

16) $\int \frac{dx}{sin2x}$; 17) $\int \frac{dx}{cosx}$; 18) $\int tg^3 x\, dx$;

19) $\int ctg^3 x\, dx$; 20)$\int sin3x \cdot sin5x dx$; 21) $\int sin4x \cdot cos6x$dx;

22) $\int cos3x \cdot cos5x dx$; 23) $\int \frac{sin^3 x+1}{cos^2 x} dx$; 24) $\int \frac{(sinx-cosx)^2}{sin2x} dx$.

Javoblar: 1)$\frac{1}{2}x - \frac{1}{12} sin6x + c$; 2)$3x + 4sinx + sin2x + c$;

3) $\frac{3x}{2} + cos2x - \frac{\sin 4x}{8} + c$; 4) $\frac{3x}{8} + \frac{\sin 2x}{4} - \frac{\sin 4x}{32} + c$; 5) $\frac{3x}{8} - \frac{\sin 2x}{4} + + \frac{\sin 4x}{32} + c$; 6) $\frac{x}{8} - \frac{\sin 4x}{32} + c$; 7) $\frac{3x}{128} - \frac{\sin 4x}{128} + \frac{\sin 8x}{1024} + c$; 8) $\frac{x}{16} - \frac{\sin 4x}{64} - \frac{\sin^3 2x}{48} + c$; 9) $-\cos x + \frac{2\cos^3 x}{3} - \frac{\cos^5 x}{5} + c$; 10) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$; 11) $-\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$; 12) $\sin x - \sin^3 x + + \frac{3\sin^5 x}{5} - \frac{\sin^7 x}{7} + c$; 13) $7x + 14\sin x + 3\sin 2x - \frac{8\sin^3 x}{3} + c$;14) $-\sin x - \frac{1}{\sin x} + c$; 15) $\cos x + \frac{1}{\cos x} + c$ 16) $\frac{1}{2}\ln|tgx| + c$;

17) $\ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + c$;18) $\frac{tg^2 x}{2} + \ln|\cos x| + c$;

19) $-\frac{ctg^2 x}{2} - \ln|\sin x| + c$; 20) $\frac{\sin 2x}{4} - \frac{\sin 8x}{16} + c$;

21) $-\frac{\cos\ 10x}{20} + \frac{\cos 2x}{4} + c$; 22) $\frac{\sin 8x}{16} + \frac{\sin 2x}{4} + c$;

23) $\frac{1}{\cos x} + \cos x + tg\, x + c$; 24) $\frac{1}{2}\ln|tgx| - x + c$.

4-§. Giperbolik funksiyalarni integrallash

Giperbolik funksiyalarni integrallash trigonometrik funksiyalarni intgrallash kabi bajariladi. Bunda quyidagi jadvaldan foydalaniladi:

$\int ch\, x\, dx = shx + c$; $\int sh\, x\, dx = chx + c$;

$\int \frac{1}{sh^2 x} dx = -cth\, x + c$; $\int \frac{1}{ch^2 x} dx = th\, x + c$.

Ba'zi hollarda $\int R(x,\sqrt{x^2-a^2})\,dx$ va $\int R(x,\sqrt{x^2+a^2})\,dx$ ko'rinishdagi integrallarni $x=acht$ va $x=asht$ almashtirishlar yordamida integrallanadi: bunda:

$$\text{agar } x=acht \text{ bo'lsa, } t=\ln\left|\frac{x+\sqrt{x^2-a^2}}{a}\right|$$

$$\text{agar } x=asht \text{ bo'lsa, } t=\text{tg}\left|\frac{x+\sqrt{x^2+a^2}}{a}\right|$$

Bulardan tashqari quyidagi o'rniga qo'yishdan ham foydalaniladi:

$$\text{agar } x=th\,t \text{ bo'lsa, } t=\frac{1}{2}\ln\frac{1+x}{1-x}.$$

1. Quyidagi integrallar hisoblansin:

1) $\int sh^2xdx$; 2) $\int sh^3xdx$; 3) $\int ch^4xdx$; 4) $\int th^3xdx$;

5) $\int ch^3x\cdot shxdx$; 6) $\int sh^4x\cdot ch^4xdx$; 7) $\int\frac{dx}{ch^2x\cdot shx}$;

8) $\int\frac{dx}{ch^2x+sh^2x}$;9) $\int\frac{dx}{thx-1}$; 10) $\int\frac{shxdx}{\sqrt{ch2x}}$; 11) $\int\frac{x^2dx}{\sqrt{x^2-3}}$;

12) $\int cth^3xdx$; 13) $\int sh^5x\cdot ch^2xdx$; 14) $\int\frac{\sqrt{x^2+4}dx}{x^2}$.

J: 1) $\frac{1}{4}sh2x-\frac{1}{2}x+c$; 2) $\frac{1}{3}ch^3x-chx+c$; 3) $\frac{3}{8}x+\frac{1}{4}sh2x++\frac{1}{32}sh4x+c$; 4) $\ln|chx|+\frac{1}{2ch^3x}+c$; 5) $\frac{1}{4}ch^4x+c$;

6) $\frac{1}{64}\left(\frac{3}{2}x-\frac{1}{2}sh4x+\frac{1}{16}sh8x\right)+c$; 7) $\ln\left|th\frac{x}{2}\right|+\frac{1}{chx}+c$;

8) $arctg(thx)+c$; 9) $-\frac{1}{2}sh^2x-\frac{1}{4}sh2x-\frac{x}{2}+c$;

10) $\frac{1}{\sqrt{2}}\ln\left|\sqrt{2}chx-\sqrt{ch2x}\right|$+c; 11) $\frac{1}{2}x\sqrt{x^2-3}+\frac{3}{2}\ln\left|x+\sqrt{x^2-3}\right|$+c;

12) $\ln|shx|-\frac{1}{2sh^2x}+c$; 13) $ch^3x\left(\frac{1}{7}ch^4x-\frac{2}{5}chx+\frac{1}{3}\right)+c$;

14) $\ln\left|x+\sqrt{x^2+4}\right|-\frac{\sqrt{4+x^2}}{x^2}+c$.

5-§. Kvadrat uchhad qatnashgan integrallarni hisoblash

Kvadrat uchhad qatnashgan integrallarni quyidagi to'rtta turga bo'lamiz:

I. $\int\frac{dx}{ax^2+bx+c}$; II. $\int\frac{dx}{\sqrt{ax^2+bx+c}}$; III. $\int\frac{Ax+B}{ax^2+bx+c}dx$;IV. $\int\frac{Ax+B}{\sqrt{ax^2+bx+c}}dx$.

I. $J_1=\int\frac{dx}{ax^2+bx+c}=\int\frac{dx}{a\left(\frac{x^2}{a}+\frac{b}{a}x+\frac{c}{a}\right)}=\int\frac{dx}{a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}\right]}=\int\frac{dx}{a\left[\left(x+\frac{b}{2a}\right)^2\pm K^2\right]}=$

$=\frac{1}{a}\int\frac{dx}{\left(x+\frac{b}{2a}\right)^2\pm K^2}$.

Oxirgi integralni osongina jadval integrallariga keltirish mumkin..

II. $J_2=\int\frac{dx}{\sqrt{ax^2+bx+c}}=\frac{1}{\sqrt{a}}\int\frac{dx}{\sqrt{\left(x+\frac{b}{2a}\right)^2\pm K^2}}=\begin{bmatrix}x+\frac{b}{2a}=t\\ dx=dt\end{bmatrix}=\frac{1}{\sqrt{a}}\int\frac{dx}{\sqrt{t^2\pm K^2}}$;

III. $J_3=\int\frac{Ax+B}{ax^2+bx+c}dx=\int\frac{\frac{A}{2a}(2ax+b)+\left(B-\frac{Ab}{2a}\right)}{ax^2+bx+c}dx=\frac{A}{2a}\int\frac{(2ax+b)}{ax^2+bx+c}dx+$

$+\left(B-\frac{Ab}{2a}\right)\int\frac{dx}{ax^2+bx+c}=\frac{A}{2a}\ln|ax^2+bx+c|+\left(B-\frac{Ab}{2a}\right)\cdot J_1$.

IV. $J_4=\int\frac{Ax+B}{\sqrt{ax^2+bx+c}}dx=\frac{A}{2a}\int\frac{d(ax^2+bx+c)}{\sqrt{ax^2+bx+c}}+\left(B-\frac{A}{2a}\right)\cdot J_2$.

1. Quyidagi integrallar hisoblansin.

I. $\int\frac{dx}{ax^2+bx+c}$ ko'rinishdagi integrallar.

1) $\int\frac{dx}{x^2+2x+5}$; 2) $\int\frac{dx}{x^2-6x-7}$; 3) $\int\frac{dx}{x^2+4x+5}$; 4) $\int\frac{dx}{x^2+4x+8}$;

5) $\int\frac{dx}{x^2-x-6}$; 6) $\int\frac{dx}{x^2+4x+29}$; 7) $\int\frac{dx}{4x-1-4x^2}$; 8) $\int\frac{dx}{2x^2-6x+9}$.

J: 1) $\frac{1}{2}arctg\frac{x+1}{2}+c$; 2) $\frac{1}{8}\ln\left|\frac{x-7}{x+1}\right|+c$; 3) $arctg(x+2)+c$;

4) $\frac{1}{2}arctg\frac{x+2}{2}+c$; 5) $\frac{1}{5}\ln\left|\frac{x-3}{x+2}\right|+c$; 6) $\frac{1}{5}arctg\frac{x+2}{5}+c$;

7) $\frac{1}{4x-2}+c$; 8) $\frac{1}{3}arctg\frac{2x-3}{3}+c$.

2. Quyidagi integrallar hisoblansin.

II. $\int\frac{dx}{\sqrt{ax^2+bx+c}}$ ko'rinishdagi integrallar.

1) $\int\frac{dx}{\sqrt{2x^2-8x+9}}$; 2) $\int\frac{(5x+3)dx}{\sqrt{x^2+4x+10}}$; 3) $\int\frac{dx}{\sqrt{x^2+4x+5}}$;

4) $\int\frac{dx}{\sqrt{4x^2-6x+5}}$; 5) $\int\frac{dx}{\sqrt{20x-16x^2-5}}$; 6) $\int\frac{dx}{\sqrt{2x-x^2}}$;

7) $\int\frac{dx}{\sqrt{2+x-x^2}}$; 8) $\int\frac{dx}{\sqrt{x^2-2x}}$; 9) $\int\frac{dx}{\sqrt{3x^2-2x-1}}$.

J: 1) $\frac{1}{\sqrt{2}}\ln\left|x-2+\sqrt{x^2-4x+\frac{9}{2}}\right|+c$; 2) $5\sqrt{x^2+4x+10}--7\ln\left|x+2+\sqrt{x^2+4x+10}\right|+c$; 3) $\ln\left|x+2+\sqrt{x^2+4x+5}\right|$ +c; 4) $\frac{1}{2}\ln\left|4x-3+2\sqrt{4x^2-6x+5}\right|+c$; 5) $\frac{1}{4}\arcsin\frac{8x-5}{\sqrt{5}}+c$;
6) $\arcsin(x-1)+c$; 7) $\arcsin\frac{2x-1}{3}+c$; 8) $\ln\left|x-1+\sqrt{x^2-2x}\right|+c$; 9) $\frac{1}{\sqrt{3}}\ln\left|3x-1+\sqrt{9x^2-6x+3}\right|+c$.

3. Quyidagi integrallar hisoblansin.

III. $\int\frac{Ax+B}{ax^2+bx+c}dx$ ko'rinishdagi integrallar.

1) $\int\frac{x}{x^2+5x+7}dx$; J: $\frac{1}{2}\ln(x^2+5x+7)-\frac{5}{\sqrt{3}}arctg\frac{2x+5}{\sqrt{3}}+c$;

2) $\int\frac{x-3}{x^2-4x+9}dx$; J: $\frac{1}{2}\ln(x^2-4x+9)-\frac{1}{\sqrt{5}}arctg\frac{x-2}{\sqrt{5}}+c$;

3) $\int\frac{2x+1}{2x^2+4x-7}dx$; J: $\frac{1}{30}\ln\left|\frac{(3x+7)^{11}}{(x-1)^9}\right|+c$;

4) $\int\frac{4x-3}{x^2+3x+4}dx$; J: $2\ln(x^2+3x+4)-\frac{18}{\sqrt{7}}arctg\frac{2x+3}{\sqrt{7}}+c$;

5) $\int\frac{3x+4}{x^2+5x}dx$; J: $\frac{4}{5}\ln|x|+\frac{11}{5}\ln|x+5|+c$;

6) $\int\frac{x}{x^2+x+1}dx$; J: $\frac{1}{2}\ln(x^2+x+1)-\frac{1}{\sqrt{3}}arctg\frac{2x+1}{\sqrt{3}}+c$.

4. Quyidagi integrallar hisoblansin.

IV. $\int\frac{Ax+B}{\sqrt{ax^2+bx+c}}dx$ ko'rinishdagi integrallar.

1) $\int\frac{3x-1}{\sqrt{x^2+2x+2}}dx$; 2) $\int\frac{2-5x}{\sqrt{4x^2+9x+1}}dx$;

3) $\int\frac{8x-11}{\sqrt{5+2x-x^2}}dx$; 4) $\int\frac{x}{\sqrt{3x^2-11x+2}}dx$;

5) $\int\frac{2-3x}{\sqrt{2x^2-7x+1}}dx$; 6) $\int\frac{3x+1}{\sqrt{x^2-4x+5}}dx$;

7) $\int\frac{4x-5}{\sqrt{3+2x-5x^2}}dx$; 8) $\int\frac{x}{\sqrt{3x^2-9x+2}}dx$.

J: 1) $3\sqrt{x^2+2x+2}-4\ln\left|x+1+\sqrt{x^2+2x+2}\right|+c$;

2) $-\frac{5}{4}\sqrt{4x^2+9x+1}+\frac{61}{16}\ln\left|8x+9+4\sqrt{4x^2+9x+1}\right|+c$;

3) $-8\sqrt{5+2x-x^2}-3\arcsin\frac{x-1}{\sqrt{6}}+c$;

4) $\frac{1}{3}\sqrt{3x^2-11x+2}+\frac{11}{6\sqrt{3}}\ln\left|x-\frac{11}{6}+\sqrt{x^2-\frac{11}{3}x+\frac{2}{3}}\right|+c;$

5) $-\frac{3}{2}\sqrt{2x^2-7x+1}-\frac{13}{4\sqrt{2}}\ln\left|4x-7+2\sqrt{4x^2-14x+2}\right|+c;$

6) $3\sqrt{x^2-4x+5}+7\ln|x-2+\sqrt{x^2-4x+5}|+c;$

7) $-\frac{4}{5}\sqrt{3+2x-5x^2}+\frac{21}{5\sqrt{5}}\arcsin\frac{5x-1}{4}+c;$

8) $\frac{1}{3}\sqrt{3x^2-11x+2}+\frac{\sqrt{3}}{2}\ln|6x-9+2\sqrt{9x^2-27x+6}|+c.$

6-§. Eng sodda ratsional kasrlar va ularni integrallash

1. $R_1(x)=\frac{A}{x-a}$; 2. $R_2(x)=\frac{A}{(x-a)^k}$; 3. $R_3(x)=\frac{Ax+B}{x^2+px+q}$ va

$R_4=\frac{Ax+B}{(x^2+px+q)^k}$ kasrlarni ***eng sodda ratsional kasrlar*** deyiladi. Bu yerda A,B, a,p,q -haqiqiy sonlar, $k=2,3,4,...$ va x^2+px+q kvadrat uchhad haqiqiy ildizlarga ega emas.

1. $\int R_1(x)dx=\int\frac{A}{x-a}dx=A\int\frac{d(x-a)}{x-a}=Aln|x-a|+c;$

2. $\int R_2(x)dx=\int\frac{Adx}{(x-a)^k}=A\int(x-a)^{-k}d(x-a)=\frac{A}{(1-k)(x-a)^{k-1}}+c$, k=2,3,4,…

3. $$\int R_3(x)dx=\int\frac{Ax+B}{x^2+px+q}dx=\int\frac{\frac{A}{2}(2x+p)-\frac{Ap}{2}+B}{x^2+px+q}dx=\frac{A}{2}\int\frac{(2x+p)dx}{x^2+px+q}+$$

$$+\left(B-\frac{A\,p}{2}\right)\int\frac{dx}{x^2+px+q}=\frac{A}{2}\ln\left(x^2+px+q\right)+\left(B-\frac{Ap}{2}\right)\int\frac{d\left(x+\frac{p}{2}\right)}{\left(x+\frac{p}{2}\right)^2+m^2}=$$

$$=\frac{A}{2}\ln|x^2+px+q|+\frac{B-\frac{Ap}{2}}{m}arctg\frac{x+\frac{p}{2}}{m}+c;$$

4. $$\int R_4dx=\int\frac{Ax+B}{(x^2+px+q)^k}dx=\int\frac{\frac{A}{2}(2x+p)+B-\frac{Ap}{2}}{(x^2+px+q)^k}dx=\frac{A}{2}\int\frac{2x+p}{(x^2+px+q)^k}dx+(B-$$

$$-\frac{Ap}{2})\int\frac{dx}{\left[(x+\frac{p}{2})^2+q-\frac{p^2}{4}\right]^k}.$$

Birinchi integral $x^2+px+q=t$, $(2x+p)dx=dt$ o`rniga qo`yish orqali ikkinchi ko`rinishdagi integralga keltiriladi.

Ikkinchi integralda (uni J_k deb belgilaymiz) $x+\frac{p}{2}=t$ deymiz va $q-\frac{p^2}{4}$ ni a^2 bilan almashtiramiz. Natijada:

$$J_k = \int \frac{dx}{\left[(x+\frac{p}{2})^2 + (q - \frac{p^2}{4})\right]^k} = \int \frac{dt}{(t^2+a^2)^k} = \frac{1}{a^2}\int \frac{a^2+t^2-t^2}{(t^2+a^2)^k}dt =$$

$$= \frac{1}{a^2}\int \frac{dt}{(t^2-a^2)^{k-1}} - \frac{1}{a^2}\int \frac{t^2dt}{(t^2+a^2)^k}.$$

Birinchi integral huddi J_k ning o`zi, biroq maxrajining daraja ko'rsatkichi bir birlikka kichik. Uni $J_{k-1}$ deb belgilaymiz. Ikkinchi integralni bo`laklab integrallaymiz:

$$\int \frac{t^2dt}{(t^2+a^2)^k}$$

$$= \frac{1}{2}\int \frac{t \cdot 2\square dt}{(t^2+a^2)^k}$$

$$= \frac{1}{2}\left[\frac{-t}{(k-1)(t^2+a^2)^{k-1}} + \frac{1}{k-1}\int \frac{dt}{(t^2+a^2)^{k-1}}\right] =$$

$$= -\frac{t}{2(k-1)(t^2+a^2)^{k-1}} + \frac{1}{2(k-1)}.\ J_{k-1}.$$

Shunday qilib, J_k integralni hisoblash uchun k ni darajasini pasaytirish formulasini hosil qildik:

$$J_k = \frac{t}{2a^2(k-1)(t^2+a^2)^{k-1}} + \frac{2k-3}{2a^2(k-1)}.J_{k-1}$$

Quyidagi integrallar hisoblansin:

1. $\int \frac{A}{x-a}dx$ ko`rinishdagi integrallar:

1)$\int \frac{dx}{x-7}$. J: $ln|x-7|$ +C;

2)$\int \frac{5dx}{x-4}$. J: $5ln|x-4| + C$;

3)$\int \frac{7dx}{2x-6}$. J: $\frac{7}{2}(ln|x-3|) + C$;

4)$\int \frac{x^3}{x-2}dx$. J: $\frac{x^3}{3} + x^2 + 4x + 8\ln|x-2| + C$;

5)$\int \frac{x^5}{x^3-a^3}dx$. J: $\frac{x^3}{3} + \frac{a^3}{3}\ln|x^3-a^3| + C$.

2.$\int \frac{Adx}{(x-a)^k}$ ko`rinishdagi integrallar:

1)$\int \frac{Adx}{(x-2)^2}$ $J{:} -\frac{1}{x-2} + C$;

2) $\int \frac{6dx}{(x-3)^3}$ $J: -\frac{3}{(x-3)^2} + C;$

3) $\int \frac{7dx}{(x-4)^4}$ $J: -\frac{7}{3(x-4)^3} + C;$

4) $\int \frac{3dx}{(x-9)^4}$ $J: -\frac{1}{(x-9)^3} + C.$

3. $\int \frac{Ax+B}{x^2+px+q} dx$ ko`rinishdagi integrallar:

1) $\int \frac{3x-1}{x^2-4x+8} dx$ $J: \frac{3}{2}\ln|x^2-4x+8| + \frac{5}{2} arctg \frac{x-2}{2} + C;$

2) $\int \frac{2x+7}{x^2+x-2} dx$ $J: \ln\left|\frac{(x-1)^3}{x+2}\right| + C;$

3) $\int \frac{3x+2}{2x^2+x-3} dx$ $J: lnC(x-1)\sqrt{2x+3};$

4) $\int \frac{x-1}{x^2-5x+6} dx$ $\square: ln\frac{C(x-2)^2}{x-3};$

5) $\int \frac{4x-2{,}4}{x^2-0{,}2x+0{,}17} dx$ $J: 2\ln(x^2-0{,}2x+0{,}17) - 5arctg\frac{10x-1}{4} + C.$

4. $\int \frac{Ax+B}{(x^2+px+q)^k} dx$ ko`rinishdagi integrallar:

1) $\int \frac{3x+2}{(x^2+2x+10)^2} dx$ $J: -\frac{3}{2(x^2+2x+10)} - \frac{x+1}{18(x^2+2x+10)} - \frac{1}{54} arctg\frac{x+1}{3} + C;$

2) $\int \frac{xdx}{(x^2+2x+2)^2}$ $J: -\frac{1}{2}\left[\frac{x+2}{x^2+2x+2} + arctg(x+1)\right] + C;$

3) $\int \frac{3x+5}{(x^2+2x+2)^2} dx$ $J: \frac{2x-1}{2(x^2+2x+2)} + arctg(x+1) + C;$

4) $\int \frac{2x-3}{(x^2-4x+8)^3} dx$ $J: \frac{3x^3-18x^2+56x-128}{128(x^2-4x+8)^2} + \frac{3}{256} arctg\frac{x-2}{2} + C.$

7-§. Ratsional kasrlarni integrallash

$R(x) = \frac{P_n(x)}{Q_m(x)}$ kasrni ***ratsional kasr*** deb ataladi. Bu yerda $P_n(x)\ va\ Q_m(x)$ lar mos ravishda n va m darajali ko`phadlar. Agar $n < m$ bo`lsa, kasr to`g`ri, $n > m$ bo`lsa, kasr noto`g`ri kasr bo`ladi. Bu holda bo`lish orqali uning butun qismi ajratilib so`ngra integrallanadi.

Agar $\frac{P_n(x)}{Q_m(x)}$ kasr to`g`ri bo`lsa, u holda quyidagicha ish tutiladi:

$Q_m(x)$ ko`phadni ko`paytuvchilarga ajratiladi.

$$Q_m(x) = a_0(x-\alpha_1)^{k_1} \dots (x-\alpha_p)^{k_p}(x^2+p_1x+q_1)^{s_1} \dots (x^2+p_ix+q_i)^{s_i}$$

bunda $\alpha_1, \alpha_2, \dots \alpha_p$ $l□r$ $Q_m(x)$ ko'phadning mos ravishda $k_1, k_2, \dots k_p$ karrali haqiqiy ildizlari, hamma kvadrat uchhadlar uchun $D_i < 0$; $k_1 + k_2 + \cdots + k_p + +2s_1 + 2s_2 + \cdots 2s_i = m$; $k_1, \dots k_p, s_1, \dots s_i \epsilon N$ ya'ni natural sonlar ; $a_0 -$ $Q_m(x)$ko`phaddagi x^n ning oldidagi koeffitsient.

Agar $R(x) = \frac{P_n(x)}{Q_m(x)}$ to`g`ri ratsional kasr maxraji $Q_m(x)$ yuqorida ko`rsatilgandek ifodalangan bo`lsa, u holda bunday kasrni $I - IV$ ko`rinishdagi eng sodda ratsional kasrlar yig`indisi sifatida yoyish mumkin. Bu yoyilmada $Q_m(x)$ ko'phadning har bir k karrali α ildiziga, ya'ni $(x-\alpha)^k$ ko`rinishdagi ko'paytuvchiga ushbu k ta kasrlar yig`indisi mos keladi:

$$\frac{A_1}{x-\alpha} + \frac{A_2}{(x-\alpha)^2} + \cdots \frac{A_k}{(x-\alpha)^k}$$

$Q_m(x)$ ko'phadning s karrali kompleks qo`shma ildizining har bir juftida, ya`ni $(x^2 + px + q)^s$ ko`rinishdagi ko'paytuvchiga ushbu S ta kasrdan iborat yig`indi mos keladi:

$$\frac{M_1x + N_1}{x^2 + px + q} + \frac{M_2x + N_2}{(x^2 + px + q)^2} + \cdots + \frac{M_sx + N_s}{(x^2 + px + q)^s}$$

Bu yerda A_1, $A_2, \dots, A_k$, M_1, M_2, M_s, N_1, $N_2, \dots, N_s$ vaqtincha noma`lum koeffitsientlar.

Agar $R(x) = \frac{P_n(x)}{Q_m(x)}$ kasrning maxraji haqiqiy va har xil ildizlarga ega bo`lsa u holda berilgan kasr faqat I-turdagi kasrlarga ajraydi.

Agar $R(x) = \frac{P_n(x)}{Q_m(x)}$ kasr maxrajining ildizlari haqiqiy va ba`zilari karrali bo`lsa, u holda u I va II- turdagi sodda kasrlarga ajraladi.

Agar $R(x) = \frac{P_n(x)}{Q_m(x)}$ kasr maxrajining ildizlari kom'leks sonlar va turlicha bo`lsa, u holda berilgan kasr III turdagi eng sodda kasrlarga ajraladi.

Agar $R(x) = \frac{P_n(x)}{Q_m(x)}$ kasr maxrajining ildizlari kompleks va karrali bo`lsa, u holda berilgan kasr III va IV turdagi kasrlarga ajraladi.

Quyidagi integrallar hisoblansin:

1. $R(x)=\frac{P_n(x)}{Q_m(x)}$ kasr maxrajining ildizlari haqiqiy va har xil:

1) $\int\frac{2x^2+41x-91}{(x-1)(x+3)(x-4)}dx J:\ln\left|\frac{(x-1)^4(x-4)^5}{(x+3)^7}\right|+C;$

2) $\int\frac{(5x^3+2)dx}{x^3-5x^2+4x}$ $J:\ \frac{1}{2}\ln|x|-\frac{7}{3}\ln|x-1|+\frac{16}{6}\ln|x-4|+C;$

3) $\int\frac{32xdx}{(2x-1)(4x^2-16x+15)}$ $J:\ln|\frac{(2x-1)2x-5)^5}{(2x-3)^6}|+C;$

4) $\int\frac{x-1}{(x-2)(x-3)}dx$ $J: ln\frac{C(x-2)^2}{x-3};$

5) $\int\frac{2x+7}{x^2+x-2}dx$ $J:\ln\left|\frac{(x-1)^3}{x+2}\right|+C;$

6) $\int\frac{3x^2+2x-3}{x^3-x}dx$ $J:\ln\left|\frac{Cx^3(x-1)}{x+1}\right|.$

2. $\boldsymbol{R(x)=\frac{P_n(x)}{Q_m(x)}}$ maxrajining ildizlari haqiqiy va ba`zilari karrali

1) $\int\frac{x+2}{x^3-2x^2}dx$ $J:\ \frac{1}{x}+\ln\left|\frac{x-2}{x}\right|+C;$

2) $\int\frac{2x^2-5x+1}{x^3-2x^2+x}dx$ $J: lnCx(x-1)+\frac{2}{x-1};$

3) $\int\frac{dx}{x^4-x^2}$ $J:\ \frac{1}{x}+\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+C;$

4) $\int\frac{5x-8}{x^3-4x^2+4x}dx$ $J: 2\ln\left|\frac{C(x-2)}{x}\right|-\frac{1}{x-2};$

5) $\int\frac{(x^3-6x^2+9x+7)}{(x-2)^3(x-5)}dx$ $J:\ \frac{3}{2(x-2)^2}+\ln|x-5|+C;$

6) $\int\frac{x^3-2x^2+4}{x^3(x-2)^2}dx$ $J:\ \frac{1}{4}\ln\left|\frac{x}{x-2}\right|-\frac{1}{x}\left(1+\frac{1}{2x}\right)-\frac{1}{2(x-2)}+C.$

3. $R(x)=\frac{P_n(x)}{Q_m(x)}$ kasr maxrajining ildizlari kom’leks va turlicha

1) $\int\frac{dx}{x^3+8}$ $J:\ \frac{1}{24}ln\frac{(x+2)^2}{x^2-2x+4}+\frac{1}{4\sqrt{3}}arctg\frac{x-1}{\sqrt{3}}+C;$

2) $\int\frac{3x^2+2x+1}{(x+1)^2(x^2+1)}dx$ $J: ln\frac{\sqrt{x^2+1}}{|x+1|}-\frac{1}{x+1}+arctgx+C;$

3) $\int\frac{dx}{(x^2-3)(x^2+2)}$ $J:\ \frac{1}{10\sqrt{3}}\ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right|-\frac{1}{5\sqrt{2}}+C;$

4) $\int\frac{dx}{x^3+4x}$ $J:\ \frac{1}{4}ln\frac{|x|}{\sqrt{4+x^2}}+C;$

5) $\int\frac{xdx}{(x^2+1)(x^2+2x+5)}$ $J:\frac{1}{10}ln\frac{x^2+1}{x^2+2x+5}+\frac{1}{20}\left(2arctgx-3arctg\frac{x+1}{2}\right)+C;$

6) $\int \frac{x^3-6}{x^4+6x^2+8}dx$ $J: ln\frac{x^2+4}{\sqrt{x^2+2}}+\frac{3}{2}arctg\frac{x}{2}-\frac{3}{\sqrt{2}}arctg\frac{x}{\sqrt{2}}+C.$

4. $R(x)=\frac{P_n(x)}{Q_m(□)}$ kasr maxrajining ildizlari kom'leks va karrali:

1) $\int \frac{(2x+1)dx}{(x^2+2x+5)^2}$ $J: -\frac{x+9}{8(x^2+2x+5)}-\frac{1}{16}arctg\frac{x+1}{2}+C;$

2) $\int \frac{dx}{(x^2-6x+10)^3}$ $J: \frac{1}{8}\left[\frac{(x-3)(3x^2-18x+32)}{(x^2-6x+10)^2}+3arctg(x-3)\right]+C;$

3) $\int \frac{x+1}{x^2+4x^2+4}dx$ $J: \frac{x-2}{4(x^2+2)}+\frac{\sqrt{2}}{8}arctg\frac{x}{\sqrt{2}}+C;$

4) $\int \frac{4xdx}{(x+1)(1+x^2)^2}$ $J: ln\frac{\sqrt{x^2+1}}{|x+1|}+\frac{x-1}{x^2+1}+C;$

5) $\int \frac{5x^2-12}{(x^2-6x+13)^2}dx$ $J: \frac{53}{16}arctg\frac{x-3}{2}+\frac{13x-159}{8(x^2-6x+13)}+C.$

8-§. Irratsional funksiyalarni integrallash

Quyida ba`zi bir irratsional funksiyalarni integrallash bilan tanishamiz:

1. $\int R\left(x,\ x^{\frac{m_1}{n_1}},\ x^{\frac{m_2}{n_2}},\ \dots,\ x^{\frac{m_k}{n_k}}\right)dx$ ko`rinishdagi integral $x=t^s$, $dx=st^{s-1}dt$ almashtirish bilan ratsional funksiyani integrallshga keltiriladi. Bu yerda s soni $\frac{m_1}{n_1},\frac{m_2}{n_2},\dots\frac{m_k}{n_k}$ kasrlarning umumiy maxrajidan iborat.

2. $\int R(x,\sqrt[n]{ax+b})dx$ ko`rinishdagi integral $ax+b=t^n$ almashtirish bilan ratsional funksiyalarni integrallashga keltiriladi.

3. $\int R\left[x,\left(\frac{ax+b}{cx+d}\right)^{\frac{m_1}{n_1}},\left(\frac{ax+b}{cx+d}\right)^{\frac{m_2}{n_2}},\dots,\left(\frac{ax+b}{cx+d}\right)^{\frac{m_k}{n_k}}\right]dx$

ko`rinishdagi integral $\frac{ax+b}{cx+d}=t^s$ almashtirish bilan ratsional funksiyani integrallashga keltiriladi.

4. $\int R\left(x,\sqrt{a^2-x^2}\right)dx$ ko`rinishdagi integral $x=asint$ almashtirish natijasida ratsional funksiyani integrallashga keltiriladi.

5. $\int R\left(x,\sqrt{a^2+x^2}\right)□x$ ko`rinishdagi integral x=atgt almashtirish yordamida ratsional funksiyani integrallashga keltiriladi.

6. $\int x^m(a+bx^n)^p\,dx$ ko`rinishdagi integrallar (m, n, p– ratsional sonlar) ***differensial binomlari integrallari*** deb atalib u quyidagi 3 ta holdagina elementer funksiyalar orqali ifodalanadi :

a)agar p -butun son bo`lsa, u holda integral $x=t^s$ o`rniga qo`yish yordamida (bunda s- kasrlar maxrajlari m va n ning eng kichik umumiy karralisi) ratsional funksiya integraliga keltiriladi;

b) agar$\frac{m+1}{n}$ - butun son bo`lsa, u holda integral $a+bx^n=t^s$ o`rniga qo`yish orqali ratsionallashtiriladi. Bunsa $s-$ soni p kasrning maxraji;

c) $\frac{m+1}{n}+p$ - butun son bo`lsa, u holda $a+bx^n=t^s x^n$ deb olamiz. Bunda $s-$ soni p kasrning maxraji.

7. $\int R(x,\sqrt{ax^2+bx+c})dx$ ko`rinishdagi integrallar Eyler almashtirishlari (L.Eyler shvetsariyalik buyuk matematik (1707-1783)) deb ataluvchi almashtirishlar yordamida integrallanadi. Bunda quyidagi uch hol bo`lishi mumkin:

1-hol. a>0 bo`lganda $\sqrt{ax^2+bx+c}=x\sqrt{a}-t$ almashtirish orqali ratsional kasrni integrallashga keltiriladi.

2-hol. c >0 bo`lganda $\sqrt{ax^2+bx+c}=xt+\sqrt{c}$ almashtirish yordamida ratsional funksiyani integrallashga keltiriladi.

3-hol. ax^2+bx+c uchhad α va β haqiqiy ildizlarga ega bo`lsa, u holda $\sqrt{ax^2+bx+c}=(x-\alpha)t$ almashtirish yordamida ratsional funksiyani integrallashga keltiriladi.

Quyidagi integrallar hisoblansin:

1. Birinchi ko`rinishdagi integrallar.

1)$\int\frac{dx}{\sqrt[3]{x}+\sqrt{x}}$; J: $6\left[\frac{\sqrt{x}}{3}-\frac{\sqrt[3]{x}}{2}+\sqrt[6]{x}-\ln(1+\sqrt[6]{x})\right]+C$;

2)$\int\frac{\sqrt{x}}{\sqrt{x}+1}dx$; J: $x-2\sqrt{x}+2\ln\left(\sqrt{x}+1\right)+C$;

3)$\int\frac{1+\sqrt[4]{x}}{x+\sqrt{x}}dx$; J: $4\sqrt[4]{x}+2\ln\left(1+\sqrt{x}\right)-4arctg\sqrt[4]{x}+C$;

4)$\int\frac{dx}{(1+\sqrt[3]{x})\sqrt{x}}$; J: $6\sqrt[6]{x}-6arctg\sqrt[6]{x}+C$;

2. Ikkinchi ko`rinishdagi integralar.

1)$\int\frac{x+1}{\sqrt[3]{3x+1}}dx$; J: $\frac{x+2}{5}\sqrt[3]{(3x+1)^2}+C$;

2)$\int \frac{xdx}{\sqrt{2x+1}+1}$; J: $\frac{2x+1}{12}\left(2\sqrt{2x+1}-3\right)+C$;

3)$\int \frac{x^3dx}{1+\sqrt[3]{x^4+1}}$; J: $\frac{3}{4}\left[\frac{\sqrt[3]{(x^4+1)^2}}{2}-\sqrt[3]{x^4+1}+\ln(\sqrt[3]{x^4+1})\right]+C$;

4)$\int \frac{dx}{\sqrt[3]{2x+1}-\sqrt{2x+1}}$; J: $\frac{3}{2}(\sqrt[6]{2x+1}+1)^2+\ln\left|\sqrt[6]{2x+1}-1\right|+C$.

3. Uchinchi ko`rinishdagi integralar.

1) $\int \frac{1}{x^2}\sqrt{\frac{1+x}{x}}dx$; J: $-\frac{2}{3}\sqrt{(\frac{1+x}{x})^3}+C$;

2)$\int \frac{1}{x}\sqrt{\frac{x-2}{x}}dx$; J: $-2\sqrt{\frac{x-2}{x}}-ln\left[|x|(1-\sqrt{\frac{x-2}{x}})^2\right]+C$;

3)$\int \sqrt{\frac{x}{2-x}}dx$; J: $2arcsin\sqrt{\frac{x}{2}}-\sqrt{2x-x^2}+C$;

4)$\int (x-2)\sqrt{\frac{1+x}{1-x}}dx$; J: $(1-0{,}5x)\sqrt{1-x^2}-arcsinx+C$.

4. To’rtinchi ko`rinishdagi integralar.

1)$\int \sqrt{a^2-x^2}dx$. J: $0{,}5\left[x\sqrt{a^2-x^2}+a^2arcsin\frac{x}{a}\right]+C$;

2)$\int x^2\sqrt{4-x^2}dx$. J: $2arcsin\frac{x}{2}-\frac{x}{4}(2-x^2)\sqrt{4-x^2}+C$;

3)$\int \sqrt{3+2x-x^2}dx$. J: $2arcsin\frac{x-1}{2}-\frac{(x-1)\sqrt{3+2x-x^2}}{2}+C$;

Ko’rsatma: $3+2x-x^2=4-(x-1)^2$ bo’lgani uchun $x-1==2\sin t$ almashtirish qilinadi.

4)$\int \frac{x^2dx}{\sqrt{(2-x^2)^3}}$. J: $\frac{x}{\sqrt{2-x^2}}-arcsin\frac{x}{\sqrt{2}}+C$.

5. Beshinchi ko`rinishdagi integralar.

1)$\int \frac{dx}{\sqrt{(4+x^2)^3}}$. J: $\frac{x}{4\sqrt{4+x^2}}+C$;

2)$\int \frac{x^2dx}{\sqrt{(a^2+x^2)^5}}$. J: $\frac{x^3}{3a^2\sqrt{(a^2+x^2)^3}}+C$;

3)$\int \frac{dx}{(x+1)^2\sqrt{x^2+2x+2}}$. J: $-\frac{\sqrt{x^2+2x+2}}{x+1}+C$;

4)$\int \frac{dx}{(x-2)^2\sqrt{x^2-4x+5}}$. J: $\frac{\sqrt{x^2-4x+5}}{2-x}+C$.

6. $\int x^m(a+bx^n)^p\,dx$ ko`rinishdagi integrallar.(1-hol: p-butun son)

1)$\int x^{\frac{1}{3}}(1-2x^{\frac{1}{2}})^3\,dx.$ J: $\frac{3}{4}\sqrt[3]{x^4}-\frac{36}{11}x^{\frac{11}{6}}-\frac{72}{7}\sqrt[3]{x^7}-\frac{48}{17}x^{\frac{17}{6}}+C;$

2)$\int \sqrt[3]{x}(2+x^{\frac{1}{2}})^2\,dx.$ J: $3\sqrt[3]{x^4}+\frac{24}{11}\sqrt[6]{x^{11}}+\frac{3}{7}\sqrt[3]{x^7}+C;$

3)$\int x^{\frac{2}{3}}(\sqrt{x}-2)^3\,dx.$ J: $\frac{6}{19}x^{\frac{19}{6}}-\frac{9}{4}\sqrt[3]{x^8}+\frac{72}{13}x^{\frac{13}{6}}-\frac{24}{5}\sqrt[3]{x^5}+C;$

7. Quyidagi integrallar hisoblansin:

1)$\int x^{-\frac{2}{3}}(1+x^{\frac{1}{3}})^{0,5}\,dx.$ J: $2(1+\sqrt[3]{x})^{\frac{3}{2}}+C;$

2)$\int x^3(1+x^2)^{0,5}\,dx.$ J: $\sqrt{(1+x^2)^3}\,\frac{(3x^2-2)}{15}+C;$

3)$\int \sqrt[3]{x}\sqrt[4]{2+\sqrt[3]{x^2}}\,dx.$ J: $\frac{2}{3}\left(\sqrt[4]{2+\sqrt[3]{x^2}}\right)^9-\frac{12}{5}\left(\sqrt[4]{2+\sqrt[3]{x^2}}\right)^5+C.$

8. Quyidagi integrallar hisoblansin:

1)$\int \frac{dx}{x^{11}\sqrt{1+x^4}}.$ J: $-\frac{\sqrt{(1+x^4)^5}}{10x^{10}}+\frac{\sqrt{(1+x^4)^3}}{3x^6}-\frac{\sqrt{1+x^4}}{2x^2}+C;$

2)$\int \frac{dx}{x^4\sqrt{1+x^2}}.$ J: $\frac{\sqrt{1+x^2}(2x^2-1)}{3x^3}+C;$

3)$\int \frac{dx}{x^2\sqrt[3]{(1-x^3)^2}}.$ J: $-\frac{\sqrt[3]{1-x^3}}{x}+C.$

9. Eylerning birinchi almashtirishidan foydalanib quyidagi integrallar hisoblansin:

1)$\int \frac{dx}{x\sqrt{x^2+4}}$ J: $\frac{1}{2}\ln\left|\frac{x+\sqrt{x^2+4}-2}{x+\sqrt{x^2+4}+2}+C;\right.$

2)$\int \frac{dx}{1+\sqrt{x^2+2x+2}},$ J: $\ln\left(x+1+\sqrt{x^2+2x+2}\right)+\frac{2}{x+2+\sqrt{x^2+2x+2}}+C;$

3)$\int \frac{dx}{x-\sqrt{x^2+2x+4}},$ J: $2\ln\left|\sqrt{x^2+2x+4}-x\right|-\frac{3}{2(\sqrt{x^2+2x+4}-x-1}-$

$$-\frac{3}{2}\ln\left|\sqrt{x^2+2x+4}-x-1\right|+C.$$

10. Eylerning ikkinchi almashtirishidan foydalanib qiyidagi integrallar hisoblansin:

1)$\int \frac{dx}{x+\sqrt{x^2-x}+1}.$ J: $2\ln|t|-\frac{1}{2}\ln|t-1|+\frac{3}{t+1}-\frac{3}{2}\ln|t+1|+C$

(bu yerda $t=\frac{\sqrt{x^2-x+1}+1}{x}$) ;

2)$\int \frac{1-\sqrt{1+x+x^2}}{x\sqrt{1+x+x^2}}dx.$ J: $\ln\left|\frac{2\sqrt{1+x+x^2}-x-2}{x^2}\right|+C;$

3) $\int \frac{dx}{x\sqrt{x^2-4x+8}}$. J: $\frac{1}{2\sqrt{2}}\ln\left|\frac{x}{4-x+\sqrt{2x^2-8x+16}}\right| + C$.

11. Eylerning uchinchi almashtirishidan foydalanib quyidagi integrallar hisoblansin:

1) $\int \frac{dx}{\sqrt{x^2-2x-8}}$. $J: ln\left|x-1+\sqrt{x^2-2x-8}\right| + C$;

2) $\int \frac{dx}{\sqrt{3-2x-x^2}}$. $J: 2arctg\sqrt{\frac{3+x}{1-x}} + C$;

3) $\int \frac{xdx}{(\sqrt{7\square-10-x^2})^3}$. J: $-\frac{2}{9}\left(-\frac{5}{t}+2t\right)+C$ $\left(bu\ yerda\ t=\frac{\sqrt{7x-10-x^2}}{x-2}\right)$.

VII BOB. ANIQ INTEGRAL

1-§. Aniq integral va uni hisoblash

$y = f(x)$ funksiya [a;b] kesmada aniqlangan va uzluksiz bo'lsin. $[a, b]$ kesmani ixtiyoriy usulda

$$a=x_0,\ x_1,\ x_2,...,\ x_{i-1},\ x_i,...,\ x_{n-1},\ x_n=b$$

nuqtalar yordamida n ta bo'lakka bo'lamiz. Bu bo'laklarning har birini uzunligi Δx_i (i=1, 2,..., n) orqali belgilaymiz, ya'ni

$\Delta x_1=x_1-x_0,\ \Delta x_2=x_2-x_1,\ \Delta x_3=x_3-x_2,...,\ \Delta x_i=x_i-x_{i-1},...,\ \Delta x_n=x_n-x_{n-1}$

Bu bo'laklarning har birida ixtiyoriy ξ_1, ξ_2,...,ξ_i,..., ξ_n nuqtalarni olamiz va

$$S_n = \sum_{i=1}^{n} f(\xi_i)\Delta x_i \quad (1)$$

yig'indini tuzamiz.

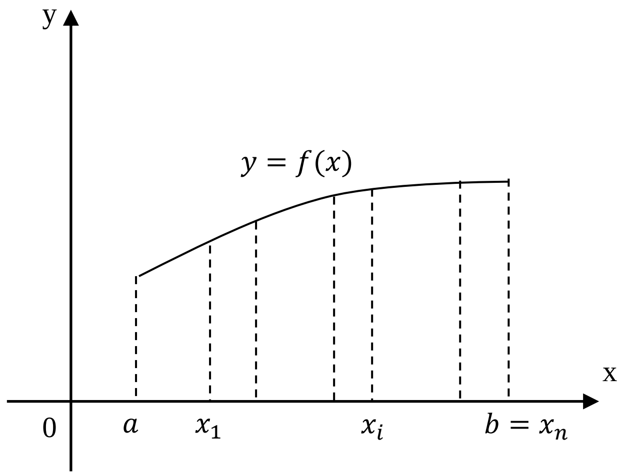
S_n yig'indi $f(x)$ funksiyaning $[a; b]$ kesmadagi ***integral yig'indisi*** deyiladi.

S_n integral yig'indining Δx_i kesmalarning eng kattasi nolga intilgandagi limiti $f(x)$ funksiyadan $[a; b]$ kesmada olingan aniq integral deyiladi.

Demak,

$$\lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(\xi_i)\Delta x_i = \int_a^b f(x)dx \qquad (2)$$

Agar $[a; b]$ da $f(x) \geq 0$ bo'lsa, u holda (2) aniq integral $x = a, x = b, y = 0$ va $y = f(x)$ chiziqlar bilan chegaralangan figuraning yuzidan iborat bo'ladi (1-rasm).



1-chizma

(2) da $\int$−integral belgisi, $f(x)$ −integral ostidagi funksiya, $x-$ integrallash o'zgaruvchisi, $f(x)dx$ - integral ostidagi ifoda, $[a;b]$ kesma integrallash kesmasi, "a" va "b" sonlari integralning ***quyi va yuqori chegaralari*** deyiladi.

Aniq integral bir qator xossalarga ega:

1. $\int_a^b kf(x)dx = k\int_a^b f(x)dx$ (bu yerda $k-$ o'zgarmas son).

2. $\int_a^b [f(x) \pm \varphi(x)]dx = \int_a^b f(x)dx \pm \int_a^b \varphi(x)dx$.

3. Agar $[a;b]$ $(a<b)$ da $f(x) \le \varphi(x)$ bo'lsa, u holda $\int_a^b f(x)dx \le \int_a^b \varphi(x)dx$.

4. Agar $f(x)$ funksiyaning $[a;b]$ dagi eng kichik va eng katta qiymatlari mos ravishda *m* va *M* bo'lsa, u holda quyidagi tenglik o'rinli bo'ladi:

$$m(b-a) \le \int_a^b f(x)dx \le M(b-a)$$

5. Agar $y=f(x)$ funksiya $[a;b]$ da uzluksiz bo'lsa, u holda bu kesmada hech bo'lmaganda shunday bitta ξ nuqta topiladiki,unda quyidagi tenglik o'rinli bo'ladi:

$$\int_a^b f(x)dx = f(\xi)(b-a)$$

6. Agar $a<c<b$ bo'lsa, u holda quyidagi tenglik o'rinli bo'ladi:

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx.$$

7. $\int_a^b f(x)dx = -\int_b^a f(x)dx$.

8. $\int_a^a f(x)dx = 0$.

Agar $F(x)$ funksiya $[a;b]$ da uzluksiz bo’lgan $f(x)$ funksiyaning boshlang’ichi bo’lsa, u holda quyidagi formula o’rinlidir:

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

bu formulani Nyuton-Leybnits formulasi deyiladi.

Agar aniq integralni hisoblashda $f(x)$ boshlang’ich funksiyani bevosita topib bo’lmasa, u holda uni $x = \varphi(t)$ almshtirish yordamida yangi t o’zgaruvchiga o’tib integrallanadi. Bu holda quyidagi formula o’rinli bo’ladi:

$$\int_a^b f(x)dx = \int_\alpha^\beta f[\varphi(t)]\varphi'(t)dt$$

bu yerda $\varphi(t)$ va $\varphi'(t)$ lar $[\alpha;\beta]$ kesmadagi uzluksiz funksiyalar, $a = \varphi(\alpha)$, $b = \varphi(\beta)$.

Ba’zi hollarda $f(x)$ boshlang’ich funksiyani topishda bo’laklab integrallash formulasi deb ataluvchi quyidagi formuladan ham foydalaniladi:

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu$$

bu yerda $u(x)$ va $v(x)$ lar $[a;b]$ kesmada differensiallanuvchi funksiyalar.

Quyidagi integrallar hisoblansin:

1) $\int_1^2 (x^2 + 3x - 2)dx$. $J: \frac{29}{6}$; 2) $\int_1^2 (x^2 + \frac{1}{x^2})\,dx$. $J: 2\frac{5}{8}$;

3) $\int_0^a (x^2 - ax)dx$. $J: -\frac{a^3}{3}$; 4) $\int_0^8 (\sqrt[3]{\square} - \sqrt{2x})dx$. $J: -\frac{28}{3}$;

5) $\int_0^1 \left(\sqrt{x} + \sqrt[3]{x^2}\right)dx$. $J: \frac{19}{15}$; 6) $\int_1^2 \frac{dx}{2x-1}$. $J: \frac{1}{2}ln3$;

7) $\int_{-2}^{-1} \frac{dx}{(11+5x)^3}$. $J: \frac{7}{72}$; 8) $\int_{0}^{1} \frac{xdx}{(x^2+1)^2}$. $J: \frac{1}{4}$;

9) $\int_{0}^{16} \frac{dx}{\sqrt{x+9}-\sqrt{x}}$. J: 14.

**<u>Ko`rsatma</u>:** Kasrning surat va maxrajini $\sqrt{x+9}+\sqrt{x}$ ga ko`paytiring.

10) $\int_{-2}^{-3} \frac{dx}{x^2-1}$. $J: \frac{1}{2} ln \frac{2}{3}$; 11) $\int_{0}^{1} \frac{dx}{x^2+4x+5}$. $J: arctg \frac{1}{7}$;

12) $\int_{0}^{1} \frac{dx}{\sqrt{4-x^2}}$. $J: \frac{\pi}{6}$; 13) $\int_{0}^{\frac{\pi}{4}} sin4xdx$. $J: \frac{1}{2}$;

14) $\int_{\frac{\pi}{8}}^{\frac{\pi}{6}} \frac{dx}{cos^2 2\,x}$. $J: \frac{\sqrt{3}-1}{2}$; 15) $\int_{-1}^{1} \frac{e^x}{1+e^{2x}} dx$. $J: arctg \frac{e^2-1}{2e}$;

16) $\int_{0}^{\frac{\pi}{4}} cos^3 t\, dt$. $J: \frac{5\sqrt{3}}{12}$; 17) $\int_{1}^{5} \frac{dx}{3x-2}$. $J: \frac{ln13}{3}$;

18) $\int_{0}^{1} \frac{dx}{(2x+1)^3}$. $J: \frac{2}{9}$; 19) $\int_{0}^{2} \frac{x+3}{x^2+4} dx$. $J: \frac{3\pi}{8}+\frac{ln2}{2}$;

20) $\int_{0}^{\pi} cos\frac{x}{2} cos\frac{3x}{2} dx$. $J: 0$; 21) $\int_{0}^{1} \frac{xdx}{x^2+3x+2}$. $J: ln\frac{9}{8}$;

22) $\int_{0}^{\frac{\pi}{4}} cos^5 xsin2xdx$. $J: \frac{2}{7}$; 23) $\int_{0}^{\frac{\pi}{4}} sin^2 x\, dx$. $J: \frac{\pi}{8}-\frac{1}{4}$;

24) $\int_{0}^{\frac{\pi}{4}} cos^2(\frac{\pi}{4}-x)\, dx$. $J: \frac{\pi+2}{8}$; 25) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \square osxcos3xdx$. $J: 0$;

26) $\int_3^4 \frac{dx}{25 - x^2}$. $J: \frac{1}{5} ln \frac{3}{2}$; 27) $\int_0^{\frac{\pi}{4}} sin^3 t dt$. $J: \frac{1}{12}(8 - 5\sqrt{2})$;

28) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} tg^3\, x\, dx$. $J: 0$; 29) $\int_4^{4\sqrt{3}} \frac{dx}{\sqrt{64 - x^2}}$. $J: \frac{\pi}{6}$;

30) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} cosec^2\, x\, dx$. $J: 1$.

Quyidagi integrallarni o’zgaruvchini almashtirish usulidan foydalanib hisoblang:

1. $\int_0^{\sqrt[5]{2}} \frac{x^9 dx}{(1 + x^5)^3}$, $J: \frac{2}{45}$; 2. $\int_3^8 \frac{x dx}{\sqrt{1 + x}}$, $J: \frac{32}{3}$;

3. $\int_0^5 \frac{x dx}{\sqrt{1 + 3x}}$, $J: 4$; 4. $\int_{ln2}^{ln3} \frac{dx}{e^x - e^{-x}}$, $J: ln \frac{3}{4}$;

5. $\int_{-\frac{\sqrt{2}}{2}}^1 \frac{\sqrt{1 - x^2}}{x^2} dx$, $J: 1 - \frac{\pi}{4}$; 6. $\int_0^1 \frac{x^2 dx}{(x + 1)^4}$, $J: \frac{1}{24}$;

7. $\int_0^{ln2} \sqrt{e^x - 1}\, dx$, $J: \frac{4 - \pi}{2}$; 8. $\int_{\sqrt{3}}^{\sqrt{7}} \frac{x^3 dx}{\sqrt[3]{(x^2 + 1)^2}}$, $J: 3$;

9. $\int_{-3}^3 x^2 \sqrt{9 - x^2}\, dx$, $J: \frac{81\pi}{8}$; 10. $\int_1^{\sqrt{3}} \frac{\sqrt{1 + x^2}}{\square^4} dx$, $J: \sqrt{2} - \frac{4}{9}\sqrt{3}$;

11. $\int_0^{\frac{\pi}{2}} \frac{dx}{5 + 4cos\, x}$, $J: \frac{2}{3} arctg \frac{1}{3}$; 12. $\int_{ln3}^{ln7} \frac{e^x \sqrt{e^x - 3}}{e^x + 1} dx$, $J: 4 - \pi$;

13. $\int_1^2 \frac{dx}{x\sqrt{1 + 3x + x^2}} \left(x = \frac{1}{t}\right)$, $J: ln \frac{5 + 2\sqrt{5}}{4 + \sqrt{11}}$;

14. $\int_0^3 \sqrt{\frac{x}{6-x}}dx, \quad (x = 6\sin^2 t) \qquad J: \frac{3}{2}(\pi - 2);$

15. $\int_{\frac{\sqrt{2}}{2}}^1 \frac{\sqrt{1-x^2}}{x^2}dx\,, \quad J: 1 - \frac{\pi}{4};$ 16. $\int_0^3 \frac{dx}{\sqrt{(16+x^2)^3}}, \quad J: \frac{3}{80};$

17. $\int_2^4 \sqrt{\frac{4-x}{x}}dx, \quad J: \pi - 2;$ 18. $\int_0^{\frac{\pi}{2}} \frac{dx}{3+2\cos x}, \quad J: \frac{2}{\sqrt{5}}arctg\frac{1}{\sqrt{5}}.$

Quyidagi integrallarni bo’laklab integrallash formulasidan foydalanib hisoblang:

1. $\int_0^2 xe^{-x}dx, \quad J: 1 - \frac{3}{e^2};$ 2. $\int_0^{\frac{\pi}{2}} x\cos x dx\,, \quad J: \frac{\pi}{2} - 1;$

3. $\int_0^{e-1} \ln(x+1)dx\,, \quad J: 1;$ 4. $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{xdx}{\sin^2 x}, \quad J: \frac{\pi}{36}(9-4\sqrt{3}) + \frac{1}{2}\ln\frac{3}{2};$

5. $\int_0^{\frac{\pi}{2}} e^{2x}\cos x dx, \quad J: \frac{e^{\pi}-2}{5};$ 6. $\int_0^{\frac{\pi}{3}} \frac{xdx}{\cos^2 x}, \quad J: \frac{\pi}{3} - \ln 2;$

7. $\int_1^{\sqrt{e}} x\ln x dx\,, \quad J: \frac{e+1}{4};$ 8. $\int_0^{\frac{\pi}{4}} e^x \sin 2x dx, \quad J: \frac{1}{5}(e^{\frac{\pi}{4}} + 2);$

9. $\int_0^{\pi} x\sin\frac{x}{2}dx, \quad J: 4;$ 10. $\int_1^e \frac{dx}{x(1+\ln^2 x)}, \quad J: \frac{\pi}{4}.$

2-§. Xosmas integrallar

Berilgan $y = f(x)$ funksiya $[a; +\infty)$ cheksiz yarim oraliqda aniqlangan va ixtiyoriy chekli $b \geq a$ uchun $[a; b]$ kesmada integrallanuvchi, ya’ni

$$F(b) = \int_a^b f(x)dx$$

integral mavjud bo’lsin.

$y=f(x)$ funksiyaning $[a;+\infty)$ cheksiz yarim oraliq bo'yicha ***birinchi tur*** xosmas integrali deb yuqori chegarasi o'zgaruvchi F(b) integralning $b\to+\infty$ bo'lgandagi limitiga aytiladi.

$y=f(x)$ funksiyaning $[a;+\infty)$ cheksiz yarim oraliq bo'yicha birinchi tur xosmas integrali

$$\int_a^{+\infty} f(x)dx \,(1)$$

ko'rinishda yoziladi va ta'rifga asosan,

$$\int_a^{+\infty} f(x)dx=\lim_{b\to+\infty}\int_a^{b} f(x)dx \,(2)$$

kabi aniqlanadi.

Agar (2) limit mavjud va chekli bo'lsa, u holda (1) xosmas integral yaqinlashuvchi, aks holda uzoqlashuvchi deyiladi.

Agar $a\le x<\infty$ cheksiz yarim oraliqda $0\le f(x)\le g(x)$ va $\int_a^{+\infty} g(x)dx$ xosmas integral yaqinlashuvchi bo'lsa, u holda $\int_a^{+\infty} f(x)dx$ xosmas integral ham yaqinlashuvchi va quyidagi tengsizlik o'rinli bo'ladi:

$$\int_a^{+\infty} f(x)dx\le\int_a^{+\infty} g(x)dx$$

Agar $a\le x<\infty$ cheksiz yarim oraliqda $0\le g(x)\le f(x)$ va $\int_a^{+\infty} g(x)dx$ xosmas integral uzoqlashuvchi bo'lsa, u holda $\int_a^{+\infty} f(x)dx$ xosmas integral ham uzoqlashuvchi bo'ladi.

Agar $x\ge a$ bo'lganda $|f(x)|\le g(x)$ va $\int_a^{+\infty} g(x)dx$ xosmas integral yaqinlashuvchi bo'lsa,u holda $\int_a^{+\infty} f(x)dx$ xosmas integral ham yaqinlashuvchi va

$$\int_a^{+\infty} f(x)dx \leq \int_a^{+\infty} |f(x)|dx \leq \int_a^{+\infty} g(x)dx$$

tengsizlik o'rinli bo'ladi.

Agar $\int_a^{+\infty} |f(x)|dx$ xosmas integral yaqinlashuvchi bo'lsa, u holda $\int_a^{+\infty} f(x)dx$ xosmas integral ***absolyut yaqinlashuvchi*** deyiladi. Agar ikkinchi integral yaqinlashuvchi, birinchi integral esa uzoqlashuvchi bo'lsa, u holda ikkinchi xosmas integral ***shartli yaqinlashuvchi*** deyiladi.

$\int_{-\infty}^{b} f(x)dx$ ham xosmas integral bo'lib, u $\int_{-\infty}^{b} f(x)dx = \lim\limits_{a\to-\infty}\int_a^b f(x)dx$ ko'rinishda aniqlanadi.

Agar $y = f(x)$ funksiya $(-\infty; +\infty)$ cheksiz oraliqda aniqlangan bo'lsa, u holda uning bu oraliq bo'yicha xosmas integrali quyidagicha aniqlanadi:

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^{c} f(x)dx + \int_{c}^{+\infty} f(x)dx = \lim_{a\to-\infty}\int_a^c f(x)dx + \lim_{b\to+\infty}\int_c^b f(x)dx$$

Agar $y = f(x)$ funksiya $[a; b]$ kesmaning biror ichki $x = c$ nuqtasida chegaralanmagan bo'lsa, u holda xosmas integral

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

kabi aniqlanadi.

Quyidagi integrallar hisoblansin:

1) $\int_1^{\infty} \frac{dx}{x^2}$; 2) $\int_1^{\infty} \frac{dx}{\sqrt{x}}$; 3) $\int_1^{\infty} \frac{d\square}{1+x^2}$;

4) $\int_0^{\infty} e^{-x}dx$; 5) $\int_0^{\infty} xe^{-x^2}dx$; 6) $\int_{-\infty}^{0} \frac{dx}{4+x^2}$;

7) $\int_{-\infty}^{+\infty} \frac{dx}{x^2+2x+2}$; 8) $\int_3^{+\infty} \frac{x^2dx}{x^3+1}$; 9) $\int_2^{+\infty} \frac{arctg\frac{x}{2}}{4+x^2}dx$;

10) $\displaystyle\int_1^{+\infty} \frac{xdx}{\sqrt[3]{(1+x^2)^2}}$; 11) $\displaystyle\int_1^{4} \frac{dx}{x^2-7x+10}$; 12) $\displaystyle\int_1^{+\infty} \frac{dx}{\square^2+x}$;

13) $\displaystyle\int_1^{+\infty} \frac{arctgxdx}{x^2}$; 14) $\displaystyle\int_1^{+\infty} \frac{dx}{(x^2+1)^2}$; 15) $\displaystyle\int_0^{2} \frac{dx}{(x-1)^2}$.

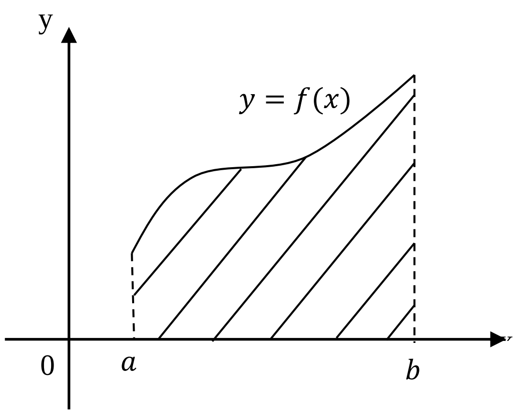
J: 1) 1; 2) uzoqlashuvchi; 3) $\frac{\pi}{4}$; 4) 1; 5) $\frac{1}{2}$; 6)$\frac{\pi}{4}$; 7) π; 8) uzoqlashadi; 9) $\frac{3\pi^2}{64}$; 10) uzoqlashadi; 11) uzoqlashadi; 12) ln2; 13)$\frac{\pi}{4}+\frac{ln2}{2}$; 14) $\frac{\pi-2}{8}$; 15) uzoqlashadi.

3-§. Aniq integralning geometrik tadbiqlari

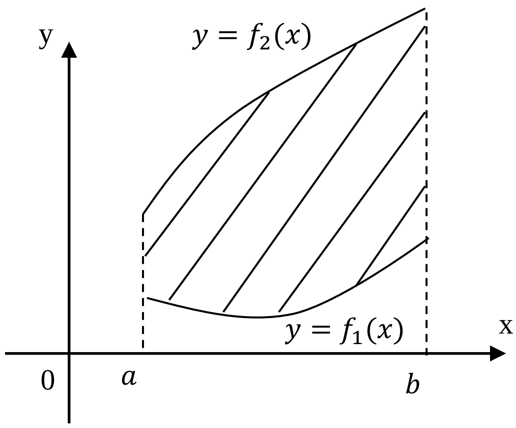
$y=f(x)$ funksiya grafigi, $x=a, x=b$ to'g'ri chiziqlar va OX o'qi bilan chegaralangan figura ***egri chiziqli trapetsiya*** deyiladi. Bunday egri chiziqni trapetsiyaning yuzi $f(x)\geq 0$ bo'lsa,

$$S=\int_a^b f(x)dx=\int_a^b ydx$$

formula bilan hisoblanadi (1-chizma).



1-chizma



2-chizma

$y_1=f_1(x)$ va $y_2=f_2(x)$ ($f_2(x)\geq f_1(x)$) egri chiziqlar, hamda $x=a$ va $x=b$ to'g'ri chiziqlar bilan chegaralangan figuraning yuzi

$$S=\int_a^b [f_2(x)-f_1(x)]dx$$

formula bilan hisoblanadi (2-chizma).

$x_1 = \varphi_1(y)$ va $x_2 = \varphi_2(y)$ $(\varphi_2(y) \geq \varphi_1(y))$ egri chiziqlar, hamda $y = c$ va $y = d$ to’g’ri chiziqlar bilan chegaralangan figuraning yuzi

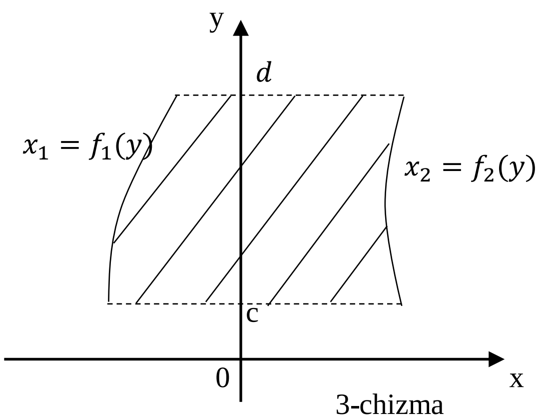
$$S = \int_c^d [\varphi_2(y) - \varphi_1(y)]dy$$

formula bilan hisoblanadi (3-chizma).

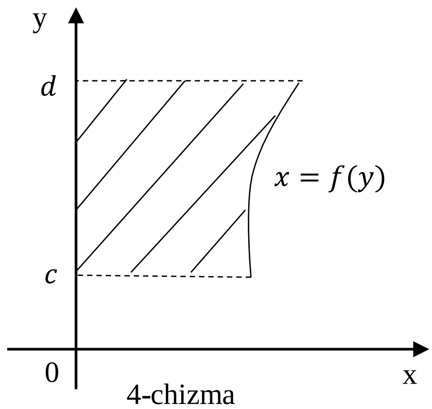
Agar egri chiziqli trapetsiya $x = f(y)$ funksiya grafigi, $y = c$,$y = d$to’g’ri chiziqlar va OY o’q bilan chegaralangan bo’lsa, u holda uning yuzi ($f(y) \geq 0$ uchun)

$$S = \int_c^d f(y)dy = \int_c^d xdy$$

formula bilan hisoblanadi (4-chizma).



3-chizma



4-chizma

Agar egri chiziq $x = x(t), y = y(t)$ parametrik tenglamalar bilan berilgan bo’lsa, u holda shu egri chiziq, $x = a, x = b$ to’g’ri chiziqlar va OX o’qi bilan chegaralangan egri chiziqli trapetsiyaning yuzi

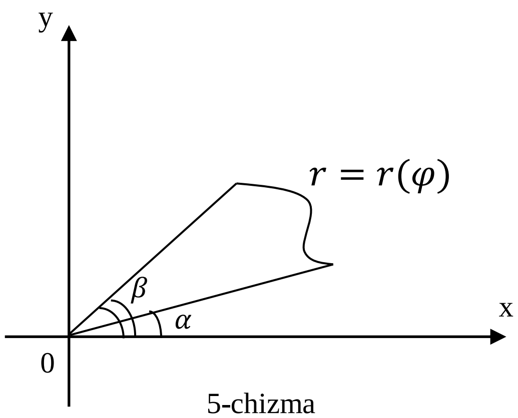
$$S = \int_{t_1}^{t_2} y(\square)x'(t)dt = \int_{t_1}^{t_2} y(t)dx(t)$$

formula bilan hisoblanadi, bunda t_1 va t_2 lar $a = x(t_1)$, $b = x(t_2)(y(t) \geq 0)$tenglamalardan topiladi.

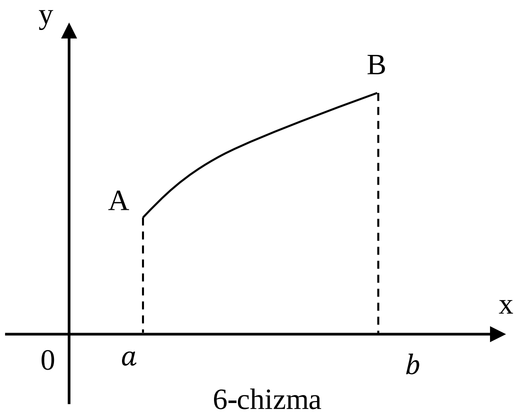
$r = r(\varphi)$ funksiya grafigi va $\varphi = \alpha,\ \varphi = \beta$ nurlar bilan chegaralangan egri chiziqli sektorning yuzi

$$S = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\varphi$$

formula bilan hisoblanadi (5-chizma).



5-chizma



6-chizma

Agar $y = (x)$ funksiya $[a; b]$ kesmada silliq (ya’ni $y' = f'(x)$ hosila uzluksiz) bo’lsa, u holda AB qismi yoyining uzunligi

$$l = \int_{a}^{b} \sqrt{1 + (y')^2}\, dx$$

formula bilan hisoblanadi (6-chizma).

Agar egri chiziq $\begin{cases} x = x(t) \\ y = y(t) \end{cases}$ parametrik tenglamalar bilan berilgan bo’lsa, u holda egri chiziq $t\epsilon[t_1; t_2]$ qismi yoyining uzunligi

$$l = \int_{t_1}^{t_2} \sqrt{[x'(t)]^2 + [y'(t)]^2}\, dt$$

formula bilan hisoblanadi.

Agar egri chiziq qutb koordinatalarida $r = r(\varphi)$ $(\alpha \le \varphi \le \beta)$ tenglama bilan berilgan bo’lsa, u holda yoy uzunligi

$$l = \int_{\alpha}^{\beta} \sqrt{r^2 + (r')^2}\, d\varphi$$

formula bilan hisoblanadi.

Agar $S(x)$ yuz biror jismning OX o'qiga perpendikulyar tekislik bilan kesishishidan hosil bo'lgan kesimi bo'lib, u $[a;b]$ kesmada uzluksiz funksiya bo'lsa, u holda shu jismning hajmi

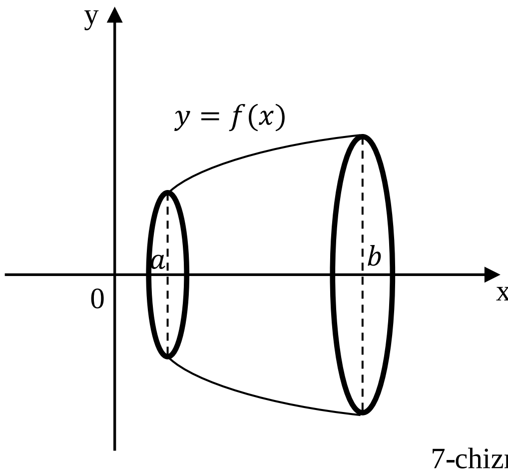
$$V = \int_a^b S(x)dx$$

formula bilan hisoblanadi.

$y = f(x)$ egri chiziq va $x = a, x = b$, $y = 0$ to'g'ri chiziqlar bilan chegaralangan egri chiziqli trapetsiya OX o'qi atrofida aylantirilsa, u holda aylanish jismining hajmi

$$V = \square \int_a^b f^2(x)dx = \pi \int_a^b y^2 dx$$

formula bilan hisoblanadi (7-chizma).



7-chizma

Agar $y_1 = f_1(x)$ va $y_2 = f_2(x)$ ($f_2(x) \geq f_1(x)$) egri chiziqlar hamda $x = a, x = b$ to'g'ri chiziqlar bilan chegaralangan figura OX o'qi atrofida aylansa, aylanish jismning hajmi

$$V = \pi \int_a^b ({y_2}^2 - {y_1}^2)dx$$

formula bo'yicha hisoblanadi.

Agar egri chiziqli trapetsiya $x = f(y)$ funksiya grafigi $y = c, y = d$ to'g'ri chiziqlar va OY o'qi bilan chegaralangan bo'lsa, u holda bu figuraning OY o'qi atrofida aylantirishdan hosil bo'lgan jismning xajmi

$$V = \pi \int_c^d x^2 dy$$

formula bo'yicha hisoblanadi.

Agar $x_1 = \varphi_1(y)$ va $x_2 = \varphi_2(y)(x_2 \geq x_1 \geq 0)$ egri chiziqlar va y=c, y=d to'g'ri chiziqlar bilan chegaralangan figura OY o'qi atrofida aylansa, u holda aylanish jismning hajmi

$$V = \pi \int_c^d (x_2^2 - x_1^2) dy$$

formula bilan hisoblanadi.

$y = f(x)$ egri chiziq yoyi $\breve{AB}$ ning OX o'qi atrofida aylanishidan hosil bo'lgan sirtning yuzi

$$P_x = 2\pi \int_{\breve{AB}} y ds \left(ds = \sqrt{dx^2 + dy^2}\right)$$

formula bilan hisoblanadi.

$x = \varphi(y)$ egri chiziq yoyi $\breve{AB}$ ning OY o'qi atrofida aylantirishdan hosil bo'lgan sirt yuzi

$$\square_y = 2\pi \int_{\breve{AB}} x ds \left(ds = \sqrt{dx^2 + dy^2}\right)$$

formula bilan hisoblanadi

Quyidagi chiziqlar bilan chegaralangan yuzalar hisoblansin:

1)$y = 4 - x^2, y = 0$; 2) $y = 3 - 2x - x^2, y = 0$;

3) $y = 6x - x^2, y = 0$; 4) $y = x^2 + 4x + 5, x = 0, y = 0$;

5) $y^2 = 2px, x = h$; 6) $xy = 4, x = 1, x = 4, y = 0$;

7) $y^2 = 2x + 4, x = 0$; 8) $y^2 = x^3, y = 8, x = 0$;

9) $y = x^2, y = 2 - x^2$; 10) $y = x^2 + 4x, y = x + 4$;

11) $y = 6x - x^2, y = 0$;

12) $y^2 = 1 - x, x = -3$;

13) $\frac{x^2}{49} + \frac{y^2}{25} = 1$;

14) $y = lnx, x = e, y = 0$;

15) $4y = x^2, y^2 = 4x$;

16) $xy = 6, x + y - 7 = 0$.

J: 1) $\frac{32}{3}$; 2) $\frac{32}{3}$; 3) 36; 4) $\frac{14}{3}$; 5) $2\sqrt{2ph}$; 6) 8ln2; 7) $\frac{16}{3}$; 8)19,2; 9)$\frac{8}{3}$; 10) $20\frac{5}{6}$; 11) 36; 12) $\frac{32}{3}$; 13) 35π; 14) 1; 15)$\frac{16}{3}$; 16) 17,5−6ln6.

Qutb koordinatalar sistemasida berilgan egri chiziqlar bilan chegaralangan yuzalar topilsin:

1) $r^2 = a^2 cos2\varphi$ lemniskata;

2) $r = a(1 - cos\varphi)$ kardioida;

3) $r = acos2\varphi$;

4) $r = asin3\varphi$;

5) $r = a(sin\varphi + cos\varphi)$;

6) $r = asin2\varphi$;

7) $r = acos3\varphi$.

J: 1) a^2; 2) $\frac{3\pi a^2}{2}$; 3) $\frac{\pi a^2}{2}$; 4) $\frac{\pi a^2}{4}$; 5) $\frac{\square a^2}{2}$; 6) $\frac{\pi a^2}{2}$; 7) $\frac{\pi a^2}{4}$.

Parametrik tenglamalar bilan berilgan quyidagi chiziqlar bilan chegaralangan yuzalar topilsin:

1) $x = a(t - sint), y = a(1 - cost)$ sikloidaning bir davri (arkasi) va OX o'q;

2) $x = acos^3 t, y = asin^3 t$ astroida;

3) $x = \frac{3at}{1+t^2}$, y $= \frac{3at^2}{1+t^3}$ Dekart yaprog'i sirtmog'ining yuzi;

4) $x = t^2 - 1, y = t^3 - t$ chiziq sirtmog'ining yuzi;

5) $x = acost, y = bsint$ ellipsning yuzi.

J: 1) 3πa^2; 2) $\frac{3\pi a^2}{8}$; 3) $\frac{3a^2}{2}$; 4) $\frac{8}{15}$; 5) πab.

Quyida berilgan egri chiziqlar yoyining uzunligi topilsin:

1) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ astroida yoyi uzunligini hisoblang;

2) $y^2 = \frac{4}{9}(2-x)^3$ egri chiziq yoyining $x = -1$ to'g'ri chiziq kesgan qismining uzunligini hisoblang;

3) $y = 2\sqrt{x}$ parabola yoyining $x = 0$ dan $x = 1$ gacha bo'lgan qismining uzunligini hisoblang;

4) $y = lnx$ egri chiziq yoyining $x = \sqrt{3}$ dan $x = \sqrt{8}$ gacha bo'lgan qismi uzunligini toping;

5) $x = \frac{1}{4}y^2 - \frac{1}{2}lny$ egri chiziqning $y = 1$ dan $y = e$ gacha yoyi uzunligini toping;

6) $y^2 = x^3$ egri chiziqning $x = \frac{4}{3}$ to'g'ri chiziq bilan kesilgan qismi uzunligini toping.

J: 1) $6a$;2)$\frac{28}{3}$; 3)$\sqrt{2} + \ln(1+\sqrt{2})$; 4) $1+\frac{1}{2}\ln\frac{3}{2}$; 5) $\frac{e^2+1}{4}$; 6) $\frac{112}{27}$.

Parametrik tenglama bilan berilgan quyidagi egri chiziqlar yoyining uzunligini toping:

1. $x = t^2,\ y = \frac{1}{3}t\ (t^2 - 1)$egri chiziqning ox o`qi bilan kesishish nuqtalari orasidagi yoyi uzunligini toping.

2. $\begin{cases} x = a(cost + tsint) \\ y = a(sint - tcost) \end{cases}$ aylana yoyilmasining $t = 0$ dan $t = T$ gacha yoyi uzunligini toping.

3.$\begin{cases} x = \frac{c^2}{a}cos^2t \\ y = \frac{c^2}{b}sin^2t,\ (c^2 = a^2 - b^2) \end{cases}$ ellips evolyotasining uzunligini toping.

4. $\begin{cases} x = \frac{1}{3}t^3 - t \\ y = t^2 + 2 \end{cases}$ egri chiziq yoyining $t = 0$ dan $t = 3$ gacha qismi uzunligini toping.

5. $\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$ egri chiziq yoyining $t = 0$ dan $t = \ln\pi$ gacha qismi uzunligini toping.

6. $\begin{cases} x = \frac{t^6}{6} \\ y = 2 - \frac{t^4}{4} \end{cases}$ egri chiziqning koordinata o’qlari bilan kesishish nuqtalari orasidagi yoyi uzunligini toping.

J: 1) $4\sqrt{3}$; 2) $\frac{1}{2}aT^2$; 3) $\frac{4(a^3-b^3)}{ab}$; 4) 12; 5) $\sqrt{2}(\pi-1)$; 6) $4\frac{1}{3}$.

Quyidagi egri chiziqlarning aylanishidan hosil bo’lgan sirtlarning yuzlarini toping.

1) $y = sinx$ egri chiziqning bitta yarim to’lqinini OX o’qi atrofida;

2) $y = \frac{x^2}{2}$ ning $y = 1{,}5$ to’g’ri chiziq bilan kesishgan qismini OY o’qi atrofida;

3) $4x^2 + y^2 = 4$ ni OY o’qi atrofida;

4) $y = \frac{x^3}{3}$ egri chiziqning $x = -2$ dan $x = 2$ gacha bo’lgan yoyini OX o’qi atrofida;

5) $y^2 = 4 + x$ egri chiziqning $x = 2$ to’g’ri chiziq bilan kesilgan qismini OX o’qi atrofida;

6) $y = tgx$ tangensoidaning $x = 0$ dan $x = \frac{\pi}{4}$ gacha qismining OX o’qi atrofida;

J: 1) $2\pi[\sqrt{2} + \ln(1+\sqrt{2})]$; 2) $\frac{14\pi}{3}$; 3) $2\pi(1+\frac{4\pi}{3\sqrt{3}})$;4) $\frac{34\sqrt{17}-2}{9}\pi$; 5) $\frac{62\pi}{3}$; 6) $\pi[\sqrt{5} - \sqrt{2} + ln\frac{2\sqrt{2}+2}{\sqrt{5}+1}]$.

Parametrik va qutb koordinatalar sistemasida berilgan quyidagi chiziqlarning aylanishidan hosil bo’lgan jismlarning sirtlari topilsin:

1. $x = acos^3t, y = asin^3t$ astroidaning absissalar o’qi atrofida aylanishidan hosil bo’lgan sirt yuzini toping. J: $\frac{12}{5}\pi a^2$.

2. $x = e^t sint, y = e^t cost$ egri chiziq yoyining $t_1 = 0$ dan $t_2 = \frac{\pi}{2}$ gacha qismi absissalar o’qi atrofida aylanishidan hosil bo’gan sirt yuzini hisoblang. J: $\frac{2\pi\sqrt{2}}{5}(e^\pi - 2)$.

3. $x = a(t - si\square t), y = a(1 - cost)$ sikloida bitta arkasining bu sikloidaga uning eng yuqori nuqtasida o’tkazilgan urinma atrofida aylanishidan hosil bo’lgan sirt yuzini hisoblang. J: $\frac{32\pi a^2}{3}$.

4. $x = \frac{t^3}{3}$, y =4$-\frac{t^2}{2}$ egri chiziq yoyining koordinata o’qlari bilan kesishgan nuqtalari orasidagi yoyining OX o’qi atrofida aylanishidan hosil bo’lgan sirt yuzini toping. J: 29.6π

5. $r = a(1 + cos\varphi)$ kardioidaning qutb o’qi atrofida aylanishidan hosil bo’lgan sirt yuzini toping. J: $\frac{32\pi a^2}{5}$.

6.$r = 2asin\varphi$ aylana qutb o’qi atrofida aylanyapti. Bunda hosil bo’lgan aylanish sirti yuzini hisoblang.J: $4\pi^2 a^2$.

7. $r^2 = a^2 cos2\varphi$ lemniskataning qutb o’qi atrofida aylanishidan hosil bo’lgan sirt yuzini hisoblang. J: $2\pi a^2\left(2 - \sqrt{2}\right)$.

8. $r = 2(1 - cos\varphi)$ kardioidaning qutb o’qi atrofida aylanishidan hosil bo’lgan sirt yuzini toping. J: $\frac{128}{5}\pi$.

Quyidagi chiziqlar bilan chegaralangan figuraning aylanishidan hosil bo’lgan jismlarning hajmini topilsin:

1. $xy = 4, x = 1, x = 4, y = 0$ chiziqlar bilan chegaralangan figuraning OX va OY o’qlari atrofida;

2. $y^2 = (x + 4)^3$ egri chiziq va $x = 0$ chiziqlar bilan chegaralangan figuraning OY o’q atrofida;

3. $y^2 = 2px$ va $x = h$ chiziqlar bilan chegaralangan figuraning OX o’q atrofida;

4. $y^2 = 4 - x$ va $y = 0$ chiziqlar bilan chegaralangan figuraning OY o’q atrofida;

5. $y^2 = x^3, x = 0,$ va $y = 8$ chiziqlar bilan chegaralangan figuraning OY o’q atrofida;

6. $y = x^2$ va $y = 4$ chiziqlar bilan chegaralangan figuraning $x = 2$ to’g’ri chiziq atrofida;

7. $(y-3)^2+3x=0$ va $x=-3$ chiziqlar bilan chegaralangan figuraning OX o'q atrofida;

8. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellips bilan chegarlangan figuraning OY o'q atrofida.

J: 1) 12π va 24π;   2) $\frac{2048\pi}{35}$;   3) πph^2;   4) $\frac{512\pi}{15}$;   5) $19{,}5\pi$;

6) $\frac{128\pi}{3}$;   7) 72π;   8) $\frac{4}{3}\pi a^2 b$.

Quyidagi egri chiziqlar bilan chegaralangan figuralarning aylanishidan hosil bo'lgan jismlarning hajmlari topilsin:

1. $x=a(t-sint), y=a(1-cost)$ sikloidaning bir arkasi va OX o'qi bilan chegaralangan figuraning OX va OY o'qlar atrofida aylanishidan hosil bo'lgan jismning hajmini toping.

2. $x=acos^3t$, $y=asin^3t$ astroidaning OY o'q atrofida aylanishidan hosil bo'lgan figuraning hajmini toping.

3. $r=a(1+cos\varphi)$ kardioidaning qutb o'qi atrofida aylanishidan hosil bo'lgan figuraning hajmini toping.

4. $r=acos^2\varphi$ egri chiziqning qutb o'qi atrofida aylanishidan hosil bo'lgan figuraning hajmini toping.

5. $r=2(1-cos\varphi)$ kardioidaning qutb o'qi atrofida aylanishidan hosil bo'lgan figuraning hajmini toping.

6. $r=2\,\mathrm{acos}\,\varphi$ egri chiziqning qutb o'qi atrofida aylanishidan hosil bo'lgan figuraning hajmini toping.

J: 1) $5\pi^2a^3$ va $6\pi^3a^3$;2) $\frac{32\pi a^2}{105}$; 3) $\frac{8\pi a^3}{3}$;   4) $\frac{4}{21}\pi a^3$; 5) $\frac{64\pi}{3}$; 6) $\frac{4}{3}\pi a^3$.

4-§. Aniq integralning mexanik tadbiqlari

$\vec{m}$ massali moddiy nuqtaning l qismiga nisbatan ***statik momenti*** deb,$M_l=md$ kattalikka aytiladi, bu yerda d moddiy nuqtadan l o'qqacha bo'lgan masofa.

Agar Oxy tekislikda massalari $m_1, m_2, \dots m_i, \dots, m_n$ bo'lgan moddiy nuqtalarning$P_1(x_1, y_1)$,$P_2(x_2, y_2)$, $P_i(x_i, y_i)$, ...,P_n (x_n, y_n) (1) sistemasi berilgan bo'lsa, u holda x_im_i va y_im_i ko'paytmalar m_i massaning Oy va Ox o'qlarga nisbatan ***statik momentlari*** deyiladi. Berilgan moddiy nuqtalar sistemasi og'irlik markazining koordinatalari quyidagi formulalardan topiladi.

$$x_c = \frac{x_1 m_1 + x_2 m_2 + \ldots + x_n m_n}{m_1 + m_2 + \ldots + m_i + \ldots + m_n} = \frac{\sum_{i=1}^{n} x_i m_i}{\sum_{i=1}^{n} m_i}, \quad y_c = \frac{y_1 m_1 + y_2 m_2 + \ldots + y_n m_n}{m_1 + m_2 + \ldots + m_i + \ldots + m_n} = \frac{\sum_{i=1}^{n} y_i m_i}{\sum_{i=1}^{n} m_i}$$

bu yerda $\sum_{i=1}^{n} x_i m_i$ va $\sum_{i=1}^{n} y_i m_i$ lar berilgan sistemaning Oy va Ox o'qlarga nisbatan statik momentlari deyiladi. Ya'ni $M_x = \sum_{i=1}^{n} x_i m_i$ va $M_y = \sum_{i=1}^{n} y_i m_i$ m massali moddiy nuqtaning l o'qlarga nisbatan ***inertsiya momenti*** deb, $J_l = md^2$ songa aytiladi, bu yerda d – nuqtadano'qqacha bo'lgan masofa.(1) moddiy sistemaning Ox va Oy o'qqa nisbatan inertsiya momentlari:

$$J_x = \sum_{i=1}^{n} y^2{}_i m_i \quad va \ J_y = \sum_{i=1}^{n} x^2{}_i m_i$$

AB yoy og'irlik markazining koordinatalari:

$$x_s = \frac{\int_a^b x ds}{\int_a^b ds}, \quad y_s = \frac{\int_a^b y ds}{\int_a^b ds}$$

formuladan topiladi.

Inertsiya momentlari esa

$$J_x = \int_a^b x^2 ds, \qquad J_y = \int_a^b x^2 ds$$

formuladan topiladi. Bu yerda $ds = \sqrt{1 + (y')^2} dx$.

Biron figura og'irlik markazining koordinatalari

$$x_s = \frac{\int_a^b xy ds}{\int_a^b y ds}, \quad y_s = \frac{\int_a^b y^2 ds}{\int_a^b y ds}$$

formulalardan, statik momentlari esa

$$M_x = \frac{1}{2}\int_a^b y^2 dx\,, \quad M_y = \int_a^b xydx$$

formulalardan topiladi. Inertsiya momentlari esa

$$J_x = \frac{1}{2}\int_a^b y^3 dx, \quad J_y = \int_a^b x^2 ydx$$

Aniq integralning mehanik tadbiqlariga doir misollar:

1. $\frac{x}{a} + \frac{y}{b} = 1$ to’g’ri chiziqning koordinata o’qlari orasida joylashgan kesmasining koordinata o’qlariga nisbatan statik momentlarini toping.

J: $M_x = \frac{b}{a}\sqrt{a^2+b^2}$, $\quad M_u = \frac{a}{2}\sqrt{a^2+b^2}$.

2. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsning birinchi kvadrantda yotuvchi yoyining Ox o’qqa nisbatan statik momentini toping.

J: $M_x = \frac{ab}{2\varepsilon} arcsin\varepsilon + \frac{b^2}{2}$ $\quad$ ($\varepsilon = \frac{\sqrt{a^2-b^2}}{a}$ – ellipsning eksentrisiteti).

3. $y = 2\sqrt{x}$ egri chiziq yoyining $x = 3$ to’g’ri chiziq kesgan qismining Ox o’qqa nisbatan statik momentini toping.J:$M_x = \frac{28}{3}$.

4. $x^2 + y^2 = 9$ aylananing birinchi kvadrantda yotgan choragining Oy o’qqa nisbatan statik momentini toping. $\quad$ J: $M_u = 9$.

5. $r = 2\,a\sin\alpha$ aylananing qutb o’qiganisbatan statik momentini toping. J: $2\pi a^2$.

6. $y = \sqrt{r^2 - x^2}$yarim aylananing og’irlik markazi koordinatalarini toping.J: $C(0,\frac{2r}{\pi})$.

7. Absissalar o’qi va $y = \sqrt{r^2 - x^2}$ yarim aylana bilan chegaralangan yarim doira og’irlik markazining koordinatalarini toping. J: $C(0,\frac{4r}{3\pi})$.

8. Koordinata o’qlari va $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ellipsning birinchi kvadrantda yotuvchi yoyi bilan chegaralangan figura og’irlik markazining koordinatalarini toping.J:$C\left(\frac{4a}{3\pi},\frac{4b}{3\pi}\right)$.

9. $y = \sin x$ sinusoida yoyi va Ox o'qi bilan chegaralangan figura $(o \leq x \leq \pi)$ og'irlik markazining koordinatalarini toping.J: $C\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

10. $x = acos^3 t, \mathrm{y} = asin^3 t$ astroidaning birinchi kvadrantda yotgan yoyining Ox va Oyo'qlarga nisbatanstatik momentlari va og'irlik markazini toping.J: $M_x = M_u = \frac{3a^2}{5}$,$C\left(\frac{2a}{5}, \frac{2a}{5}\right)$.

11. $x = a(t - \sin t)$, $y = a(1 - \cos t)$ sikloidabirinchi arkasi og'irlik markazining koordinatalarini toping.J: $C\left(\pi a, \frac{4a}{3}\right)$.

12. Astroidaning birinchi kvadrantdagi qismi va koordinata o'qlari bilan chegaralangan figura og'irlik markazining koordinatalarini toping.

J:$C\left(\frac{256a}{315\pi}, \frac{256a}{315\pi}\right)$.

13.$r = a(1 + \cos \alpha)$ kardioida bilan chegaralanganfiguraog'irlik markazining Dekart koordinatalarini toping.J:$C\left(\frac{5a}{6}, 0\right)$.

14. $\mathrm{r}^2 = a^2 \cos 2\alpha$ Bernulli lemniskatasining o'ng sirtmog'i bilanchegaralangan figura og'irlik markazining Dekart koordinatalarini toping.J:$C\left(\frac{\pi a\sqrt{2}}{8}, 0\right)$.

15. R radiusli yarim aylananing uning diametriga nisbatan inertsiya momentini toping.J: $J_x = \frac{MR^2}{2}(\,bu\, yerda\; M = \pi R)$.

16. $y = 4\sqrt{x}$ parabola yoyining $x = 4$ to'g'ri chiziq kesgan qismining absissalar o'qiga nisbatan inertsiya momentini toping.

J: $32(6\sqrt{2} - \ln(3 + 2\sqrt{2}))$.

17. $\mathrm{y} = 2 - x^2$va$y = x^2$chiziqlar bilan chegaralanganfiguraning koordinata o'qlariga nisbatan inertsiya momentini toping.

J:$J_x = \frac{356}{105},\ J_y = \frac{8}{15}$.

18. $x = a(t - \sin t),\ y = a(1 - cost)$ siklonda bitta arkasining har ikkala koordinata o'qlariga nisbatan inertsiya momentini toping.

J.. $J_x = \frac{256a^3}{15},\ J_y = 16a^3(\pi^2 - \frac{128}{45})$

19. $x = acost,\ y = a\sin t$ doira chorak yuzining ox o'qqa nisbatan inertsiya momentini toping.J. $\frac{\pi a^4}{16}$.

5-§. Aniq integralning fizik tadbiqlari

$V = f(t)(f(t) \geq 0)$ o'zgaruvchi tezlik bilan to'g'ri chiziq bo'ylab harakatlanayotgan nuqtaning $[a, b]$ vaqt oralig'ida bosib o'tgan yo'li $S = \int_a^b f(t)dt$ formula bilan hisoblanadi.

$[a, b]$kesmada o'zgaruvchan $F = f(x)$ kuchning bajargan ishi

$$A = \int_a^b f(t)dt$$

formula bilan hisoblanadi.

Suyuqlikning gorizantal yuzga bosim kuchi *P* bu yuzning cho'kish chuqurligi x ga, ya'ni yuzdan suyuqlik sirtigacha bo'lgan masofaga bog'liqdir.

Gorizontal yuzga bo'lgan bosim kuchi R $= 9{,}807\rho sx$ formula bilan hisoblanadi.

Bu yerda $\rho -$ suyuqlikning zichligi, $S -$ yuz sirti, $x -$ yuzning cho'kish chuqurligi.

Vertikal yuzga suyuqlikning bosim kuchi $R = \int_a^b 9{,}807\rho x f(x)dx$ integral bilan hisoblanadi.

1. Nuqtaning tezligiV $= (100 + 8t)\frac{m}{cek}$. Bu nuqta $[0; 10]$ vaqt oralig'ida qanday masofani o'tadi? J: 1400 m.

2. Nuqtaning tezligi$\upsilon = (2t^2 - 3t)\frac{m}{sek}$ga teng. Harakat boshlangandan keyin $t = 4\ sek$. ichida nuqta bosib o'tgan S yo'lni toping.J: $66\frac{2}{3}$ m.

3. $48\frac{km}{soat}$ tezlik bilan harakatlanayotgan avtomobil tormoz berib tezlikni kamaytira boshladi va 3 sek. dan keyin to'xtadi. Avtomobil butunlay to'xtaguncha qancha masofani bosib o'tkanini toping.J: 20 m.

4. 294 m balandlikdan pastga vertikal yo'nalishda $19{,}6\frac{m}{sek}$ boshlang'ich tezlik bilan jism tashlandi. Necha sekunddan keyin jism yerga kelib tushadi? J: 6 sek.

5. Agar prujinani 1 sm ga qisish uchun 1 kg kuch kerak bo'lsa, prujinani 8 smga qisishga sarf bo'ladigan F kuch bajaradigan ishni toping. J: 0,32 (kg m).

6. Agar prujinani 2 smga cho'zish uchun 3 N kuch kerak bo'lsa, prujinani 5 smga cho'zishda bajaradigan ishni hisoblang. J: 0,1875 (J).

7. Uzunligi 1 m, kesimining radiusi 2 mm bo'lgan mis simni 1mm cho'zishda bajaradigan ishni hisoblang. J: $0{,}024\pi$ (kg m).

8. Massasi m bo'lgan jismni yerdan h balandlikka ko'tarish uchun sarf etish kerak bo'lgan ish aniqlansin. J: $\frac{mgRh}{R+h}$.

9. Vertikal to'g'on trapetsiya shaklida bo'lib, yuqori asosi $a = 6{,}4\ m$ pastki asosi $b = 6{,}4\ m$ balandligi esa $h = 3m$. Suvning butun to'g'onga bosim kuchini toping. J: 22,2 T.

10.Asosining radiusi R, balandligi H bo'lgan vertikal doiraviy konus uchi bilan suvga shunday bostirilganki, uning asosi suv sathida joylashgan. Konusga suvning bosim kuchini toping. J: $\frac{\pi R^2 H}{3}$.

11. To'g'on yuqori asosi 20 m, quyi asosi 10m va balandligi 6 m bo'lgan trapetsiya shaklida. Suvning to'g'onga bo'lgan bosimi aniqlansin.J: 240 T.

6-§. Aniq integralni taqribiy hisoblash

$\int_a^b f(x)dx$ aniq integral qiymatini hisoblash masalasi integral ostidagi $f(x)$ funksiyaning biror $F(x)$ boshlang'ich funksiyasini topish va uningqiymatlarini hisoblash masalasiga keltiriladi. Ammo ayrim aniq integrallar uchun bu usullarni qo'llashda bag'zi bir muammolar bo'lishi mumkin:

1) $F(x)$ boshlang'ich funksiyani topish murakkab;

2) $f(x)$ boshlang'ich funksiyani ko'rinishi murakkab, uning $F(a)$ va $F(b)$ qiymatlarini hisoblash qiyinchilik tug'diradi;

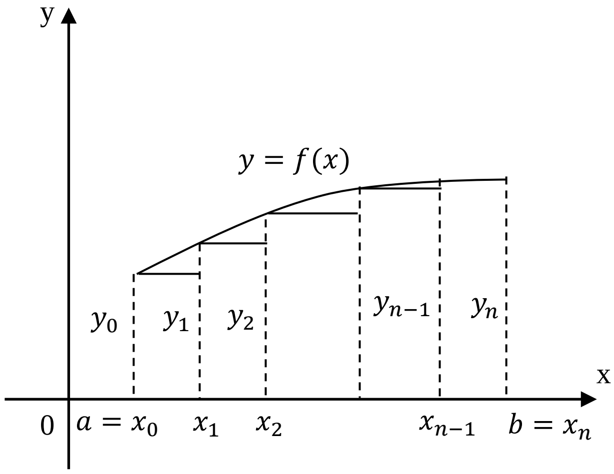
3) $f(x)$ boshlang'ich funksiya elementar funksiyalarda ifodalanmaydi;

4) Integral ostidagi funksiya jadval ko'rinishida berilgan.

Bunday hollarda aniqintegralni taqribiy hisoblashmasalasi paydo bo'ladi. Bunda bir necha formulalar mavjud:

1. ***To'g'ri to'rtburchaklar formulasi***(1-chizma).

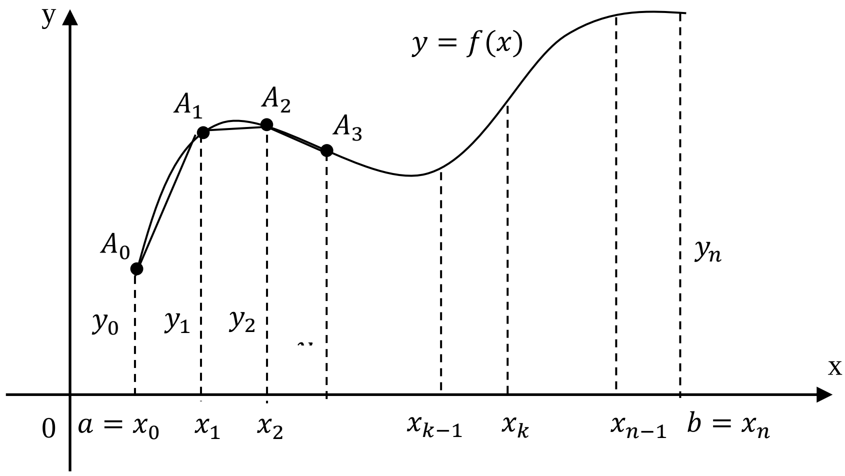
$$\int_a^b f(x)dx \approx \frac{b-a}{n(y_0+y_1+y_2+\cdots+y_{n-1})}, \qquad \int_a^b f(x)dx \approx \frac{b-a}{n(y_1+y_2+y_3+\cdots+y_n)}.$$



1-chizma

2. ***Trapettsiyalar formulasi*** (2-chizma).

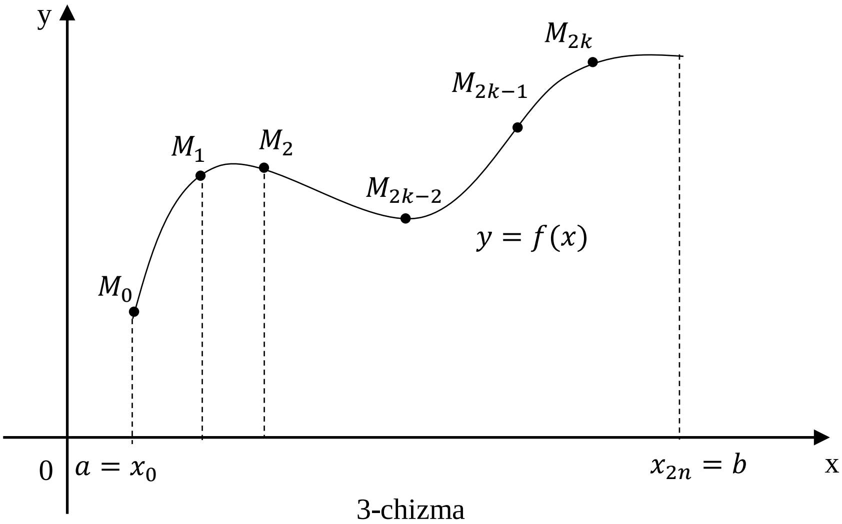
$$\int_a^b f(x)dx \approx \frac{b-a}{n}\left(\frac{y_0+y_n}{2} + y_1 + y_2 + y_3 + \cdots + y_{n-1}\right)$$



2-chizma

3. ***Parabolalar formulasi*** (Simpson formulasi) (3-chizma).

$$\int_a^b f(x)dx \approx \frac{b-a}{6n}[y_0 + y_{2n} + 4(y_1 + y_3 + \dots + y_{2n-1}) + (y_2 + y_4 + \dots + y_{2n-2})]$$



3-chizma

Aniq integralni taqribiy hisoblash ga doir misollar.

1. Quyidagi aniq integrallarni to'g'ri to'rtburchaklar formulasi bilan taqribiy hisoblang.

1) $\int_0^1 \frac{dx}{1+x^2}$ ni 0,06 dan katta bo'lmagan xatolik bilan hisoblang.

J: 0,75998.

2) $\int_1^2 \sqrt[3]{x}dx$ integralni $n = 8$ deb hisoblang. J: 0,0167.

3) $\int_0^1 \frac{dx}{2-\sqrt{x}}$ ni 0,07 dan oshmaydigan xatolik bilan taqribiy hisoblang. J: 1,3702 ortig'i bilan, 1,3037 kami bilan.

2. Quyidagi integrallarni trapetsiyalar formulasi bilan taqribiy hisoblang.

1) $\int_0^1 \frac{dx}{1+x^2}$ integralni trapetsiyalar formulasi bo'yicha $n = 10$ deb hisoblang. J: 0,78498.

2) $\int_0^1 \frac{dx}{1+x^3}$ integralni trapetsiyalar formulasidan foydalanibhisoblang. Bunda $n = 8$ deb olinsin. J: 0,836.

3. Quyidagi integrallarni parabolalar formulasidan foydalanib hisoblang.

1) $\int_0^1 \frac{dx}{1+x^2}$ integralnin $= 2$ deb Simnson(parabolalar) formulasi bo’yicha taqribiy hisoblang.J: 0,7854.

2) $\int_0^1 \frac{dx}{1+x^2}$ integralniSimpsonformulasi bo’yicha taqribiy hisoblang.J:0,3217.

VIII BOB. KO’P O’ZGARUVCHILI FUNKSIYA

1-§. Ko’p o’zgaruvchili funksiya, uning limiti va uzluksizligi

Agar biror D to’plamning har bir (x, y) haqiqiy sonlar juftligi biror qoida bilan E to’plamdagi yagona z haqiqiy songa mos qo’yilgan bo’lsa, u holda D to’plamda ikki o’zgaruvchining funksiyasi Z berilgan deyiladi va quyidagi ko’rinishlarda belgilanadi.

$Z = f(x, y), Z = F(x, y), Z = \varphi(x, \square)$ va hokazolar.

D to’plam funksiyaning aniqlanish sohasi deyiladi.

E to’plam funksiyaning qiymatlar sohasi deyiladi.

$Z = f(x, y)$ funksiyaning Oxyz to’g’ri burchakli koordinatalar sistemasidagi tasviri (grafigi) biror sirtdan iborat bo’ladi.

Istalgan chekli sondagi o’zgaruvchining funksiyasi ham yuqoridagi kabi ta’riflanadi va quyidagicha belgilanadi.

U $= f(x, y, z)$ –uch o’zgaruvchili funksiya;

$V = f(x, y, z, u) -$ to’rt o’zgaruvchili funksiya;

Y $= f(x_1, x_2, x_3, \dots, x_n) - n$ o’zgaruvchili funksiya.

Agar har qanday kichik $\varepsilon > 0$ soni uchun unga bog’liq shunday r$(\varepsilon) = r > 0$ son topilsaki $M_0(x_0, y_0)$ nuqtaning r(ε) radiusli atrofiga tegishli bo’lgan barcha$M(x, y) \neq M_0(x_0, y_0)$ nuqtalar uchun

$$|f(x, y) - A| < \varepsilon$$

tengsizlik bajarilsa, u holda $M(x, y)$nuqta $M_0(x_0 y_0)$nuqtaga intilganda Z$=$ $f(x, y)$ funksiya A ga teng limitga intiladi deyiladi.

Ikkio’zgaruvchili funksiya Z$= f(x, y)$ ning $x \to x_0$, $y \to y_0$ holdagi limiti $\lim\limits_{\substack{x \to x_0 \\ y \to y_0}} f(x, y) = A$ yoki $\lim\limits_{m \to m_0} f(M) = A$ kabi yoziladi.

Ikki o’zgaruvchili funksiyaning limiti uchun bir o’zgaruvchili funksiya limitining ilgari ko’rib o’tilgan barcha xossalari saqlanib qoladi.

$M_0(x_0, y_0)$ nuqta Z$= f(x, y)$ funksiyaning $D\{f\}$ aniqlanish sohasidagi biror nuqta bo’lib, o’zgaruvchi $M(x, y)$ nuqta funksiyaning aniqlanish sohasida qolgan holda $M_0(x_0, y_0)$nuqtaga ixtiyoriy usulda intilganda

$$\lim_{\substack{x\to x_0\\ y\to y_0}} f(x,y) = f(x_0, y_0) \text{ yoki } \lim_{M\to M_0} f(M) = f(M_0)$$

tenglik o'rinli bo'lsa, $Z = f(x,y)$funksiya $M_0(x_0, y_0)$ nuqtada ***uzluksiz*** deyiladi.

Agar biror $M_0(x_0, y_0)$ nuqtada yuqoridagi tenglik o'rinli bo'lmasa, bu nuqtada berilgan $Z = f(x,y)$ funksiya ***uzilishga ega*** deyiladi. $M_0(x_0, y_0)$ nuqta ***uzilish nuqtasi*** deyiladi.

1.$F(x,y) = \frac{x-2y}{2x-y}$ funksiya berilgan. 1)$F(3{,}1)$;2)$F(1;3)$;3) $F(1;2)$;4) $F(2;1)$; 5) $F(a;a)$; 6)$F(a;-a)$ lar topilsin.

J: 1) $\frac{1}{5}$; 2) 5; 3) Mavjud emas; 4) 0; 5) -1; 6) 1.

2. $F(x,y) = \frac{x}{x-y}$ funksiya uchun $F(a,b) + F(b,a) = 1$ ekani ko'rsatilsin.

3. $F(x,y) = \sqrt{x^4 + y^4} - 2xy$ funksiya uchun $F(tx,ty) == t^2F(x,y)$ ekani ko'rsatilsin.

4. $Z = e^{\sin(x+y)}$funksiyaning$x = y = \frac{\pi}{2}$ bo'lgandagi qiymatini toping. J: 1.

5. $Z = y^{x^2-1} + x^{y^2-1}$funksiyaning$x = y = 2$ bo'lgandagi qiymatini toping. J: 16.

6. $Z = (\frac{arctg(x+y)}{arctg(x-y)})^2$ funksiyaning $x = \frac{1+\sqrt{3}}{2}, y = \frac{1-\sqrt{3}}{2}$ bo'lgandagi qiymatini toping. J: $\frac{9}{16}$.

7. Quyidagi funksiyalarning aniqlanish sohalari topilsin.

1) $Z = x^2 + y^2$; 2) $Z = \frac{4}{x^2+y^2}$; 3) $Z = \sqrt{4 - x^2 - y^2}$;

4) $Z = \ln(1 - x^2 - y^2)$; 5) $Z = \sqrt{9 - x^2 - y^2}$; 6) $Z = \sqrt{xy}$;

7) $Z = \frac{1}{\sqrt{1-x^2-y^2}}$; 8) $Z = \frac{xy}{y-x}$; 9) $u = ln\sqrt{1 - x^2 - y^2 - z^2}$;

10) $u = \sqrt{1 - x^2 - y^2 - z^2}$; 11) $Z = \sqrt{x^2 + y^2 - 9}$;

12) $u = \frac{1}{1-x^2-y^2-z^2}$.

J: 1) Tekislikdagi barcha nuqtalar; 2) Tekislikning $O(0{,}0)$ nuqtadan farqli barcha nuqtalari; 3) Markazi $O(0{,}0)$ nuqtada va radiusi 2 ga teng bo'lgan doiradan iborat; 4) Markazi $O(0{,}0)$ nuqtada va radiusi 1 ga teng bo'lgan aylananing ichki nuqtalari; 5) Markazi $O(0,0)$ nuqtada va radiusi 3 ga teng bo'lgan doiradan iborat; 6) I va III koordinata tekisliklaridagi nuqtalar to'plami; 7) Markazi $O(0,0)$ nuqtada va radiusi 1 ga teng bo'lgan aylananing ichki nuqtalari; 8) $y \neq x$ bo'lgan nuqtalar; 9) Markazi $O(0,0,0)$ nuqtada va radiusi 1 ga teng bo'lgan sferaning ichki nuqtalari; 10) Markazi $O(0,0,0)$ nuqtada va radiusi 2ga teng bo'lgan sferadagi va ichki nuqtalari; 11) Markazi $O(0,0)$ nuqtada va radiusi 3ga teng doira aylanasidagi va undan tashqaridagi nuqtalari; 12) Markazi $O(0,0,0)$ nuqtada va radiusi 1 ga teng bo'lgan sferada yotmagan nuqtalari.

8.$M(x,y)$nuqta $M_0(x_0,y_0)$ nuqtaga ixtiyoriy usul bilan intilganda quyidagi limitlar hisoblansin:

1)$\lim\limits_{\substack{x\to 1\\ y\to 2}} (3x+5y+1)$; 2) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{2x+5xy-3y+1}{x^2+y^3+2}$; 3) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{2-\sqrt{xy+4}}{xy}$;4) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\sin xy}{x\square}$; 5) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$; 6) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\sqrt{x^2y^2+1}-1}{x^2+y^2}$;7) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{\sin(x^3+y^3)}{x^2+y^2}$; 8) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{1-cos(x^2+y^2)}{(x^2+y^{2)}x^2y^2}$; 9) $\lim\limits_{\substack{x\to 3\\ y\to 0}} \frac{tg\, xy}{y}$; 10) $\lim\limits_{\substack{x\to 0\\ y\to 0}} \frac{x}{x+y}$.

J: 1) 14; 2) $\frac{1}{2}$; 3) $-\frac{1}{4}$; 4) 1; 5) 2; 6) 0; 7) 0; 8) limitga ega emas; 9) 1; 10) limit mavjud emas.

9. Quyidagi funksiyalarning uzilish nuqtalari topilsin.

1) $Z = \frac{2}{x^2+y^2}$; 2) $Z = \frac{1}{\sin^2\pi x+\sin^2 \pi y}$; 3) $Z = \frac{1}{x-y}$;

4) $Z = \frac{1}{\sin\pi x} + \frac{1}{\sin \pi y}$; 5) $Z = \frac{y^2+2x}{y^2-2x}$; 6) $Z = \frac{10x}{(x-1)^2 +(y-1)^2}$;

7) $Z = \frac{3y}{2x-y}$; 8) $Z = \frac{x^2}{x^2-2y^2 -4}$.

J: 1) $O(0,0)$; 2) Koordinatalari butun sonlardan iborat barcha nuqtalarda; 3) $y = x$ to'g'ri chiziqdagi nuqtalarda; 4) $x = m$, $y = n$ to'g'ri chiziqlarda $(m\epsilon z, n\epsilon z)$;5) $y^2 = 2x$ parabolada yotgan nuqtalarda;6) $M(1,-1)$; 7) $y = 2x$ to'g'ri chiziqda yotgan nuqtalarda; 8) $x^2-2y^2 = 4$ giperbolada yotgan nuqtalarda.

2-§. Ko'p o'zgaruvchili funksiyaning xususiy va to'la orttirmalari. Xususiy hosilalar. To'liq differensial

Agar $z = f(x, y)$ funksiyada x o'zgaruvchiga Δx orttirma berib, y ni o'zgartirishsiz qoldirsak, u holda $z = f(x, y)$ funksiya $\Delta_x z$ orttirma oladi. Bu orttirma z funksiyaning x o'zgaruvchi bo'yicha xususiy orttirmasi deyiladi va u quyidagicha yoziladi:

$$\Delta_x z = f(x + \Delta x, y) - f(x, y)$$

Huddi shunday z funksiyaning y o'zgaruvchi bo'yicha xususiy orttirmasi $\Delta_y z$ deyiladi va u quyidagicha yoziladi:

$$\Delta_y z = f(x, y + \Delta y) - f(x, y)$$

Agar $\lim\limits_{\Delta x \to 0} \frac{\Delta_x z}{\Delta x}$ chekli limit mavjud bo'lsa, u holda unga $z = f(x, y)$ funksiyaning erkli o'zgaruvchi ***x bo'yicha xususiy hosilasi*** deyiladi va $\frac{\partial z}{\partial x}$, z'_x yoki $f'_x(x, y)$ lardan biri bilan bilan belgilanadi.

Agar $\lim\limits_{\Delta y \to 0} \frac{\Delta_y z}{\Delta y}$ chekli limit mavjud bo'lsa, u holda unga $z = f(x, y)$ funksiyaning erkli o'zgaruvchi ***y bo'yicha xususiy hosilasi*** deyiladi va $\frac{\partial z}{\partial y}$, z'_y yoki $f'_y(x, y)$lardan biribilan belgilanadi.

Huddi shunday uch, to'rt va hokazo o'zgaruvchili funksiyalarning xususiy hosilalari haqida ham gapirish mumkin.

Xususiy hosilalar uchun bir o'zgaruvchi funksiyasini differensiallashning qoida va formulalari saqlanadi.

Agar x va y o'zgaruvchilar mos ravishda Δx va Δy orttirmalar olsa, u holda $z = f(x, y)$ funksiya $\Delta z = f\big((x + \Delta x), (y + \Delta y)\big) - f(x, y)$ orttirma oladi. Bu orttirmaga ***to'la orttirma*** deyiladi.

z$= f(x, y)$ funksiyaning to'liq orttirmasini Δx va Δy larga nisbatan chiziqli bo'lgan bosh qismi funksiyaning to'liq differensiali deyiladi va dz bilan belgilanadi.

z $= f(x,y)$ funksiyaning to'liq differensiali quyidagi formula bo'yicha hisoblanadi:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy, \quad \text{bu yerda} dx = \Delta x, dy = \Delta y.$$

To'liq differensialdan ko'pincha funksiyaning taqribiy qiymatlarini hisoblashda ham foydalaniladi. Chunki $\Delta z \approx d□$, ya'ni

$$f\big((x+\Delta x),(y+\Delta y)\big) \approx f(x,y) + dz$$

$d_x z = \frac{\partial z}{\partial x}dx$ va$d_y z = \frac{\partial z}{\partial y}dy$ larga ***xususiy differensiallar*** deyiladi.

1. Quyidagi funksiyalarni xususiy va to'la orttirmalari topilsin.

1) z= $x+y$; 2) z= xy; 3) z= x^2+y^2; 4) z= $\frac{x}{y}$;

5) z= $x^2+3xy-4y$;6) z= x^2y+xy^2; 7) u= $x+y+z$;

8) u $= xyz$; 9) u $= xy+yz$.

2. Quyidagi funksiyalarning xususiy hosilalari topilsin.

1) z= $x-y$; 2) z= x^3y-xy^3; 3) z= $(5x^2y-y^3+7)^3$;

4) z= $x\sqrt{y}+\frac{y}{\sqrt[3]{x}}$; 5) z= $arctg\frac{x}{y}$; 6) z= $ln(x^2+y^2)$; 7) z= $e^{-\frac{x}{y}}$; 8) z= $ln\, tg^{\frac{x}{y}}$;9) z= $ln(x+lny)$;10) z= $e^{xy(x^2+y^2)}$;

11) $z = arctg\frac{y}{1+x^2}$; 12) z= $arctg\frac{y}{x}+ arctg\frac{x}{y}$;13) z= $\sqrt[x]{e^y}$;

14) z= $x^3+5xy^2-y^3$;15) z= $(5x^3y^2+1)^3$; 16) u= $\frac{x}{y}+\frac{y}{z}-\frac{z}{x}$; 17) u $= xyz$; 18) u $= xy+yz+zx$; 19) u= $\sin(x^2+y^2+z^2)$; 20) $u = \ln(x+y+z)$; 21) u $= \sqrt{x^2+y^2+z^2}$.

3. Quyidagi funksiyalar xususiy hosilalarining qiymatlarini argumentlarning berilgan qiymatlarda hisoblang.

1) f$(\alpha,\beta) = \cos(m\alpha-n\beta)ni\alpha = \frac{\pi}{2m}, \beta = 0$ da;

2) z= $ln(x^2+y^2)$ ni$x = 2, y = -1$ da;

3) u $= \sin^2(3x+2y-z)$ ni $M(1,-1,1)$ nuqtada.

J: 1)$-m$va$n$; 2) $\frac{4}{3}$va$\frac{2}{3}$;3) 0; $2sin2 \approx 1{,}82$; $0{,}84$.

4. z= $x\, ln\frac{x}{y}$funksiya $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$ = z tenglikni qanoatlantirishi ko'rsatilsin.

5. z= x^yfunksiya $\frac{x}{y}\frac{\partial z}{\partial x}+\frac{1}{\ln x}\frac{\partial z}{\partial y}=2$z tenglikni qanoatlantirishi ko'rsatilsin.

6.z= $e^{\frac{x}{y}}\ln y$funksiya xz'_x+y$z'_y=\frac{z}{\ln y}$tenglikni qanoatlantirishi ko'rsatilsin.

7. u = $\sqrt{x^2+y^2+z^2}$ funksiya $(u'_x)^2+(u'_y)^2+(u'_z)^2=1$ tenglikni qanoatlantirishi ko'rsatilsin.

8. u = x + $\frac{x-y}{y-z}$ funksiya $u'_x+u'_y+u'_z=1$ tenglikni qanoatlantirishi ko'rsatilsin.

9. Quyidagi funksiyalarni xususiy differensiallari topilsin.

1) z= $xy^3-3x^2y^2+2y^4$;2) z= $\sqrt{x^2+y^2}$;

3) z= $\frac{xy}{x^2+y^2}$;4) y = ln $(x^3+2y^3-z^3)$.

J: 1) $d_x z=(y^3-6xy^2)dx$, $d_y z=(3xy^2-6x^2y+8y^3)dy$

2) $d_x z=\frac{xdx}{\sqrt{x^2+y^2}}$, $d_y z=\frac{ydy}{\sqrt{x^2+y^2}}$;3) $d_x z=\frac{y(y^2-x^2)dx}{(x^2+y^2)^2}$, $d_y z=\frac{x(x^2-y^2)dy}{(x^2+y^2)^2}$;

4) $d_x u=\frac{3x^2dx}{x^3+2y^3-z^3}$, $d_y u=\frac{6y^2dy}{x^3+2y^3-z^3}$, $d_z u=-\frac{3z^2dz}{x^3+2y^3-z^3}$.

10. z= $\sqrt[3]{x+y^2}$ funksiya berilgan $x=2, y=5, \Delta y=0{,}01$ bo'lgandagi $d_y z$ topilsin. (J: $\frac{1}{270}$)

11. $z=\sqrt{\ln xy}$ funksiya berilgan. $x=1,\ y=1{,}2,\ \Delta x=0{,}016$ bo'lgandagi$d_x z$xususiy differensial topilsin. (J: $\approx 0{,}0187$)

12. $u=p-\frac{qr}{p}+\sqrt{p+q+r}$ funksiya berilgan $p=1$, $q=3$, $r=5$bo'lgandagi $d_p u$ xususiy differensial topilsin. (J: $\approx\frac{97}{600}$)

13. Quyidagi funksiyalarning to'liq differensiallari topilsin.

1) z= $x^2y^4-x^3y^3+x^4y^2$; 2) z= $\frac{1}{2}\ln(x^2+y^2)$; 3) z= $\frac{x+y}{x-y}$;

4) z=arcsin$\frac{x}{y}$; 5) z= $sin(xy)$; 6) z= $arctg\frac{x+y}{1-xy}$; 7) z= $\frac{x^2+y^2}{x^2-y^2}$;

8) z= $arctg(xy)$; 9) u = $\sqrt{x^2+y^2+z^2}$; 10) u = $2x^{yz}$.

J:1) $xy[(2y^3-3xy^2+4x^2y)dx+(4y^2x-3yx^2+2x^3)dy]$; 2) $\frac{xdx+ydy}{x^2+y^2}$;

3) $\frac{2(xdy-ydx)}{(x-y)^2}$; 4) $\frac{ydx-xdy}{y\sqrt{y^2-x^2}}$;5) $(xdy+ydx)\cos(xy)$; 6) $\frac{dx}{1+x^2}+\frac{dy}{1+y^2}$;7) $\frac{4xy(xdy-ydx)}{(x^2-y^2)^2}$;

8) $\frac{xdy+ydx}{1+x^2y^2}$;9) $\frac{xdx+ydy+zdz}{\sqrt{x^2+y^2+z^2}}$;

10) $du=2x^{yz}(\frac{yz}{x}dx++zlnxdy+ylnxdz)$.

14. $z=arctg\frac{x}{y}$ funksiyaning x=1, $y=3$, $dx=0{,}01$; $dy=-0{,}05$bo’lgandagi to’liq diffensiali topilsin. J: $-0{,}008$.

15. $z=\frac{y}{x}$ funksiyaning $x=2,\ y=1,\ dx=0{,}1;\ dy=0{,}2$ bo’lgandagi to’liq diffensiali topilsin. J: 0,075.

16. u $=e^{xy}$ funksiyaning $x=1,y=2,dx=-0{,}1;\ dy=0{,}1$ bo’lgandagi to’liq diffensiali topilsin. J: $-0{,}739$.

17. z $=xy$ funksiyauchun $x=5,y=4,\ \Delta x=0{,}1;\ \Delta y=-0{,}2$ bo’lganda dzvaΔzhisoblansin.

18. Quyidagilarni taqribiy hisoblang.

1) $1{,}02^{3{,}01}$; 2) $\ln(0{,}09^3+0{,}99^3)$;3)$\sqrt{(4{,}05)^2+(2{,}93)^2}$;

4) $\sqrt{(1{,}02)^3+(1{,}97)^3}$; 5) $\sin 28°\cdot cos61°$; 6) $1{,}08^{3{,}96}$

J: 1) 1,06; 2) $-0{,}03$; 3) 4,998;4) 2,95; 5) 0,227; 6) 1,32.

3-§. Murakkab va oshkormas funksiyalarning xususiy hosilalari

Agar z= $f(x,y)$, $x=x(t)$, y $=y(t)$ funksiyalar differensiallanuvchi bo’lsa, u holda $z=f(x(t),y(t))$ murakkab funksiya bo’lib, uning hosilasi

$$\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{dx}{dt}+\frac{\partial z}{\partial y}\cdot\frac{dy}{dt}$$

formuladan topiladi.

Agar z= $f(x,y)$, y $=y(x)$ bo’lsa, u holda z= $f(x,y(x))$ dan x bo’yicha hosilasi

$$\frac{\partial z}{\partial x}=\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t}$$

formuladan topiladi va u ***to’la hosila*** deyiladi.

Agar $x = x(u,v), \mathrm{y} = y(u,v)$ bo'lsa, u holda $z = f(x,y)$ ning xususiy hosilalari

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}; \quad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v}$$

formulalardan topiladi.

Agar $F(x,y) = 0$ tenglama biror $y(x)$ funksiyani oshkormas ko'rinishda aniqlasa va $\mathrm{F'}_y(x,y) \neq 0$ bo'lsa, u holda quyidagi formula o'rinlidir:

$$\frac{\partial y}{\partial x} = -\frac{\mathrm{F'}_x(x,y)}{\mathrm{F'}_y(x,y)}$$

Agar $F(x,y,z) = 0$ tenglama ikki o'zgaruvchili $z = f(x,y)$ funksiyani oshkormas ko'rinishda aniqlasa va $\mathrm{F'}_z(x,y,z) \neq 0$ bo'lsa, u holda quyidagi formulalar o'rinlidir:

$$\frac{\partial z}{\partial x} = -\frac{\mathrm{F'}_x(\mathrm{x,y},z)}{\mathrm{F'}_y(\mathrm{x,y},z)}; \quad \frac{\partial z}{\partial y} = -\frac{\mathrm{F'}_y(\mathrm{x,y},z)}{\mathrm{F'}_z(\mathrm{x,y},z)}.$$

1. $z = \frac{\mathrm{y}}{x}$,$x = e^t, y = 1 - e^{2t}$bo'lsa, $\frac{dz}{dt}$ topilsin. J: $\frac{dz}{dt} = -2cht.$

2. $z = \frac{\mathrm{x}}{y}$, $x = e^t, y = \ln t$bo'lsa, $\frac{dz}{dt}$ topilsin. J: $\frac{tye^t - x}{y^2 t} = -2cht.$

3. $\mathrm{y} = \mathrm{u}^2\mathrm{e}^\mathrm{v}$, $\mathrm{u} = sinx$, $v = cosx$ bo'lsa, $\frac{dy}{dx}$ topilsin.J: $2u\mathrm{e}^\mathrm{v}\mathrm{cosx} +$ $+\mathrm{u}^2\mathrm{e}^\mathrm{v}(-sinx)$.

4.$u = \mathrm{e}^{\mathrm{z}-2\mathrm{y}}$, z= $sinx$, $y = x^3$bo'lsa,$\frac{du}{dx}$topilsin.

J: $\mathrm{e}^{z-2\mathrm{y}}(\mathrm{cos}x - 6x^2)$

5. Agar $\mathrm{y} = x^2$ bo'lsa, $z = arctg\frac{\mathrm{y}}{x}$ funksiyaning to'liq hosilasini toping.J: $\frac{1}{1+x^2}$.

6.Agar$f(x) = arcsin\frac{\mathrm{x}}{y}, y = \sqrt{x^2 + 1}$bo'lsa, $\frac{df}{dx}$ topilsin.J:$\frac{1}{x^2+1}$.

7. Agar $\mathrm{z} = \sqrt{x^2 + y^2}$, $y = sin^2 x$ bo'lsa, $\frac{dz}{dx}$ topilsin. J: $\frac{1}{\sqrt{x^2+y^2}}(x +$ $+ysin2x)$.

8. Agar z= $ln(x^2 + y^2)$, $y = e^{x^2}$bo'lsa, $\frac{dz}{dx}$ topilsin. J:$\frac{2x}{x^2+y^2}(1 + +2ye^{x^2})$.

9. Agar z= $arctg\frac{x+y}{1-xy}$, $y = cosx$bo'lsa, $\frac{dz}{dx}$ topilsin. J:$\frac{1}{1+x^2} - -\frac{1}{1+y^2}sinx$.

10. Agar z$=\frac{x^2}{y}$, $x=u-2v, y=v+2u$bo’lsa,$\frac{\partial z}{\partial u}$ va $\frac{\partial z}{\partial v}$ lar topilsin.J: $\frac{\partial z}{\partial u}=\frac{2x}{y}\left(1-\frac{x}{y}\right)$, $\quad \frac{\partial z}{\partial v}=-\frac{x}{y}\left(4+\frac{x}{y}\right)$.

11. Agar$p=u^2\ln v$, u $=\frac{x}{y}$, $v=3x-2y$ bo’lsa,$\frac{\partial p}{\partial x}va\frac{\partial p}{\partial y}$ lar topilsin.J: $\frac{\partial p}{\partial x}=\frac{u}{vy}(3x+2vln\, v)$, $\frac{\partial p}{\partial y}=-\frac{2xu}{vy^2}(y+v\ln v)$.

12. Agar z $=arctg\frac{u}{v}$, $u=xsiny$, $v=xcosy$ bo’lsa, $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial y}$ lar topilsin.J: $\frac{\partial z}{\partial x}=0$; $\frac{\partial z}{\partial y}=1$.

13. Agar$z=ln(u^2+v)$, $u=e^{x+y^2}$, $v=x^2+y$bo’lsa,$\frac{\partial z}{\partial x}va\frac{\partial z}{\partial y}$ lar topilsin.J: $\frac{2}{u^2+v}\left(ue^{x+y^2}+x\right)$; $\frac{1}{u^2+v}(2uye^{x+y^2}+1)$.

14. Oshkormas ko’rinishda $(x^2+y^2)^3-3(x^2+y^2)+1=0$tenglama bilan berilgan $y(x)$ funksiyaning hosilasini toping. J: $-\frac{x}{y}$.

15. Oshkormas ko’rinishda $x^2-2y^2+3z^2-yz+y=0$ tenglama bilan berilgan $z(x,y)$ funksiyaning xususiy hosilalarini toping.

J: $\frac{\partial z}{\partial x}=-\frac{2x}{6z-y}$,; $\frac{\partial z}{\partial y}=-\frac{1-4y-z}{6z-y}$.

16. Oshkormas holda berilgan quyidagifunksiyalarning xususiy hosilalari topilsin.

1) $x^2+y^2+z^2-6x=0$; 2) $z^2=xy$;

3) $\cos(ax+by-cz)=k(ax+by-cz)$;

4) $x^2+y^2+z^2-2zx=a^2$.

J: 1)$\frac{\partial z}{\partial x}=\frac{3-x}{z}, \frac{\partial z}{\partial y}=-\frac{y}{z}$; 2) $\frac{\partial z}{\partial x}=\frac{y}{2z}, \frac{\partial z}{\partial y}=\frac{x}{2z}$;

3) $\frac{\partial z}{\partial x}=\frac{a}{c}, \frac{\partial z}{\partial y}=\frac{b}{c}$; 4) $\frac{\partial z}{\partial x}=1, \frac{\partial z}{\partial y}=\frac{y}{x-z}$.

17. $2\sin(x+2y-3z)=x+2y-3z$ bo’lsa, $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=1$ bo’lishi ko’rsatilsin.

4-§. Yuqori tartibli xususiy hosilalar va to'liq differensiallar

z$= f(x,y)$ funksiyaning ***ikkinchi tartibli xususiy hosilalari*** deb, birinchi tartibli xususiy hosilalardan olingan xususiy hosilalarga aytiladi. Ikkinchi tartibli xususiy hosilalar quyidagicha belgilanadi:

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial^2 z}{\partial x^2}=z''_{xx}=f''_{xx}(x,y);\ \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial^2 z}{\partial x\partial y}=z''_{xy}=f''_{xy}(x,y);$$

$$\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right)=\frac{\partial^2 z}{\partial y\partial x}=z''_{yx}=f''_{yx}(x,y);\ \frac{\partial}{\partial y}(\frac{\partial z}{\partial y})=\frac{\partial^2 z}{\partial y^2}=z''_{yy}=f''_{yy}(x,y).$$

$f''_{xy}\ va\ f''_{yx}$ xususiy hosilalar ***aralash hosilalar*** deyiladi.

Uchinchi tartibli va boshqa yuqori tartibli xususiy hosilalar ham shunga o'xshash ta'riflanadi va belgilanadi.

Hosila olish tartibi bilangina farqlanuvchi aralash hosilalar uzluksiz bo'lsa, ular o'zaro teng bo'ladi.

$$\frac{\partial^2 z}{\partial x\partial y}=\frac{\partial^2 z}{\partial y\partial x},\quad \frac{\partial^3 z}{\partial x^2\partial y}=\frac{\partial^3 z}{\partial x\partial y\partial z}=\frac{\partial^3 z}{\partial y\partial x^2}$$

Yuqori tartibli to'liq differensiallar quyidagicha aniqlanadi:

$$d^2z=\frac{\partial^2 z}{\partial x^2}dx^2+2\frac{\partial^2 z}{\partial x\partial y}dxdy+\frac{\partial^2 z}{\partial y^2}dy^2=(\frac{\partial}{\partial x}dx+\frac{\partial}{\partial y}dy)^2z;$$

$$d^3z=\frac{\partial^3 z}{\partial x^3}dx^3+3\frac{\partial^3 z}{\partial x^2\partial y}dx^2dy+3\frac{\partial^3 z}{\partial x\partial y^2}dxdy^2+\frac{\partial^3 z}{\partial y^3}dy^3=(\frac{\partial}{\partial x}dx+\frac{\partial}{\partial y}dy)^3z;$$

....

$$d^nz=(\frac{\partial}{\partial x}dx+\frac{\partial}{\partial y}dy)^nz.$$

1. Quyidagi funksiyalarning ikkinchi tartibli xususiy hosilalari topilsin.

1) z$= xy$;2) z$= e^{ax+by}$; 3) z$= e^{xy}$;

4) z$= x^3-2x^2$y$+3y^2$;5) u$= e^{xyt}$;6) z$= x^2$y$+y^3$.

J: 1) $\frac{\partial^2 z}{\partial x^2}=0,\frac{\partial^2 z}{\partial x\partial y}=1,\frac{\partial^2 z}{\partial y^2}=0$; 2) $z_{xx}''=a^2e^{ax+by}$; $z''_{xy}=abe^{ax+by}$, $z_{yy}''=b^2e^{ax+by}$;3) $z_{xx}''=y^2e^{xy}$, $z''_{xy}==e^{xy}(xy+1)$, $z_{yy}''=x^2e^{xy}$; 4) $z''_{xx}=6x-4y$, $z''_{\square y}=-4x$, $z''_{yy}=6$; 5) $u_{xx}''=y^2t^2e^{xyt}$, $u''_{xy}=u_{yx}''=t(1+xyt)e^{xyt}$, $u_{xt}''=u_{tx}''=y(1+xyt)e^{xyt}$,$u''_{yt}=u''_{ty}=x(1+$

$xyt)e^{xyt}$, $u''_{yy} = y^2t^2e^{xyt}$, $u_{tt}'' = x^2e^{xyt}\cdot y^2$; 6) $z''_{xx} = 2y$, $z''_{xy} = z''_{yx} = 2x$, $z_{yy}'' = 6y$.

2. $u = \sin(xyz)$ funksiya uchunu'''_{xyz}topilsin.

J:$(1 - x^2y^2z^2)\cos(xyz) - 3xyz\sin(xyz)$.

3. u $= \ln(x + y)$ funksiya uchunu'''_{xxy} topilsin.

4.u $= 2^{xyz}$ funksiya uchun u'''_{xyz} topilsin.

5.z $= \ln\frac{x}{y}$ va $z = arctg(x + 2y)$ funksiyalar uchun $\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial^2 z}{\partial y\partial x}$ tenglikni o'rinli bo'lishi ko'rsatilsin.

6.$z = e^{xy}$ funksiya $x\frac{\partial z}{\partial x} + y\,\frac{\partial z}{\partial y} - z\frac{\partial^2 z}{\partial x\partial y} = -2z$tenglamani qanoatlantirishini tekshiring.

7.u $= \frac{y}{x}$ funksiyaning 3-tartibli xususiy hosilalari topilsin.

8.u $= x^4 + 3x^2y^2 - 2y^4$ funksiyaning 4-tartibli xususiy hosilalari topilsin.

9. u $= \frac{y}{\sqrt[3]{t}}$ funksiyaning 3-tartibli xususiy hosilalari topilsin.

10. 1) z= $\frac{y^2}{x^2}$; 2) z= $xln\frac{y}{x}$bo'lsa, d^2ztopilsin.

J: 1) $\frac{2}{x^4}(3y^2dx^2 - -4xydxdy + x^2dy^2)$; 2) $-\frac{(ydx-xdy)^2}{xy^2}$.

11. z= $y\ln x$ funksiya berilgan d^2zva d^3ztopilsin.

J:$d^2z = -\frac{y}{x^2}dx^2 + \frac{2}{x}dxdy, \quad d^3z = \frac{2y}{x^2}dx^3 - \frac{3}{x^2}dx^2dy$

12. z= x^2y^2funksiya berilgan.d^2ztopilsin.

J: $d^2z = 2y^2dx^2 + 8xydxdy + 2x^2dy^2$.

13. z= $\cos(x + 2y^2\ )$ funksiya berilgan.d^3ztopilsin.

14. z= x^3y^3funksiya berilgan.d^2ztopilsin.

J: $6xy^3dx^2 + 18x^2y^2dxdy + 6x^3ydy^2$.

5-§. Sirtga urinma tekislik va normal. Yo'nalish bo'yicha hosila. Gradient

Agar sirt z= $f(x, y)$ tenglama bilan berilgan bo'lsa, u holda $M_0(x_0, y_0, z_0)$ nuqtada bu sirtga o'tkazilgan urinma tekislik tenglamasi:

$$z - z_0 = f'_x(x_0, y_0)(x \quad - x_0) + f'_y(x_0, y_0)(y \quad - y_0)$$

normal tenglamasi:

$$\frac{x - x_0}{f'_x(x_0 y_0)} = \frac{y - y_0}{f'_y(x_0 y_0)} = \frac{\mathrm{z} - z_0}{1}$$

dan iborat bo'ladi.

Agar sirt $F(x, y, z)$ tenglama bilan berilib, $M_0(x_0, y_0, z_0)$ nuqta shu sirtda yotuvchi nuqta bo'lsa, u holda sirtga M_0 nuqtada o'tkazilgan urinma tekislik tenglamasi

$$(x \quad - x_0)F'_x(M_0) + (y \quad - y_0)F'_y(M_0) + (\mathrm{z} - z_0)F'_z(M_0) = 0$$

tenglama bilan aniqlanadi. M_0 nuqtada sirtga o'tkazilgan normal

$$\frac{x - x_0}{F'_x(M_0)} = \frac{y - y_0}{F'_y(M_0)} = \frac{\mathrm{z} - z_0}{F'_z(M_0)}$$

tenglama bilan aniqlanadi.

$$\frac{\partial u}{\partial l} = \frac{\partial F}{\partial x}\cos\alpha + \frac{\partial^2 F}{\partial y}\cos\beta + \frac{\partial^2 F}{\partial z}\cos\gamma$$

hosila $\mathrm{y} = F(x, y, z)$ funksiyaning berilgan $l_0\{\cos\alpha, \cos\beta, \cos\gamma\}$ ***yo'nalish bo'yicha hosilasi*** deyiladi.

$\mathrm{u} = F(x, y, z)$ skalyarning ***gradienti*** deb, $grad\, u = \frac{\partial u}{\partial x} i + \frac{\partial u}{\partial y} j + + \frac{\partial u}{\partial z} k$ vektorga aytiladi.

1. $\mathrm{z} = x^2 - xy + y^2 - x + 2y$ sirtga $M_0(1,1,1)$ nuqtada o'tkazilgan urinma tekislik va normal tenglamalari tuzilsin.

J:$x - 2y + z = 0$va$\frac{x-1}{1} = \frac{y-1}{-2} = \frac{z-1}{1}$.

2.$z = x^3 + y^3 + \mathrm{z}^3 + xy\mathrm{z} - 6 = 0$ sirtga $M_0(1, 2, -1)$ nuqtada o'tkazilgan urinma tekislik va normal tenglamalari tuzilsin.

J: $x + 11y + 5z - 18 = 0\ va\ \frac{x-1}{1} = \frac{y-2}{11} = \frac{z+1}{5}$.

3. $z = x^2 + 2y^2$ sirtga $M_0(1,1,3)$ nuqtada o'tkazilgan urinma tekislik tenglamasi tuzilsin.J: $2x + 4y - z = 3$.

4. Absissalar o'qining musbat yo'nalishi bilan 60° li burchak hosil qiluvchi l vektor yo'nalishi bo'yicha $f(x, y) = x^3 - y^3$ funksiyaning M(1,1) nuqtadagi hosilasi topilsin. J: $\frac{\partial f}{\partial l} = \frac{3}{2}(1 - \sqrt{3})$.

5. Absissalar o'qining musbat yo'nalishi bilan α burchak hosil qiluvchi l vektor yo'nalishi bo'yicha $f(x,y) = 3x^2 - 6xy + y^2$ funksiyaning M$(-\frac{1}{3}, -\frac{1}{2})$ nuqtadagi hosilasi topilsin.

J: $\frac{\partial f}{\partial l} = \cos\alpha + \sin\alpha$.

6. $u = x^2 + y^2 + z^2$ funksiyaningS$= 2i + j + 3k$ vektor yo'nalishi bo'yicha M(1,1,1) nuqtadagi hosilasi topilsin.J: $\frac{12}{\sqrt{14}}$.

7. $u = x^2 + y^2 + z^2$funksiyaning S $= i + j + k$ vektor yo'nalishi bo'yicha M(1,1,1) nuqtadagi hosilasi topilsin.J: $2\sqrt{3}$.

8. $u = x^2 + y^2 + z^2$ funksiyaningM(1,1,1) nuqtadagi gradienti topilsin.J: $|gradu|_M = 2\sqrt{3}$.

9. $u = \frac{x^2}{2} + \frac{y^2}{3}$funksiyaningM(2,4) nuqtadagi gradienti topilsin.

J: $gradu = 2i + \frac{8}{3}j$.

6-§. Ko'p o'zgaruvchili funksiyaning ekstremumlari. Teylor formulasi

Agar $z = f(x,y)$ funksiyaning $P_0(x_0, y_0)$ nuqtadagi qiymati uning bu nuqtani biror atrofidagi istalgan $P(x,y)$ nuqtasidagi qiymatidan katta, ya'ni $f_0(x_0 y_0) > f(x,y)$ bo'lsa, u holda $z = f(x,y)$ funksiya $P_0(x_0, y_0)$ nuqtada ***maksimumga*** ega deyiladi.

Agar $z = f(x,y)$ funksiyaning $P_0(x_0 y_0)$ nuqtadagi qiymati uning bu nuqtani biror atrofidagi istalgan $P(x,y)$ nuqtasidagi qiymatidan kichik, ya'ni$f_0(x_0, y_0) < f(x,y)$ bo'lsa, u holda $z = f(x,y)$ funksiya $P_0(x_0, y_0)$ nuqtada ***minimumga*** ega deyiladi.

Funksiyaning maksimumi yoki minimumi uning ***ekstremumlari*** deyiladi.

Funksiya ekstremumga ega bo'lgan nuqta uning ***ekstremum nuqtasi*** deyiladi.

Ekstremumning zaruriy sharti. Agar $P_0(x_0, y_0)$ nuqta z$= f(x,y)$ uzluksiz funksiyaning ekstremum nuqtasi bo'lsa, u holda $f'_x(x_0, y_0) = 0$, $f'_y(x,y) = 0$ bo'ladi yoki bu hosilalarning aqalli bittasi mavjud bo'lmaydi.

Bu shartlar bajariladigan nuqtalar ***kiritik nuqtalar*** deyiladi. Har qanday kritik nuqta ham ekstremum nuqtasi bo'lavermaydi.

Ikkinchi tartibli hosilalarning $P_0(x_0, y_0)$ kritik nuqtadagi qiymatlarini mos ravishda A, B va C lar bilan belgilaymiz, ya'ni

$$A = f''_{xx}(x_0, y_0), \quad B = Z''_{xy}(x_0, y_0), \quad C = Z_{yy}{}''(x_0, y_0)$$

belgilab, $\Delta = AC - B^2$ diskriminantni tuzamiz.

<u>Ekstremumning yetarli sharti.</u>

$a)$ Agar $\Delta > 0$ bo'lsa, $Z = f(x, y)$ funksiya $P_0(x_0, y_0)$ nuqtada ekstremumga ega bo'lib, bunda A< 0 (yoki$C < 0$) bo'lganda P_0 nuqta maksimum nuqtasi, A> 0 (yoki $C > 0$) bo'lganda minimum nuqtasi bo'ladi.

b) Agar $\Delta < 0$ bo'lsa, P_0 nuqtada ekstremum mavjud emas.

c) Agar $\Delta = 0$ bo'lsa, ekstremum mavjud bo'lishi ham mavjud bo'lmasligi ham mumkin.

Chegaralangan yopiq D sohada differensiallanuvchi funksiya o'zining eng katta va eng kichik qiymatiga yo D soha ichida yotuvchi kiritik nuqtada yo bu soha chegarasida erishadi.

Yopiq D sohada funksiyaning eng katta va eng kichik qiymatini topish uchun:*a*) soha ichida va uning chegarasida yotgan barcha kritik nuqtalar topiladi;
b) Funksiyaning bu nuqtalardagi va chegaradagi qiymatlari hisoblanadi;
c) topilgan qiymatlar orasidan eng katta va eng kichik qiymatlar ajratiladi.

$z = f(x, y)$ funksiyaning ***<u>shartli ekstremumi</u>*** deb, bu funksiyaning x va y o'zgaruvchilarning bog'lanish tenglamasi deb ataluvchi $\varphi(x, y) = 0$ tenglama bilan bog'langanlik shartida erishadigan ekstremumga aytiladi.

Ushbu $\Phi(x, y, \lambda) = f(x, y) + \lambda\varphi(x, y)$ funksiya Lagranj funksiyasi deyiladi. Bu yerda $\lambda -$biror o'zgarmas ko'paytuvchi. Shartli ekstremumni topish $\Phi(x, y, \lambda)$ funksiyaning oddiy ekstremumini izlashga keltiriladi, Lagranj funksiyasi ekstremumining zaruriy sharti quyidagicha bo'ladi:

$$\begin{cases} \frac{\partial \Phi}{\partial x} = 0, \\ \frac{\partial \Phi}{\partial y} = 0, \\ \frac{\partial \Phi}{\partial \lambda} = 0. \end{cases} \text{yoki} \begin{cases} \frac{\partial f}{\partial x} + \lambda \frac{\partial \varphi}{\partial x} = 0, \\ \frac{\partial f}{\partial y} + \lambda \frac{\partial \varphi}{\partial y} = 0, \\ \varphi(x, y) = 0. \end{cases}$$

Agar $P_0(x_0, y_0)$, $\lambda_0 -$ bu sistemaning istalgan yechimi va

$$\Delta=-\begin{vmatrix} 0 & \varphi'_x(x_0y_0)\varphi'_y(x_0y_0) \\ \varphi'_x(x_0y_0) & \Phi''_{xx}(x_0,y_0,\lambda_0)\Phi''_{xy}(x_0,y_0,\lambda_0) \\ \varphi'_y(x_0y_0)\Phi''_{xy}(x_0,y_0,\lambda_0)\Phi''_{yy}(x_0,y_0,\lambda_0) \end{vmatrix}$$

bo'lsa, $\Delta< 0$da$z = f(x,y)$ funksiya $P_0(x_0y_0)$ nuqtada shartli maksimumga, $\Delta> 0$da shartli minimumga ega bo'ladi.

Agar $z = f(x,y)$ funksiya $P_0(x_0,y_0)$ nuqta atrofida $(n+1)-$ tartibgacha uzluksiz xususiy hosilalarga ega bo'lsa, u holda qaralayotgan nuqta atrofida ushbu ***Teylor formulasi*** o'rinlidir:

$$f(\square,y) = f(x_0,y_0) + \frac{1}{1!}\left[f'_x(x_0,y_0)(x-x_0) + f'_y(x_0,y_0)(y-y_0)\right] +$$

$$+\frac{1}{2!}\left[f''_{xx}(x_0,y_0)(x-x_0)^2 + 2f''_{xy}(x_0,y_0)(x-x_0)(y-y_0) + f''_{yy}(x_0,y_0)(y-y_0)^2\right] +$$

$$\dots + \frac{1}{n!}\left[(x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right]^n f(x_0,y_0) + R_n(x,y),$$ bu yerda

$$R_n(x,y) = \frac{1}{(n+1)!}\left[(x-x_0)\frac{\partial}{\partial x} + (y-y_0)\frac{\partial}{\partial y}\right]^{n+1} f\left(\left(x_0 + \theta(x-x_0)\right),\left(y_0 +\right.\right.$$

$$\left.\left.+\theta(y-y_0)\right)\right), \quad 0 < \theta < 1.$$

Teylor formulasida $x_0 = y_0 = 0$ bo'lsa, u holda hosil bo'lgan formula ***Makloren formulasi*** deyiladi.

1.Quyidagi funksiyalarning ekstremumlari topilsin.

1) $z = xy(x + y - 2)$; 2) $z = x^2 - xy + y^2 + 9x - 6y + 20$;

3) $z = y\sqrt{x} - y^2 - x + 6y$; 4) $z = x^3 + 8y^3 - 6xy + 1$;

5) $z = 2xy - 4x - 2y$; 6) $z = e^{\frac{x}{2}}(x + y^2)$;

7) $z = 3x + 6y - x^2 - xy - y^2$;8) $z = x^2 + y^2 - 2x - 4\sqrt{xy} - 2y + 8$;

9) $z = 2x^3 - xy^2 + 5x^2 + y^2$; 10) $z = 3x^2 - 2x\sqrt{y} + y - 8x + 8$;

11) z= $x^2 + xy + y^2 - 3x - 6y$; 12) $z = \frac{1}{2}xy + (47 - x - y)\left(\frac{x}{3} + \frac{y}{4}\right)$;

13) $z = xy^2(1 - x - y)$; 14) $z = x^3 + y^3 - 15xy$.

J: 1) $z_{min} = z\left(\frac{2}{3},\frac{2}{3}\right) = -\frac{8}{27}$; 2) $z_{min} = -1$; 3) $z_{max}(4,4) = 12$;

4) $z_{min}\left(1,-\frac{1}{2}\right) = 0$;5) Ekstremum mavjud emas;6) $z_{min}(-2,0) = -\frac{2}{e}$;

7) $z_{min}(0{,}3) = 9$; 8) $z_{min}(2{,}2) = 0$; 9)$z_{min}(0{,}0) = 0$; 10)$z_{min}(2{,}4) = 0$; 11) $z_{min} = -9$; 12) $z_{max} = 282$;

13) $z_{max} = \frac{1}{64}$; 14) $z_{min} = -125$.

2. Quyidagi funksiyalarning ko'rsatilgan sohadagi eng katta va eng kichik qiymatlarini toping.

1) $z = x^2 + y^2 - xy + x + y$; $x \leq 0,\ y \leq 0,\ x + y \geq -3$;

2) $z = x^2 - xy + y^2 - 4x$; $x \geq 0,\ y \geq 0,\ 2x + 3y \leq 12$;

3) $z = x^2 + 3y^2 + x - y$; $x \leq 1,\ y \leq 1,\ x + y \geq 1$;

4) $z = x^2 + 2xy - 4x + 8y$; $x \geq 0,\ y \geq 0,\ x \leq 1, y \leq 2$;

5) $z = x^2 - 2y^2 + 4xy - 6x + 5$; $\square \geq 0,\ y \geq 0,\ x + y \leq 3$.

J:1) $z_{eng\ katta} = 6$, $z_{eng\ kichik} = -1$;2) $z_{eng\ kichik} = -\frac{16}{3}$, $z_{eng\ katta} = 16$;3)$z_{eng\ kichik} = 1$,$z_{eng\ katta} = 4$; 4)$z_{eng\ kichik} = -3$, $z_{eng\ katta} = 17$;
5) $z_{eng\ kichik} = -9$, $z_{eng\ katta} = 5$.

3.Quyidagi funksiyalarning shartli ekstremumlari topilsin.

1) $z = x + 2y$ni $x^2 + y^2 = 5$shartda;

2) $z = x^2 + y^2 - xy + x + y - 4$ ni $x + y = 3$shartda;

3) $z = xy$ ni $2x + 3y - 5 = 0$shartda;

4) $z = x^2 + y^2$ni $\frac{x}{4} + \frac{y}{3} = 1$shartda;

5) $z = 6 - 4x - 3y$ ni $x^2 + y^2 = 1$shartda;

6) $z = cos^2x + cos^2y$ ni $y - x = \frac{\pi}{4}$shartda;

7) $z = \frac{1}{x} + \frac{1}{y}$ ni$x + y = 2$ shartda;

8) $z = xy^2$ ni $x + 2y = 1$shartda.

J: 1) $z_{min} = -5$,$z_{max} = 5$;2) $z_{min} = -\frac{19}{4}$;3) $z_{max} = \frac{25}{24}$;
4) $z_{min} = \frac{144}{25}$; 5) $z_{max} = 11$, $z_{min} = 1$; 6) $z_{max} = \frac{2+\sqrt{2}}{2}$, $z_{min} = \frac{2-\sqrt{2}}{2}$; 7) $z_{min} = 2$; 8) $z_{min} = 0$, $z_{max} = \frac{1}{27}$.

4. $z = x^3 - 5x^2 - xy + y^2 + 10x + 5y - 4$ funksiyani $P_0(2,-1)$ nuqta atrofida Teylor formulasi bo'yicha yoying.

5. $z = x^3 - 2y^3 + 3xy$ funksiyani $P_0(1, 2)$ nuqta atrofida Teylor formulasi bo'yicha yoying.

6. $z = e^{x+y}$ funksiyani $P_0(1, -1)$ nuqta atrofida uchinchi tartibli hadlargacha Teylor formulasi bo'yicha yoying.

7. $z = e^x \sin y$ funksiyani $P_0(0, 0)$ nuqta atrofida Teylor formulasi bo'yicha yoying.

IX BOB. ODDIY DIFFERENSIAL TENGLAMALAR

1-§. Differensial tenglamalar bo'yicha asosiy tushunchalar. Birinchi tartibli tenglamalar. O'zgaruvchilari ajralgan va ajraladigan tenglamalar. Bir jinsli va bir jinsli tenglamaga keltiriladigan tenglamalar. To'la differensialli tenglamalar

Erkli o'zgaruvchi x, noma'lum funksiya y va uning turli tartibli hosilalari $y', y'', y''', \dots, y^{(n)}$ lar orasidagi bog'lanishni ifodalovchi tenglik ***oddiy differensial tenglama*** deb ataladi.

Noma'lum funksiyaning differensial tenglamada qatnashuvchi hosilalarining eng yuqori tartibi bu ***differensial tenglamaning*** tartibi deyiladi.

Umumiy holda $n-$ tartibli differensial tenglama

$$F(x, y, y', y'', y''', \dots., y^{(n)}) = 0 \quad (1)$$

ko'rinishda yoziladi.

Agar bu tenglamani $y^{(n)}$ ga nisbatan yechish mumkin bo'lsa, u holda uni $y^{(n)} = f(x, y, y', y'', y''', \dots., y^{(n-1)})$ (2)ko'rinishda yoziladi.

Agar biror $\varphi(x)$funksiya n marta differensiallanuvchi bo'lib, bu funksiya va uning hosilalari (1) yoki (2) tenglamaga qo'yilganda bu tenglama ayniyat ko'rinishiga kelsa, unda $\varphi(x)$ funksiya (1) yoki (2) ***tenglamaning yechimi*** deyiladi.

(1) yoki (2) tenglamaning yechimini topish uni ***integrallash***topilgan$y = \varphi(x)$ yechim esa uning ***integrali*** deyiladi.

$y' = f(x, y)$ (3) $-$ tenglamani birinchi tartibli hosilaga nisbatan yechilgan tenglama deyiladi. Uni $y' = \frac{dy}{dx}$ ekanligini e'tiborga olib, boshqacha $\frac{dy}{dx} = f(x, y)$ yoki $dy = f(x, y)dx$ ko'rinishda yozish mumkin.

$(3) -$ differensial tenglamani berilgan x_0 nuqtada y_0 qiymatni qabul qiluvchi $\mathrm{y} = y(x)$ yechimini topish masalasi ***Koshi masalasi*** deyiladi.

Bundagi shart $y(x_0) = y_0$ yoki $y|_{x=x_0} = y_0$ (4) ko'rinishida yoziladi va uni ***boshlang'ich shart*** deb ataladi.

Bitta ixtiyoriy o’zgarmas c soniga bog’liq $y=\varphi(x,c)$ funksiya birinchi tartibli (3)−differensial tenglamaning umumiy yechimi deyiladi, agar u quyidagi ikkita shartni qanoatlantirsa:

1) bu funksiya co’zgarmas sonning har bir qiymatida (3)–tenglamaning yechimi bo’ladi;

2) berilgan (4)−boshlang’ich shartda c o’zgarmasning shunday c_0 qiymati topiladiki, $y=\varphi(x,c_0)$ funksiya bu boshlang’ich shartni qanoatlantiradi.

Birinchitartibli differensial tenglamaning umumiy yechimi ko’pincha $F(y,x,c)=0$ ko’rinishida oshkormas holda topilishi mumkin.Bunday hollarda $F(y,x,c)\ =0$ ga differensial tenglamaning ***umumiy integrali*** deyiladi.

$y'=f(x)$ (5) –tenglamaeng sodda birinchi tartibli differensial tenglama bo’lib, uning yechimi $y=\int f(x)dx$dan iborat bo’ladi.

$$y'=-\frac{N(x)}{M(y)}\rightarrow\frac{dy}{dx}=-\frac{N(x)}{M(y)}\rightarrow M(y)dy+N(x)dx=0$$

(6) –tenglamaga ***o’zgaruvchilari ajralgan*** birinchi tartibli differensial tenglama deyiladi.Bu tenglama har ikkala qismini hadma-had integrallash orqali yechiladi, ya’ni

$$\int M(y)dy+\int N(x)dx=C \quad (6)$$

$$f_1(x)\cdot f_2(y)dx+f_3(x)\cdot f_4(y)dy=0 \quad (7)$$

tenglama ***o’zgaruvchilari ajraladigan*** differensial tenglama deyiladi.Bu tenglamani yechish uchun $f_2(y)\neq 0,\ f_3(x)\neq 0$ shartda tenglamani har ikkala qismini hadma –had $f_2(y)\cdot f_3(x)$ ifodaga bo’lamiz va natijada oldin ko’rib o’tilgan

$$\frac{f_1(x)}{f_3(x)}dx+\frac{f_4(y)}{f_2(y)}dy=0$$

tenglamaga ega bo’lamiz.Bundan esa (7) tenglamaning umumiy yechimi uchun

$$\int\frac{f_1(x)}{f_3(x)}dx+\int\frac{f_4(y)}{f_2(y)}dy=C$$

formulaga ega bo’lamiz.

Agar $f(x,y)$ funksiya ixtiyoriy o'zgarmas λ soni uchun $f(\lambda x, \lambda y) == f(x,y)$ shartni qanoatlantirsa, u holda bu funksiya va y o'zgaruvchilarga nisbatan ***bir jinsli funksiya*** deyiladi.

Agar $f(x,y)$ funksiya bir jinsli funksiya bo'lsa, u holda uni $f(x,y) = g\left(\frac{y}{x}\right)$ ko'rinishda yozish mumkin.

Agar birinchi tartibli $y' = f(x,y)$ tenglamada $f(x,y)$ bir jinsli funksiya bo'lsa, u holda uni ***bir jinsli differensial tenglama*** deyiladi. Bu tenglama $u = \frac{y}{x}$yokiy $= ux$ almashtirish bilan yechiladi.

$y' = \frac{dy}{dx} = \frac{ax+by+c}{a_1x+b_1y+c_1}$ $(*)$ ko'rinishdagi tenglamalar bir jinsli tenglamalarga keltiriladi. Bu yerda $c_1 \neq 0$, $c_2 \neq 0$. $x = x_1 + h$, y=y_1+k almashtirishqilamiz. U holda $\frac{dy}{dx} = \frac{dy_1}{dx_1}$ $(*)'$. x, y va$\frac{dy}{dx}$larning ifodalarini $(*)'$ tenglamaga qo'ysak,

$$\frac{dy_1}{dx_1} = \frac{ax_1 + by_1 + ah + bk + c}{a_1x_1 + b_1y_1 + a_1h + b_1k + c_1} (*)''$$

hosil bo'ladi. h va k ni $\begin{cases} ah + bk + c = 0 \\ a_1h + b_1k + c_1 = 0 \end{cases}$ tengliklar o'rinli bo'ladigan qilib tanlaymiz, ya'ni h va k ni yuqoridagi tenglamalar sistemasining yechimi kabi aniqlaymiz. Bu shartda $(*)''$ tenglama

$$\frac{dy_1}{dx_1} = \frac{ax_1 + by_1}{a_1x_1 + b_1y_1}$$

bir jinsli tenglamaga aylanadi. Bu tenglamani yechib, so'ngra $(*)$ formulaga muvofiq, yana$x$ va y larga o'tsak, $\frac{dy_1}{dx_1} = \frac{ax_1+by_1}{a_1x_1+b_1y_1}$ tenglama hosil bo'ladi. Bu bir jinsli tenglamadir. Uni yechish usuli bizga ma'lum.

Agar $M(x,y)dx + N(x,y)dy = 0$ (9) tenglamada $M(x,y)$ va $N(x,y)$ funksiyalar tekislikdagi biror D sohada uzluksiz, differensiallanuvchi bo'lib, ularning xususiy hosilalari uchun

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

shart bajarilib, bu hosilalar ham D sohada uzluksiz bo'lsa, unda tenglama ***to'liq differensialli*** tenglama deyiladi.

To’liq differensialli (9) tenglamaning chap tomonini

$$U(x,y) = \int_{x_0}^{x} M(t,y)dt + \int_{y_0}^{y} N(x_0,s)ds \text{ (9’)}$$

formula bilan topiladigan funksiyaning to’liq differensiali ko’rinishida yozish mumkin. Bu holda (9) tenglamaning umumiy integrali $U(x,y)=C$ tenglik bilan oshkormas ko’rinishda ifodalanadi.

$U(x,y)$ ni topish uchun y ni o’zgarmas deb qaraymiz. U holda $dy = 0$ ekanligidan $du = M(x,y)dx$ bo’ladi. Bu tenglikni$x$ bo’yicha integrallasak,

$$U = \int M(x,y)dx + \varphi(y)$$

Oxirgi tenglikni y bo’yicha differensiallaymiz va natijani $N(x,y)$ ga tenglaymiz. Chunki$\frac{\partial U}{\partial y} = N(x,y)$.

$$\int \frac{\partial M}{\partial y} dx + \varphi'(y) = N(x,y) \text{yoki } \varphi'(y) = N(x,y) - \int \frac{\partial M}{\partial y} dx$$

Buifodani y bo’yicha integrallab, $\varphi(y)$ni topamiz:

$$\varphi(y) = \int \left(N(x,y) - \int \frac{\partial M}{\partial y} dx \right) dy + c.$$

Demak, $U(x,y) = \int M(x,y)dx + \int \left(N(x,y) - \int \frac{\partial M}{\partial y} dx \right) dy + c.$

Bu ifodani ixtiyoriy o’zgarmasga tenglab, tenglamaning umumiy integralini hosil qilamiz.

Agar $M(x,y)dx + N(x,y)dy = 0$ tenglamada

$$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$$

bo’lsa, u holda tenglama to’la differensialli tenglama bo’lmaydi. Bu holda ba’zi shartlar bajarilganda shunday $\mu(x,y)$ funksiyani topish mumkinki uning uchun $\mu M dx + \mu N dy = dU$ bo’ladi. Bu $\mu(x,y)$ funksiya ***integrallovchi ko’paytuvchi*** deyiladi.

Quyidagi hollarda integrallovchi ko’paytuvchini topish oson:

1) $\frac{\frac{\partial M}{\partial y} - \frac{\partial \square}{\partial x}}{N} = \Phi(x)$ bo’lgandaln$\mu = \int \Phi(x)dx$ bo’ladi;

2) $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \Phi_1(y)$ bo’lganda, $\ln \mu = \int \Phi_1(y)dy$ bo’ladi.

1.Quyidagi funksiyalar berilgan differensial tenglamalarning yechimi ekanligi tekshirilsin.

1) $y = \sqrt{x}$funksiya $2yy' = 1$ tenglamani;

2) $ln\, xlny = c$ funksiya $ylnydx + xlnxdy = 0$ tenglamani;

3) S$= -t - \frac{1}{2}sin2t$funksiya$\frac{d^2s}{dt^2} + tgt \cdot \frac{ds}{dt} = sin2t$tenglamani;

4) y $= Ce^{-2x}$funksiya$y' + 2y = 0$tenglamani;

5) $y = C_1x + C_2x^2$funksiya$x^2y'' - 2xy' + 2y = 0$tenglamani;

6) $x^2 + 2xy = C$funksiya$(x + y)dx + xdy = 0$tenglamani.

2.Quyidagi o’zgaruvchilari ajraladigan differensial tenglamalarning umumiy integrallari topilsin.

1) $(x + 1)^3dy - (y - 2)^2$dx$= 0$; 6) $xyy' = 1 - x^2$;

2) $\sec^2 x\ \sec y dx = -\text{ctg} x\ \sin y dy$; 7) $yy' = \frac{1-2x}{y}$;

3) $(\sqrt{xy} + \sqrt{x})y' - y = 0$;8) $xy' + y = y^2$;

4) $yy' + x = 1$; 9) $y' + \sqrt{\frac{1-y^2}{1-x^2}} = 0$;

5) $(y + xy)dx + (x - xy)dy = 0$; 10) $y' = 10^{x+y}$.

J: 1) $-\frac{1}{y-2} + \frac{1}{2(x+1)^2} = C$; 2) $\text{tg}^2 x + \sin^2 y = C$;

3) $2\sqrt{y} + ln|y| - 2\sqrt{x} = C$; 4) $(x - 1)^2 + y^2 = C^2$;

5) $x - y + ln(xy) = C$; 6) $x^2 + y^2 = lnCx^2$;

7) $y = \sqrt[3]{C + 3x - 3x^2}$; 8) $Cx = \frac{y-1}{y}$;

9) $x\sqrt{1 - y^2} + y\sqrt{1 - x^2} = C$; 10) $10^x + 10^{-y} = C$.

3. Quyidagi o’zgaruvchilari ajratiladigan differensial tenglamalarning berilgan boshlang’ich shartlari qanoatlantiruvchi xususiy integrallari topilsin.

1) 2y$'\sqrt{x} = y$ tenglamani $x = 4$ bo’lganda $y = 1$ bo’ladigan;

2) $y' = (2y + 1)ctg\, x$tenglamani$x = \frac{\pi}{4}$bo’lganda$y = \frac{1}{2}$ bo’ladigan;

3) $x^2y' + y^2 = 0$tenglamani$x = -1$bo’lganda$y = 1$ bo’ladigan;

4) y$'= 2\sqrt{y}lnx$tenglamani$x = e$bo’lganda$y = 1$ bo’ladigan;

5) $(1+x^2)y' + y\sqrt{1+x^2} = xy$tenglamani$x = 0$bo'lganda$y$=1 bo'ladigan;

6) $y^2 + x^2y' = 0$tenglamani$x = -1$bo'lganda$y = 1$ bo'ladigan;

7) $2(1+e^x)yy' = e^x$tenglamani$x = 0$bo'lganda$y = 0$ bo'ladigan;

8) $(1+x^2)y^3dx - (y^2-1)x^3dy = 0$ tenglamani $x = 1$ bo'lganda $y = -1$ bo'ladigan.

J: 1) $y = e^{\sqrt{x-2}}$; 2) $y = 2sin^2x - \frac{1}{2}$; 3) $y = -x$; 4) $\sqrt{y} = xlnx - -x + 1$;

5) $y = \frac{\sqrt{1+x^2}}{x+\sqrt{1+x^2}}$; 6) $x + y = 0$; 7) $2ye^{y^2} = e^x + 1$;

8) $x^{-2} + y^{-2} = 2\left(1 + ln\left|\frac{x}{y}\right|\right)$.

4. Quyidagi bir jinsli tenglamalar yechilsin.

1) $(x^2+y^2)dx - 2xydy = 0$; 2) $y - xy' = yln\frac{x}{y}$;

3)$y - xy' = x + yy'$;4) $ydy + (x-2y)dx = 0$;

5) $ydx + (2\sqrt{xy} - x)dy$;6) $xy + y^2 = (2x^2 + xy)y'$;

7) $yy' = 2y - x$; 8) $x^2y' = y^2 + xy$.

J: 1) $y^2 = x^2 - cx$; 2) $y = xe^{c_1x}$; 3) $arctg\,\frac{y}{x} + lnc\sqrt{x^2+y^2}$=0; 4) $x = (y-x)\ln c(y-x)$; 5) $\sqrt{x} + \sqrt{y}\ln cy = 0$; 6) $y^2 = cxe^{-\frac{y}{x}}$;

7) $y - x = ce^{\frac{y}{y-x}}$; 8) $y = \frac{x}{c-lnx}$.

5. Quyidagi bir jinsli tenglamalarni berilgan boshlang'ich shartlar bo'yicha xususiy integrallari topilsin.

1) $y + \sqrt{x^2+y^2} - xy' = 0$tenglamani $x = 1$bo'lganda$y = 0$ bo'ladigan;

2) $xy' = y\left(1 + ln\frac{y}{x}\right)$ tenglamani $x = 1$ bo'lganda $y = \frac{1}{\sqrt{e}}$ bo'ladigan;

3) $y' = \frac{y^2}{x^2} - \frac{y}{x})$ tenglamani $x = -1$ bo'lganda $y = 1$ bo'ladigan;

4) $(y^2 - 3x^2)dy + 2xydx = 0$tenglamani $x = 1$bo'lganda$y$=-2 bo'ladigan;

5) $xy' - y = xtg\frac{y}{x}$tenglamani $x = 1$ bo'lganda $y = \frac{\pi}{2}$ bo'ladigan;

6) $xy' = xe^{\frac{y}{x}} + y$ tenglamani $x = 1$ bo'lganda $y = 0$ bo'ladigan.

J: 1) $y = \frac{x^2-1}{2}$; 2) $y = xe^{cx}$; 3) $y = \frac{2x}{1-cx^2}$; 4) $3y^3 = 8(x^2-y^2)$;

5) $y = x\arcsin x$;   6) $y = -x\, ln|1 - lnx|$.

6.Bir jinsli tenglamaga keltiriladigan quyidagi tenglamalar yechilsin.

1) $y' = -\frac{2x+y+1}{x+2y-1}$;   2) $(x - 2y - 3)y' + (2x + y - 1) = 0$;

3) $(x - y + 4)dy + (x + y - 2)dx = 0$; 4) $y' = \frac{2x-y+1}{x-2y+1}$;

5) $(2x - y + 4)dy + (x - 2y + 5)dx = 0$;   6) $y' = \frac{2x+y-1}{4x+2y+5}$;

J: 1) $x^2 + x\square + y^2 + x - y = c$; 2) $x^2 + xy - y^2 - x + 3y = c$;3) $x^2 + 2xy - y^2 - 4x + 8y = c$; 4) $x^2 - xy + y^2 + x - y$ =c;
5) $(x + y - 1)^2$=$c(x - y + 3)$;6) $10y - 5x + 7\ln|10x + 5y + 9|$=$c$.

7. Quyidagi to'liq differensialli differensial tenglamalar yechilsin.

1) $(4 - \frac{y^2}{x^2})dx + \frac{2y}{x}dy = 0$;   2) $3x^2e^ydx + (x^3e^y - 1)dy = 0$;

3) $e^{-y}dx + (1 - xe^{-y})dy = 0$;

4) $2xcos^2ydx + (2x - x^2sin2y)dy = 0$;

5) $(3x^2 + 6xy^2)dx + (\ 6x^2y + 4y^3)dy = 0$;

6) $(3x^2 + 2y)dx + (2x - -3)dy = 0$.

J: 1) $4x^2 + y^2 = cx$;2) $x^3e^y - y = c$;3) $y + xe^{-y} = c$;
4) $x^2cos^2y + y =$ c; 5) $x^2 + 3x^2y^2 + y^4 = c$; 6) $x^3 + 2xy - 3y$=c.

8. Quyidagi differensial tenglamalarning integrallovchi ko'paytuvchilari topilsin va ularyechilsin.

1) ($x^2 - y)dx + xdy$=0; 2) $2xtg\, y\, dx + (x^2 - 2\sin y)dy$=0;

3) ($e^{2x} - y^2)dx + ydy$=0; 4) $(1 + 3x^2 \sin y)dx - xctgydy$=0;

5) ($x^2 - 3y^2)dx + 2xydy = 0$;   6) $y^2dx + (yx - 1)\, dy = 0$.

J: 1) $\mu = \frac{1}{x^2}$, $x + \frac{y}{x}$=c;2) $\ln \mu = ln \cos y$, $x^2 \sin y + \frac{1}{2}\cos 2y$=$c$;

3) $\mu = e^{-2x}$, $y^2 = (c - 2x)e^{2x}$;   4) $\mu = \frac{1}{\sin y}$, $\frac{x}{\sin y} + x^3 = c$;

5) $\mu = \frac{1}{x^4}$, $y^2 = cx^3 + x^2$;   6) $\mu = \frac{1}{y}$,$xy - ln\, y = 0$.

2-§. Birinchi tartibli chiziqli tenglama. Bernulli tenglamasi

$y' + p(x)y = q(x)$ (1) ko'rinishidagi tenglama ***birinchi tartibli chiziqli differensial tenglama*** deb ataladi.

Bu yerda $p(x)$ va $q(x)$lar x ning uzluksiz funksiyalari yoki o'zgarmas sonlar. Agar $q(x) \neq 0$ bo'lsa, tenglama chiziqli ***bir jinsli bo'lmagantenglama,*** agar $q(x) = 0$ bo'lsa, tenglama ***bir jinsli tenglama*** deyiladi.

(1) tenglamani umumiy yechimini $y = u(x) \cdot v(x) = uv$ (Bernulli usuli) ko'rinishda izlanadi. U holda $y' = u'(x) \cdot v(x) + u(x) \cdot v'(x)$ bo'lib, berilgan tenglama $u'v + uv' + puv = q$ yoki $u'v + u(v' + +pv) = q$ ko'rinishga keladi.

$v' + pv = 0$ deb olib uni yechamiz:

$v' = -pv$, $\frac{v'}{v} = -p$, $\frac{dv}{v} = -pdx$, $\int \frac{dv}{v} = -\int pdx$, $\ln v = -\int pdx + +\ln c$, $ln\frac{v}{c} = -\int pdx$, $\frac{v}{c} = e^{-\int pdx}$, $v = ce^{-\int pdx}$. Buni o'rniga qo'yib, (c=1 deb oldik) $u'e^{-\int pdx} = q$, $u' = qe^{\int pdx}$, $u = \int qe^{\int pdx}dx + C_1$. Uning bu qiymatini $y = uv$ ga qo'yib, berilgan tenglamani umumiy yechimini topamiz: $y = uv = e^{-\int pdx}\left[\int qe^{\int pdx} + C_1\right]$.

(1) tenglamani umumiy yechimini $y' + py = 0$ tenglamani umumiy yechimidan foydalanib ham topish mumkin.

$y' + py = 0$, $y' = -py$, $\frac{dy}{dx} = -py$, $dy = -pydx$, $\frac{dy}{y} = -pdx$, $\int \frac{dy}{y} = -\int pdx + \ln c$, $\ln y = -\int pdx + \ln c$, $ln\frac{y}{c} = -\int pdx$, $\frac{y}{c} = e^{-\int pdx}$, $y = ce^{-\int pdx}$.

Bu yechimdagi c ni x ning funksiyasi deb, ya'ni $c(x)$ deb olsak u dastlabki tenglamaning yechim bo'lmasmikan deb $c(x)$ ni topamiz, ya'ni $y = c(x)\, e^{-\int pdx}$deb olamiz.

U holda $y' = c'(x)\, e^{-\int pdx} - \mathrm{c}(x)e^{-\int pdx} \cdot p$ bo'lib, berilgan tenglamadan $c'(x)\, e^{-\int pdx} - \mathrm{c}(x)e^{-\int pdx} \cdot p + p \cdot c(x)e^{-\int pdx}$=$q$yokic$'(x)e^{-\int pdx} = q$ tenglama hosil bo'ladi. Bundan esa $c'(x) = \frac{q}{e^{-\int pdx}} == qe^{\int pdx}$, $c(x) = \int qe^{\int pdx}dx + c_1$ kelib chiqadi.Demak berilgan tenglamaning yechimi quyidagicha,

$$y = c(x)e^{-\int pdx} = e^{-\int pdx}\left[\int qe^{\int pdx}dx + c_1\right]$$

$y' + py = qy^n$ ko'rinishdagi tenglama ***Bernulli tenglamasi*** deyiladi. Bu yerda $n \neq 0, n \neq 1, n = 0$ yoki $n = 1$ bo'lganda yuqorida ko'rib o'tilgan tenglamalar hosil bo'ladi. Shuning uchun $n \neq 0$ va $n \neq 1$. Bernulli tenglamasini yechish uchun tenglamani har ikkala qismini hadma–had y^n ga bo'lamiz. U holda berilgan tenglama

$$\frac{y'}{y^n} + p \cdot \frac{y}{y^n} = qy' \cdot y^{-n} + p \cdot y^{1-n} = q$$

ko'rinishga keladi. Endi $z = y^{1-n}$ belgilash qilamiz. U holda $z' = (1-n)y^{-n} \cdot y^1$ bo'lib, undan

$$y' = \frac{z'}{(1-n)y^{-n}}$$

kelib chiqadi. Belgilashlarni va y' ni o'rnilariga qo'yamiz:

$y^1 \cdot y^{-n} + p \cdot y^{1-n} = q, \frac{z\prime}{(1-n)y^{-n}} \cdot y^{-n} + p \cdot z = q,\ \ \frac{z'}{1-n} + pz = q,$

$z' + (1-n)pz = q(1-n)$ bu tenlama esa z ga nisbatan chiziqli tenglama bo'lib, uni yechish usuli bizga ma'lum. Bundan z ni keyin esa y ni topamiz.

Bernulli tenglamasini yangi z o'zgaruvchi kiritmay, chiziqli tenglama sifatida $y = uv$ o'rniga qo'yishdan foydalanib ham yechish mumkin.

1. Quyidagi chiziqli tenglamalar yechilsin.

1) $y' - \frac{3}{x}y = x$; 2) $y' - \frac{2}{x}y = \frac{e^{-x^2}}{x}$; 3) $y'\cos x - y\sin x = \sin 2x$;

4) $(2x+1)y' + y = x$; 5) $y' - y\,\mathrm{tg}x = \mathrm{ctg}x$;

6) $(x^2 - x)y' + y = x^2(2x-1)$; 7) $(1+x^2)y' - 2xy = (1+x^2)^2$.

J: 1) $y = cx^3 - x^2$; 2) $y = \frac{c - e^{-x^2}}{2x^2}$; 3) $y = \frac{c - cos2x}{2cosx}$;

4) $y = \frac{x-1}{3} + \frac{c}{\sqrt{2x+1}}$;5) $y = 1 + \frac{\ln C\,tg\frac{x}{2}}{\cos x}$; 6) $y = \frac{x(x^2 - x + c)}{x-1}$;

7) $y = (x+c)(1+x^2)$.

2. $(2x - y^2)y' = 2y$ tenglamaning $y|_{x=1} = 1$ boshlang'ich shartni qanoatlantiruvchi xususiy yechimini toping.

Ko'rsatma: Berilgan tenglamani $(2x-y^2)\frac{1}{x'}=2y$ deb olinadi va uni x ga nisbatan chiziqli tenglama deb yechiladi.J: $x=\frac{1}{2}y(3-y)$.

3. $y'-ytgx=sec\,x$ tenglamani $y|_{x=0}=0$ shartni qanoatlantiruvchi xususiy yechimini toping. J: $y=\frac{x}{cosx}$.

4. Bernulli tenglamasiga doir quyidagi tenglamalar yechilsin.

1) $y'x+y=-xy^2$; 2) $y'-xy=-y^3e^{-x^2}$;3) $y'+xy=xy^3$;

4) $y'+\frac{y}{x}=y^2\frac{\ln x}{x}$; 5) $y'+xy=x^3y^3$; 6) $x^2y^2y'+\ xy^3\ =1$.

J: 1) $y=\frac{1}{xlncx}$; 2) $y^2=\frac{e^{x^2}}{2x+c}$; 3) $y^2=\frac{1}{1+ce^{x^2}}$; 4) $y=\frac{1}{cx+1+lnx}$;

5) $y=\frac{1}{\sqrt{x^2+1+ce^x}}$; 6) $y=\sqrt[3]{\frac{3}{2x}+\frac{c}{x^3}}$.

5. $(1-x^2)y'-xy=xy^2$ tenglamani $y|_{x=0}=0{,}5$ boshlang'ich shartni qanoatlantiruvchi xususiy yechimini toping. J: $y=\frac{1}{3\sqrt{1-x^2}-1}$.

6. $y'-7y=e^{3x}y^2$ tenglamani $y|_{x=0}=2$ boshlang'ich shartni qanoatlantiruvchi xususiy yechimini toping. J: $y=\frac{10e^{7x}}{e^{10x}-6}$.

3-§. Yuqori tartibli differensial tenglamalar

n−tartibli differensial tenlamani odatda

$$F(x,y,y',y'',\dots\dots y^{(n)})=0 \quad (1)$$

yoki uni n−tartibli hosilaga nisbatan yechish mumkin bo'lsa,

$$y^{(n)}=\ F\big(x,y,y',y'',\dots\dots y^{(n-1)}\big)=0$$

ko'rinishda yozish mumkin. Bunday tenglamalar uchun birinchi tartibli tenglamaning yechimi haqidagi teoremaga o'xshash yechimning mavjudligi va yagonaligi haqidagi quyidagi teorema o'rinlidir.

Teorema. Agar

$$y^{(n)}=\ f\big(x,y,y',y'',\dots\dots y^{(n-1)}\big)=0$$

tenglamada $f\big(x,y,y',y'',\dots,y^{(n-1)}\big)=0$ funksiya va uning $y,y',y'',\dots,y^{(n-1)}$ argumentlari bo'yicha olingan xususiy hosilalari $x=x_0$, $y=y_0$, $y'=y_0'$, $y''=y_0''$, ... , $□^{(n-1)}=y_0{}^{(n-1)}$

qiymatlarni o’z ichiga olgan biror sohadagi uzluksiz funksiyalardan iborat bo’lsa, bu holda berilgan tenglamaning

$$\begin{cases} y|_{x=x_0} = y_0 \\ y'|_{x=x_0} = y'_{x_0} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ y^{(n-1)}|_{x=x_0} = y^{(n-1)}_{x_0} \end{cases} \qquad (2)$$

shartlarni qanoatlantiruvchi $y = y(x)$ yechimi mavjud va yagonadir. Bu yerdagi (2) shartlar boshlang’ich shartlar deb ataladi.

Ta’rif. n − tartibli differensial tenlamaning umumiy yechimi deb, n ta $c_1, c_2, \dots, c_n$ o’zgarmas miqdorga bog’liq bo’lgan

$$y = \varphi(x, c_1, c_2, \dots, c_n)$$

funksiyaga aytiladi va bu funksiya;

a) $c_1, c_2, \dots, c_n$ ixtiyoriy o’zgarmas miqdorlarning har qanday qiymatlarida ham tenglamani qanoatlantiradi;

b) berilgan $\begin{cases} y|_{x=x_0} = y_0 \\ y'|_{x=x_0} = y'_{x_0} \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ y^{(n-1)}|_{x=x_0} = y^{(n-1)}_{x_0} \end{cases}$

boshlang’ich shartlarda $c_1, c_2, \dots, c_n$ o’zgarmas miqdorlarni shunday tanlab olish mumkinki, $y = \varphi(x, c_1, c_2, \dots, c_n)$ funksiya berilgan boshlang’ich shartlarni qanoatlantiradi.

Umumiy yechimni oshkormas holda aniqlovchi $\varphi(x, y, c_1, c_2, \dots, c_n) = 0$ ko’rinishdagi funksiya differensial tenglamaning ***umumiy integrali*** deyiladi.

Umumiy yechimdan $c_1, c_2, \dots, c_n$ larning tayin qiymatlarida hosil bo’ladigan har qanday funksiya ***xususiy yechim*** deyiladi.

Xususiy yechimning grafigi berilgan differensial tenglamaning ***integral egri chizig’i*** deyiladi.

Eng sodda n-tartibli differensial tenglama

$$y^{(n)} = f(x) \quad (3)$$

ko’rinishda bo’ladi. Bu tenglama n marta ketma−ket integrallash orqali yechiladi.

$y^{(n)} = f(x, y^{(k)} \dots\dots, y^{(n-1)})$ (4) ko'rinishdagi tenglamada noma'lum funksiya va uning $(k-1)-$ tartibgacha hosilalari qatnashmaydi. Bunday tenglamaning tartibini $y^{(k)} = p(x)$ o'rniga qo'yish bilan pasaytiriladi.

$y^{(n)} = f\left(y, y', y'', \dots\dots y^{(n-1)}\right) = 0$ (5) ko'rinishdagi tenglamada x erkli o'zgaruvchi qatnashmaydi. Bunday tenglamaning tartibini $y' = p(y)$ o'rniga qo'yish orqali pasaytiriladi.

y″$= f(x, y)$ (6) ko'rinishdagi tenglama noma'lum y funksiyani oshkor holda o'z ichiga olmaydi. Bu tenglama $y' = p$ o'rniga qo'yish orqali birinchi tartibli tenglamaga keltiriladi.

y″$= f(y, y')$ (7) ko'rinishdagi tenglama x erkli o'zgaruvchini oshkor holda o'z ichiga olmaydi. Bu tenglamada $y' = p$ deb olib, va y″$= p'p == \frac{dp}{dy} \cdot p$ dan foydalanib, birinchi tartibli tenglamaga keltiriladi va yechiladi.

1.Quyidagi tenglamalar yechilsin.

1) $y''' = e^{2x}$; 2) $y''' = \frac{6}{x^2}$; 3) $y'' = x\sin x$; 4) $y^{IV} = \square$;

5) $y'' = 2\sin x \cdot \cos^2 x - \sin^3 x$; 6)y″$= arctg\, x$; 7) $y'' = \frac{1}{1+x^2}$;

8) $y'' = \ln x$.

J : 1) y $= \frac{e^{2x}}{8} + c_1 x^2 + c_2 x + c_3$; 2) $y = -6x \ln x + 6x + c_1 \frac{x^3}{6} + +c_2 x + c_3$; 3) y $= c_1 x + c_2 - x \sin x - 2\cos x$; 4) $y = \frac{x^5}{120} + c_1 \frac{x^3}{6} + +c_2 \frac{x^2}{2} + c_3 x + c_4$;5) y $= \frac{1}{3} sin^3 x$ + $c_1 x + c_2$; 6) $y = \frac{arctg\, x}{2}(x^2 - 1) - -\frac{x}{2}\ln(1 + x^2) + c_1 x + c_2$;7) y $= c_1 x + x\, arctg\, x - ln\sqrt{1 + x^2} + c_2$; 8) $y = \frac{x^2}{2}\left(\ln x - \frac{3}{2}\right) + c_1 x + c_2$.

2.Quyidagi tenglamalarni berilgan boshlang'ich shartlarni qanoatlantiruvchi xususiy yechimlari topilsin.

1) y″$= xe^{-x}$, y $\Big|_{x=0} = 1$, y′ $\Big|_{x=0} = 0$;

2) y‴$= xe^{-x}$, y $\Big|_{x=0} = 0$, y′ $\Big|_{x=0} = 2$,y″ $\Big|_{x=0} = 2$;

3) y‴$= x\sin x$, y $\Big|_{x=0} = 0$, y′ $\Big|_{x=0} = 0$,y″ $\Big|_{x=0} = 2$;

4) $y''= 3x^2$, $y\,\Big|_{x=0} = 2, y'\,\Big|_{x=0} = 1$;

5) $y'' = \frac{1}{cos^2x}$, $x = \frac{\pi}{4}$ bo'lganda$y = \frac{ln2}{2}, y' = 1$;

6) $y''= 4\cos 2x y\,\Big|_{x=0} = 0, y'\,\Big|_{x=0} = 0.$

J: 1) $y = xe^{-x} + 2e^{-x} + x - 1$; 2) $y = -(x+3)e^{-x} + \frac{3}{2}x^2 + 3$;

3) $y = x\cos x - 3\sin x + x^2 + 2x$; 4) $4y = x^4 + 4x + 8$;

5) $y = c_1x + c_2 - \ln\cos x$; 6) $y = 1 - \cos 2x$.

3. Quyidagi ($F(x, y', y'') = 0$ ko'rinishidagi) tenglamalar yechilsin.

1) $y'' = \frac{y'}{x} ln\frac{y'}{x}$; 2) $(1-x^2)y'' - xy' = 2$; 3) $y'' = \frac{y'}{x} + x$;

4) $x^3y'' + x^2y' = 1$; 5) $(1+x^2)y'' + 2xy' = x^3$;

6) $y''tgx = y' + 1$; 7) $xy'' = y'ln\frac{y'}{x}$;

8)$(1+x^2)y'' + 1 + (y')^2 = 0$.

J: 1) $y = \frac{x}{c_1}e^{c_1x+1} - \frac{1}{(c_1)^2}e^{c_1x+1} + c_2$;2) $y = (\arcsin x)^2 + +c_1\arcsin x + c_2$;

3) $y = \frac{x^3}{3} + c_1x^2 + c_2$; 4) $y = \frac{1}{x} + c_1lnx + c_2$; 5) $y = \frac{x^3}{12} - \frac{x}{4} + c_1arc\ tg\ x + c_2$;

6) $y = c_2 - c_1\cos x - x$;

7) $y = \frac{1}{c_1}xe^{1+c_1x} - \frac{1}{(c_1)^2}e^{1+c_1x} + c_2$;

8) $y = (1 + c_1^{-2})\ln(1 + c_1x) + c_1^{-1}x + c_2$.

4. $y'' - \frac{y'}{x-1} = x(x-1)$ tenglamani $y(2) = 1,\ y'(2) = -1$ shartlarni qanoatlantiruvchi yechimi topilsin.

J: $y = \frac{1}{24}(3x^4 - 4x^3 - 36x^2 + 72x + 8)$.

5. $(y''x - y')y' = x^3$ tenglamani $y(1) = 1$, $y'(1) = 0$ shartlarni qanoatlantiruvchi yechimi topilsin.

J: $225(y-1)^2 = 8(x-1)^2(3x+2)^2$.

6.Quyidagi ($F(y, y', y'') = 0$ ko'rinishdagi) tenglamalar yechilsin.

1) $y'' = \frac{1+y'^2}{y}$; 2) $y''(2y+3) - 2y'^2 = 0$; 3) $yy'' + y'^2 = 0$;

4) $y'' + 2y(y')^3 = 0$; 5) $2yy'' = 1 + (y')^2$; 6) $y''y^3 = 1$;

7) $yy'' - (y')^2 = y^2 \ln y$; 8) $y'' + ay = b$.

J:1) $\frac{1}{c_1} ln\left|c_1 y + \sqrt{c^2y^2 - 1}\right| = \pm(x + c_2)$;2) $\frac{1}{2} ln|2y + 3| = c_1x + c_2$;

3) $y^2 = c_1x + c_2$; 4) $y^3 + c_1y + c_2 = 3x$;

5) $(c_1x + c_2)^2 = 4(c_1y - 1)$; 6) $c_1y^2 = 1 + (c_1x + c_2)^2$;

7) $\ln y = c_1e^x + c_2e^{-x}$; 8) $ay = b + c_1\sin(x\sqrt{a} + c_2)$

7. Quyidagi($F(y, y', y'') = 0$ ko'rinishdagi) tenglamalarni berilgan boshlang'ich shartlarni qanoatlantiruvchi yechimlari topilsin.

1) $yy'' - (y')^2 = 0$, $y(0) = 1$, $y'(0) = 2$;

2) $y'' = y'e^y$, $y(0) = 0$, $y'(0) = 1$;

3) $yy'' = (y')^2 - (y')^3$, $y(1) = 1$, $y'(1) = -1$;

4) $y^3y'' + 1 = 0$, $y(1) = 1$, $y'(1) = 0$;

5) $yy'' + (y')^2 = 1$, $y(0) = 1$, $y'(0) = -1$;

6) $3yy'y'' = 1 + (y')^2$, $y(0) = 1$, $y'(0) = 0$;

J: 1)$y = e^{2x}$; 2) $y = -ln|1 - x|$; 3) $y - x = 2ln|y|$;

4) $y^2 + x^2 = 2x$; 5) $x + y - 1 = 0$; 6) $2x = 3(y - 1)^{\frac{2}{3}}$.

4-§. Bir jinsli chiziqli tenglamalar

Ta'rif. Agar n– tartiblidifferensial tenglama noma'lum y funksiya va uning $y', y'', \dots, y^{(n-1)}, y^n$ hosilalariga nisbatan, birinchi darajali bo'lsa, bunday tenglama chiziqli differensial tenglama deyiladi va u

$$y^n + a_1y^{(n-1)} + \cdots + a_nu = f(x)(1)$$

ko'rinishda yoziladi. Bu yerda $a_1, a_2, \dots, a_n$ va $f(x)$ lar x ning ma'lum funksiyalari yoki o'zgarmas sonlar. (1) tenglamaning o'ng tomonida turgan $f(x)$funksiya ***tenglamaning o'ng tomoni*** deyiladi.

Agar $f(x) \neq 0$ bo'lsa, u holda tenglama ***bir jinsli bo'lmagan chiziqli tenglama*** yoki ***o'ng tomonli tenglama*** deyiladi.

Agar $f(x) \equiv 0$bo'lsa, u holda tenglama

$$y^n + a_1y^{(n-1)} + \cdots + a_ny = 0 \quad (2)$$

ko'rinishida bo'ladi va ***bir jinsli chiziqli*** yoki ***o'ng tomonsiz tenglama*** deyiladi.

$$y'' + a_1y' + a_2y = 0 \quad (3)$$

tenglama ***2-tartibli bir jinsli chiziqli tenglama*** deyiladi. Bu tenglama uchun quyidagi teoremalar o'rinlidir.

Teorema. Agar $y_1, va\ y_2$ 2-tartibli(3) tenglamaning ikkita xususiy yechimi bo'lsa, u holda $y_1 + \ y_2$ ham bu tenglamaning yechimi bo'ladi.

Teorema. Agar y_1(3) tenglamaning yechimi bo'lib, c ixtiyoriy o'zgarmas son bo'lsa,u holda cy_1 ham bu tenglamaning yechimi bo'ladi.

Ta'rif. Agar $[a, b]$ kesmada (3) tenglama y_1 va y_2 yechimining nisbati o'zgarmas miqdorga teng bo'lmasa, ya'ni

$$\frac{y_1}{y_2} \neq c$$

bo'lsa, u holda y_1 va y_2 yechimlar $[a, b]$ kesmada chiziqli bog'liq bo'lmagan yechimlar deyiladi. Aks holda chiziqli bog'liq yechimlar deyiladi.

Ta'rif. Agar y_1 va y_2 lar x ning funksiyasi bo'lsa, u holda

$$W(y_1, y_2) = \begin{vmatrix} y_1 y_2 \\ y_1' y_2' \end{vmatrix} = y_1 y'_2 - y'_1 y_2$$

determinantVronskiy determinanti yoki berilgan funksiyalarning vronskiani deyiladi (Yu. Vronskiy (1778-1854) polyak matematigi).

Teorema. Agar y_1 vay_2 (3) tenglamaning ikkita chiziqli erkli yechimi bo'lsa, u holda

$$y = c_1 y_1 + c_2 y_2$$

(3) tenglamaning umumiy yechimi bo'ladi.

Teorema. Agar ikkinchi tartibli bir jinsli chiziqli tenglamaning bitta xususiy yechimi ma'lum bo'lsa, u holda umumiy yechimni topish funksiyalarni integrallashga keltiriladi.

Agary_1 (3) tenglamaning biror xususiy yechimi bo'lsa, u holda u bilan chiziqli erkli y_2 xususiy yechimi

$$y_2 = y_1 \int \frac{e^{-\int a_1 dx}}{{y_1}^2} dx \qquad (4)$$

formuladan topiladi.

Teorema. Agar $y_1(x)$ (3) tenglamaning xususiy yechimi bo'lsa, u holday $=$ $y_1 \cdot z$ formula bilan z o'zgaruvchini kiritib,tenglamaning tartibini bittaga pasaytirish mumkin.

Teorema. Agar y_1 va y_2 funksiyalar $[a, b]$ kesmada chiziqli bog'liq bo'lsa, u holda bu kesmada Vronksiy determinanti aynan nolga teng bo'ladi.

Teorema. Agar bir jinsli chiziqli (3) tenglama ning y_1 va y_2 yechimlari uchun tuzilgan $W(y_1, y_2)$ Vronskiy determinanti tenglamaning koeffitsientlari uzluksiz bo'lgan $[a, b]$ kesmadagi biror $x = x_0$ qiymatida nolga teng bo'lmasa, u holda u x ning bu kesmadagi hech bir qiymatida nolga aylanmaydi.

Teorema. Agar (3) tenglamaning y_1 va y_2 yechimlari $[a, b]$ kesmada chiziqli erkli bo'lsa, bu yechimlardan tuzilgan W Vronksiy determinanti ko'rsatilgan kesmaning hech bir nuqtasida nolga aylanmaydi.

Ikkinchi tartibli chiziqli bir jinsli (3) tenglamaning (a, b) oraliqda chiziqli bog'liq bo'lmagan yechimlari to'plamiga bu tenglamaning ***fundamental yechimlar sistemasi*** deyiladi.

1. $y_1 = shx$ va $y_2 = chx$ lar $y'' - y = 0$ tenglamaning xususiy yechimlari ekanligi tekshirilsin va ular fundamental sistema hosil qilishi ko'rsatilsin.

2. Xususiy yechimlari. $y_1 = \frac{1}{\sqrt{x}} sinx,\ \ y_2 = \frac{1}{\sqrt{x}} cosx$ bo'lgan $y'' + \frac{1}{x} y' + \left(1 - \frac{1}{4x^2}\right) y = 0 \ \ (x \neq 0)$ tenglamaning umumiy yechimini yozish mumkinmi?

3.Quyida berilgan funksiyalar chiziqli bog'liq yoki chiziqli bog'liq emasligi tekshirilsin.

1) $y_1 = x + 1,\ \ \ y_2 = 2x + 1,\ \ y_3 = x + 2;$

2) $y_1 = 2\,x^2 + 1,\ \ y_2 = \ x^2 - 1,\ \ y_3 = x + 2;$

3) $y_1 = \sqrt{x},\ \ y_2 = \sqrt{x + a},\ \ y_3 = \sqrt{x + 2a};$

4) $y_1 = 4\,x^2,\ \ y_2 = \ x^3;$

5) $y_1 = \sin x,\ \ \ y_2 = \cos x;$

6) $y_1 = x,\ \ \ y_2 = |x|;$

7) $y_1 = \ x^5,\ \ \ y_2 = \ x^5 + 1;$

8) $y_1 = \sin^2 x, y_2 = \cos x$.

4. $y'' - \frac{3x^2}{x^3+1}y' + \frac{3x}{x^3+1}y = 0$ tenglama berilgan: 1) $y_1 = x$ bu tenglamaning yechimi bo'lishi ko'rsatilsin; 2) umumiy yechim topilsin.

Javob: $y = c_1 x + c_2(\frac{x^2}{2} - 1)$.

5. $y'' - \frac{2}{x}y' + \frac{2}{x^2}y = 0$ tenglama berilgan: 1) $y_1 = x^2$ tenglamaning yechimi ekanligi ko'rsatilsin; 2) umumiy yechim topilsin. Javob: $y = c_1 x^2 + c_2 x$.

6. $y'' + \frac{x}{1-x}y' - \frac{1}{1-x}y = 0$ tenglama berilgan: 1) $y_1 = e^x$ tenglamaning yechimi ekanligi ko'rsatilsin; 2) umumiy yechim topilsin. Javob: $y = c_1 e^2 + c_2 x$.

7. $y'' + \frac{2}{x}y' + y = 0$ tenglama berilgan: 1) $y_1 = \frac{\sin x}{x}$ tenglamaning yechimi ekanligi ko'rsatilsin; 2) umumiy yechim topilsin.

Javob: $y = c_2 \frac{\sin x}{x} - c_1 \frac{\cos x}{x}$.

5-§. O'zgarmas koeffitsientli ikkinchi tartibli bir jinsli chiziqli tenglamalar

$$y'' + py' + qy = 0 \qquad (1)$$

tenglamaga o'zgarmas koeffitsientli ikkinchi tartibli bir jinsli chiziqli tenglama deyiladi, bu yerda p va q o'zgarmas sonlar. Tenglamani umumiy integralini topish uchun uning ikkita chiziqli erkli xususiy yechimini topish yetarlidir. Xususiy yechimlarni $y = e^{kx}$ $(k = c$) (2) ko'rinishda izlaymiz. Bu holda$y' = ke^{kx}, y'' = k^2 e^{kx}$bo'ladi. Bularni (1) tenglamaga qo'yamiz. U holda (1) tenglama

$$e^{kx}(k^2 + pk + q) = 0$$

ko'rinishga keladi. Bundan $k^2 + pk + q = 0$ (3) tenglama hosil bo'ladi. Bu tenglama (1) tenglamaning harakteristik tenglamasi deyiladi. U kvadrat tenglama bo'lib, uning ildizlari

$$k_1 = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q}; k_1 = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q}$$

lardan iborat bo'ladi. Bunda quyidagi hollar bo'lishi mumkin:

I. $k_1$va k_2 –haqiqiy va bir-biriga teng bo'lmagan sonlar;

II. k_1 va k_2 –haqiqiy va bir-biriga teng sonlar;

III. k_1 va $□_2$ –kompleks sonlar.

Birinchi holda $y_1 = e^{k_1x}$ va $y_2 = e^{k_2x}$ lar xususiy yechimlar bo'lib, umumiy yechim (integral) $y = c_1e^{k_1x} + c_2e^{k_2x}$ bo'ladi.

Ikkinchi holda $y_1 = e^{kx}$ va $y_2 = xe^{kx}$ lar xususiy yechimlar bo'lib, umumiy yechim (integral) $y = □_1e^{kx} + c_2xe^{kx}$ yoki $y = e^{kx}(c_1 + c_2x)$ dan iborat bo'ladi.

Uchinchi holda umumiy yechim (integral) $y = c_1cos\beta x + c_2sin\beta x$ bo'ladi.

1. Quyidagi (xarakteristik tenglamaning ildizlari haqiqiy va har xil) tenglamalar yechilsin:

1) $y" + y' - 2y = 0;$ 2) $y" - 7y' + 6y = 0;$

3) $y" - 5y' + 6y = 0;$ 4) $y" - 5y' + 4y = 0;$

5) $y" + 13y' + 42y = 0;$ 6) $y" + 4y' + 3y = 0;$

7) $y" - 4y' + 3y = 0;$ 8) $y" + 3y' + 2y = 0$

Javob: 1) $y = c_1e^x + c_2e^{-2x}$; 2) $y = c_1e^{6x} + c_2e^x$;

3) $y = c_1e^{2x} + c_2e^{3x}$; 4) $y = c_1e^x + c_2e^{4x}$;

5) $y = c_1e^{-6x} + c_2e^{7x}$; 6) $y = c_1e^{-x} + c_2e^{-3x}$;

7) $y = c_1e^x + c_2e^{3x}$; 8) $y = c_1e^{-x} + c_2e^{-2x}$.

2. $y" - 4y' + 3y = 0$ tenglamani $y(0) = 6$, $y'(0) = 10$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $y = 4e^x + 2e^{3x}$.

3. $y" + 5y' + 6y = 0$ tenglamani $y(0) = 1$, $y'(0) = -6$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $y = 4e^{-3x} - 3e^{-2x}$.

4. Quyidagi (xarakteristik tenglamaning ildizlari haqiqiy va o'zaro teng) tenglamalar yechilsin:

1) $y" - 4y' + 4y = 0;$ 2) $y" - 2y' + y = 0;$

3) $y" + 4y' + 4y = 0;$ 4) $y" - 6y' + 9y = 0;$

5) $4y" - 20y' + 25y = 0.$

Javob: 1) $y = (c_1 + c_2x)e^{2x}$;2) $y = (c_1 + c_2x)e^x$;

3) $y = (c_1 + c_2x)e^{2x}$;4) $y = (c_1 + c_2x)e^{3x}$;

5) $y = (c_1 + c_2x)e^{2,5x}$.

5. $y" - 10y' + 25y = 0$ tenglamani $y(0) = 0,\ y'(0) = 1$ shartlarni qanoatlantiruvchi yechimi topilsin.Javob: $y = xe^{5x}$.

6. $4y" + 4y' + y = 0$ tenglamani $y(0) = 2,\ y'(0) = 2$ shartlarni qanoatlantiruvchi yechimi topilsin.Javob: $y = e^{-\frac{x}{2}}(2 + x)$.

7. $y" - 2y' + y = 0$ tenglamani $y(2) = 1,\ y'(2) = -2$ shartlarni qanoatlantiruvchi yechimi topilsin.Javob: $y = (7 - 3x)e^{x-2}$.

8. Quyidagi (harakteristik tenglamaning ildizlari kompleks sonlar) tenglamalar yechilsin:

1) $y'' + 8y' + 25y = 0$; 2) $y" + y = 0$;

3) $y" - 4y' + 5y = 0$; 4) $y" + 4y' + 8y = 0$;

5) $y" - 4y' + 13y = 0$; 6) $y" + 25y = 0$;

7) $y" + 6y' + 13y = 0$; 1) $4y" - 8y' + 5y = 0$;

Javob:1) $y = e^{-4x}(c_1cos3x + c_2sin3x)$; 2) $y = c_1cosx + c_2sinx$;

3) $y = e^{2x}(c_1cosx + c_2sinx)$; 4) $y = e^{-2x}(c_1cos2x + c_2sin2x)$;

5) $y = e^{2x}(c_1cos3x + c_2sin3x)$; 6) $y = c_1cos5x + c_2sin5x$;

7) $y = e^{-3x}(c_1cos3x + c_2sin3x)$; 8) $y = e^{x}\left(c_1cos\frac{x}{2} + c_2sin\frac{x}{2}\right)$.

9. $y" - 2y' + 10y = 0$ tenglamani $y\left(\frac{\pi}{6}\right) = 0$, $y'\left(\frac{\pi}{6}\right) = e^{\frac{\pi}{6}}$ shartlarni qanoatlantiruvchi yechimi topilsin.Javob: $y = -\frac{1}{3}e^{x}cos3x$.

10. $9y" + y = 0$ tenglamani $y\left(\frac{3\pi}{2}\right) = 2$, $y'\left(\frac{3\pi}{2}\right) = 0$ shartlarni qanoatlantiruvchi yechimi topilsin.Javob: $y = 2sin\frac{x}{3}$.

11. $y" + 9y = 0$ tenglamani $y(0) = 0$, $y'\left(\frac{\pi}{4}\right) = 1$ shartlarni qanoatlantiruvchi yechimi topilsin.Javob: $y = \sqrt{2}cos3x$.

6-§. n-tartibli o’zgarmas koeffitsientli bir jinsli chiziqli tenglamalar

$$y^{(n)} + a_1y^{(n-1)} + \ ... + a_ny = 0 \qquad (1)$$

tenglamaga n-tartibli o'zgarmas koeffitsientli bir jinsli chiziqli tenglama deyiladi. Bu yerda $a_1, a_2, \dots, a_n$ lar o'zgarmas sonlar.

Ta'rif. Agar $[a, b]$ kesmadagi x ning barcha qiymatlari uchun

$$\varphi_n(x) = A_1\varphi_1(x) + A_2\varphi_2(x) + \ \dots + A_{n-1}\varphi_{n-1}(x)$$

tenglik o'rinli bo'lsa, unda $\varphi_n(x)$ funksiya $\varphi_1(x)$, $\varphi_2(x)$, …, $\varphi_{n-1}(x)$, $\varphi_n(x)$ funksiyalar orqali ***chiziqli bog'liq funksiyalar***deyiladi. Bu yerda A_1, A_2, …,A_n lar hammasi bir vaqtda nolga teng bo'lmaydigan o'zgarmas sonlar.

Ta'rif. Agar n ta $\varphi_1(x)$, $\varphi_2(x)$, …, $\varphi_{n-1}(x)$, $\varphi_n(x)$ funksiyalarning hech biri qolganlari orqali chiziqli ifoda etilmasa, u funksiyalar ***chiziqli erkli funksiyalar*** deb ataladi.

Agar $\varphi_1(x)$, $\varphi_2(x)$, …, $\varphi_{n-1}(x)$ funksiyalar chiziqli bog'liq bo'lsa, u holda hammasi ham nolga teng bo'lmagan shunday $c_1, c_2, \dots, c_n$ sonlar topiladiki, $[a, b]$ kesmadagi x ning hamma qiymatlari uchun

$$c_1\varphi_1(x) + c_2\varphi_2(x) + \ \dots + c_n\varphi_n(x) \equiv 0$$

ayniyat bajariladi.

Teorema. Agar $y_1, y_2, \dots, y_n$ funksiyalar (1) tenglamaning chiziqli erkli yechimlari bo'lsa, u holda $y = c_1y_1 + c_2y_2 + \ \dots + c_ny_n$ (2) uning umumiy yechimi bo'ladi, bunda $c_1, c_2, \dots, c_n$ ixtiyoriy o'zgarmas sonlar.

Agar (1) tenglamaning koeffitsiyentlari o'zgarmas sonlar bo'lsa, bu holda uning umumiy yechimi ikkinchi tartibli tenglamaning umumiy yechimini topgandek topiladi.

1) harakteristik tenglamani tuzamiz:

$$k^n + a_1k^{n-1} + a_2k^{n-2} + \ \dots + a_n = 0$$

2) harakteristik tenglamaning

$$k_1, k_2, \dots, k_n$$

ildizlarini topamiz.

3) Quyidagilarga asoslanib ildizlarning xarakteriga ko'ra chiziqli erkli xususiy yechimlarni topamiz:

a) Har bir karrali k ildizga e^{kx} xususiy yechim mos keladi;

b) Har bir juft ikkita $k^{(1)} = \alpha + \beta i$ va $k^{(2)} = \alpha - \beta i$ qo'shma kompleks bir karrali ildizlarga ikkita $e^{\alpha x} cos\beta x$ va $e^{\alpha x} sin\beta x$ xususiy yechimlar to'g'ri keladi;

c) Har bir r karrali haqiqiy k ildizga r ta chiziqli erkli

$$e^{kx}, xe^{kx}, x^2 e^{kx}, \dots, x^{r-1} e^{kx}$$

xususiy yechim to'g'ri keladi;

d) Har bir μ karrali juft $k^{(1)} = \alpha + \beta i$ va $k^{(2)} = \alpha - \beta i$ qo'shma kompleks ildizga 2μ ta

$$e^{\alpha x} cos\beta x, xe^{\alpha x} cos\beta x, \dots, x^{\mu-1} e^{\alpha x} cos\beta x,$$

$$e^{\alpha x} sin\beta x, xe^{\alpha x} sin\beta x, \dots, x^{\mu-1} e^{\alpha x} sin\beta x$$

xususiy yechimlar to'g'ri keladi;

4) n ta chiziqli erkli $y_1, y_2, \dots, y_n$ xususiy yechimlarni topgandan so'ng berilgan chiziqli tenglamaning umumiy yechimini tuzamiz:

$$y = c_1 y_1 + c_2 y_2 + \dots + c_n y_n$$

1. Quyidagi funksiyalar chiziqli bog'liq yoki chiziqli erkli ekanligi aniqlansin:

1) $y_1 = e^x, y_2 = e^{2x}, y_3 = 3e^x$;

2) $y_1 = 1,\ y_2 = x,\ \ y_3 = x^2$;

3) $y_1 = e^{k_1 x}, y_2 = e^{k_2 x}, \dots, y_n = e^{k_n x}, \dots$.

Javob: 1) chiziqli bog'liq; 2) chiziqli erkli; 3) chiziqli erkli.

2. Quyidagi tenglamalar yechilsin:

1) $y^{IV} - y = 0$;

2) $y^{IV} - 13y^{II} + 36y = 0$;

3) $y^{IV} - 2y^{III} + y^{II} = 0$;

4) $y^{IV} + 5y^{II} + 4y = 0$;

5) $y^{V} - 16y^{I} = 0$;

6) $y^{IV} - 8y^{II} + 16y = 0$;

7) $y^{III} - y^{I} = 0;\ y(0) = 3,\ y^{I}(0) = -1,\ y^{II}(0) = 1$.

8) $y^{V} = y^{I}; y(0) = 0, y^{I}(0) = 1, y^{II}(0) = 0, y^{III}(0) = 1, y^{IV}(0) = 2$.

Javoblar:

1) $y = c_1 e^x + c_2 e^{-x} + c_3 cosx + c_4 sinx$;

2) $y = c_1e^{3x} + c_2e^{-3x} + c_3e^{2x} + c_4e^{-2x}$

3) $y = c_1 + c_2x + c_3e^x + c_4xe^x$;

4) $y = c_1cosx + c_2sinx + c_3cos2x + c_4sin2x$;

5) $y = c_1 + c_2e^{2x} + c_3e^{-2x} + c_4cos2x + c_5sin2x$;

6) $y = e^{2x}(c_1 + c_2x) + e^{-2x}(c_3 + c_4x)$;

7) $y = 2 + e^{-x}$;

8) $y = e^x + cosx - 2$;

7-§. Ikkinchi tartibli o'zgarmas koeffitsientli chiziqli bir jinslimas chiziqli tenglamalar

$$y'' + a_1y' + a_2y = f(x) \qquad (1)$$

tenglama ikkinchi tartibli o'zgarmas koeffitsientli chiziqli bir jinslimas tenglamadir. Bu tenglamaning umumiy yechimi quyidagi teorema bilan aniqlanadi:

Teorema. (1) tenglamaning umumiy yechimi bu tenglamaning biror y^* xususiy yechimi bilan unga mos bir jinsli

$$y'' + a_1y' + a_2y = 0 \qquad (2)$$

tenglamaning $\bar{y}$ umumiy yechimi yig'indisi kabi aniqlanadi. Ya'ni,

$$y = y^* + \bar{y} \qquad (3)$$

(1) tenglamaning biror xususiy yechimini o'zgarmasni variatsiyalash usuli bilan topamiz. Bunda dastlab (2) tenglamaning umumiy yechimini yozamiz:

$$y = c_1y_1 + c_2y_2 \qquad (4)$$

$c_1$va c_2 ni x ning hozircha noma'lum funksiyalari deb hisoblab, (1) tenglamaning xususiy yechimini (4) ko'rinishda izlaymiz.

(4) tenglikni differensiallaymiz:

$$y' = c_1y_1' + c_2y_2' + c_1'y_1 + c_2'y_2$$

$c_1$va c_2 larni $c_1'y_1 + c_2'y_2 = 0$ (5) tenglik bajariladigan qilib tanlaymiz. Agar bu qo'shimcha shartni e'tiborga olsak, u holda birinchi tartibliy' hosila

$$y' = c_1y_1' + c_2y_2'$$

ko'rinishni oladi. Endi bundan y'' ni topamiz:

$$y'' = c_1y_1'' + c_2y_2'' + c_1'y_1' + c_2'y_2'$$

y, y', y'' larni (1) tenglamaga qo'yamiz:

$$c_1y_1'' + c_2y_2'' + c_1'y_1' + c_2'y_2' + a_1(c_1y_1' + c_2y_2') + a_2(c_1y_1 + c_2y_2) = f(x)$$

yoki $c_1(y_1'' + a_1y_1' + a_2y_1) + c_2(y_2'' + a_1y_2' + a_2y_2) + c_1'y_1' + c_2'y_2' = f(x)$

ni hosil qilamiz. Bu yerdagi birinchi ikkita qavs ichidagi ifodalar nolga tengligi ravshan. Demak, oxirgi tenglik

$$c_1'y_1' + c_2'y_2' = f(x) \qquad (6)$$

ko'rinishga keladi. Shunday qilib, c_1 va c_2 funksiyalar (5) va (6) tenglamalarning sistemasini qanoatlantirsa, ya'ni

$$c_1'y_1 + c_2'y_2 = 0, c_1'y_1' + c_2'y_2' = f(x)$$

bo'lsa, (4) funksiya (1) tenglamaning yechimi bo'ladi. Ammo bu sistemaning determinanti chiziqli erkli y_1 va y_2 funksiyalarning Vronskiy determinanti bo'lgani uchun u nolga teng bo'lmaydi. Demak, sistemani yechib c_1' va c_2' ni x ning ma'lum funksiyalari sifatida aniqlaymiz:

$$c_1' = \varphi_1(x), \qquad c_2' = \varphi_2(x).$$

Bularni integrallab,

$$c_1 = \int \varphi_1(x)dx + \overline{c_1},\ c_2 = \int \varphi_2(x)dx + \overline{c_2}$$

larni hosil qilamiz. c_1 va c_2 larni topilgan ifodalarini (4) tenglikka qo'yib, (1) tenglamaning umumiy yechimini topamiz.

Xususiy yechimni topishda quyidagi teoremaning natijalaridan foydalanish qulaydir.

Teorema. $y'' + a_1y' + a_2y = f_1(x) + f_2(x)$ (7) tenglamaning y^* yechimini $y^* = y_1^* + y_2^*$ yig'indi shaklida tasvirlash mumkin, bunda y_1^* va y_2^* lar mos ravishda

$$y_1'' + a_1y_1' + a_2y_1 = f_1(x) \quad (8) \quad \text{va} \quad y_2'' + a_1y_2' + a_2y_2 = f_2(x) \quad (9)$$

tenglamalarning yechimlari.

Agar $y'' + py' + qy = f(x)$ (10) tenglamada p va q lar o'zgarmas sonlar bo'lsa, u holda (10) tenglama ikkinchi tartibli o'zgarmas koeffitsientli chiziqli bir jinslimas tenglama deyiladi.

Yuqorida biz bir jinslimas tenglama yechimini topishning umumiy usulini ko’rdik. Ba’zan o’zgarmas koeffitsientli tenglamani yechishda xususiy yechimni osonroq topish mumkin bo’ladi. Quyida o’ng tomon $f(x)$ ning ba’zi bir ko’rinishlarida xususiy yechimni qanday izlash mumkinligini ko’rib chiqamiz:

I. O’ng tomon ko’rsatkichli funksiya bilan ko’phad ko’paytmasidan iborat, ya’ni

$$f(x) = e^{\alpha x} P_n(x) \qquad (11)$$

ko’rinishda bo’lgan hol. Bu yerda $P_n(x)$ – n-darajali ko’phad.

Bunda quyidagi xususiy hollar bo’lishi mumkin:

a) α soni $k^2 + pk + q = 0$ xarakteristik tenglamaning ildizi bo’lmagan hol. Bu holda xususiy yechimni

$$y^* = (A_0 x^n + A_1 x^{n-1} + \ \dots + A_n) e^{\alpha x} = Q_n(x) e^{\alpha x} \qquad (12)$$

ko’rinishda izlash kerak.

b) α xarakteristik tenglamaning oddiy (bir karrali) ildizi bo’lgan hol. Bu holda y^* ni

$$y^* = x Q_n(x) e^{\alpha x}$$

ko’rinishda qidiriladi.

c) α soni xarakteristik tenglamaning ikki karrali ildizi bo’lgan hol. Bu holda xususiy yechimni

$$y^* = x^2 Q_n(x) e^{\alpha x}$$

ko’rinishda qidiriladi.

II. O’ng tomon $f(x) = P(x) e^{\alpha x} cos\beta x + Q(x) e^{\alpha x} sin\beta x$ ko’rinishda bo’lgan hol.

Bunda xususiy yechimni ko’rinishi quyidagicha bo’lishi mumkin:

a) agar $\alpha + \beta i$ xarakteristik tenglamaning ildizi bo’lmasa, u holda (10) tenglamaning xususiy yechimini

$$y^* = u(x) e^{\alpha x} cos\beta x + v(x) e^{\alpha x} sin\beta x$$

ko’rinishda izlash kerak. Bu yerda $u(x)$ va $v(x)$ -darajasi $P(x)$ va $Q(x)$ ko’phadlarning eng yuqori darajasiga teng bo’lgan ko’phadlardir.

b) agar $\alpha + \beta i$ xarakteristik tenglamaning ildizi bo'lmasa, u holda xususiy yechimni

$$y^* = x[u(x)e^{\alpha x}cos\beta x + v(x)e^{\alpha x}sin\beta x]$$

ko'rinishda izlanadi.

III. O'ng tomon $f(x) = Mcos\beta x + Nsin\beta x$ bo'lgan hol. Bunda M va N – o'zgarmas sonlar.

a) agar βi xarakteristik tenglamaning ildizi bo'lmasa, xususiy yechimni

$$y^* = Acos\beta x + Bsin\beta x$$

ko'rinishda izlash kerak.

b) agar βi xarakteristik tenglamaning ildizi bo'lsa, xususiy yechimni

$$y^* = x(Acos\beta x + Bsin\beta x)$$

ko'rinishda izlash kerak.

$$y^{(n)} + a_1 y^{(n-1)} + \ ... + a_n y = f(x)$$

tenglama n-tartibli bir jinslimas chiziqli tenglamadir. Bu yerda $a_1, a_2, ..., a_n$ va $f(x)$ lar x ning uzluksiz funksiyalari yoki o'zgarmas sonlar. Bu tenglamaga mos bir jinsli

$$y^{(n)} + a_1 y^{(n-1)} + \ ... + a_n y = 0$$

tenglamaning

$$\bar{y} = c_1 y_1 + c_2 y_2 + \ ... + c_n y_n$$

umumiy yechimi ma'lum bo'lsin. Bu holda ham quyidagi teorema o'rinlidir.

Teorema. Agar $\bar{y}$ bir jinsli tenglamaning umumiy yechimi, y^* esa bir jinslimas tenglamaning xususiy yechimi bo'lsa, u holda $y = \bar{y} + y^*$ funksiya berilgan bir jinslimas tenglamaning umumiy yechimi bo'ladi.

Bu yerda ham bir jinslimas tenglamaning xususiy yechimini

$$\bar{y} = c_1 y_1 + c_2 y_2 + \ ... + c_n y_n$$

ifodadagi $c_1, c_2, ..., c_n$ larni x ning funksiyalari deb qarab o'zgarmas miqdorlarni variatsiyalash usuli bilan topiladi.

n-tartibli o'zgarmas koeffitsientli chiziqli bir jinslimas tenglamaning xususiy yechimlari ba'zan ancha sodda topiladi.

I. Differensial tenglamaning o’ng tomonida

$$f(x) = P(x)e^{\alpha x}$$

funksiya turgan bo’lsin. Bu yerda $P(x)x$ ga nisbatan ko’phad. Bunda ikki hol bo’lishi mumkin:

a) agar α xarakteristik tenglamaning ildizi bo’lmasa, u holda xususiy yechimni

$$y^* = Q(x)e^{\alpha x}$$

ko’rinishda izlash mumkin, bunda $Q(x) -$ koeffitsiyentlari noma’lum bo’lgan va darajasi $P(x)$ ning darajasi bilan bir xil bo’lgan ko’phad;

b) α xarakteristik tenglamaning μ karrali ildizi bo’lsa, bu holda bir jinslimas tenglamaning xususiy yechimini

$$y^* = x^{\mu}\, Q(x)e^{\alpha x}$$

ko’rinishda izlash mumkin bo’lib, bunda Q(x)- darajasi P(x) ning darajasi bilan bir xil bo’lgan ko’phad.

II. Tenglamaning o’ng tomoni

$$f(x) = Mcos\beta x + Nsin\beta x$$

ko’rinishda (M va N – o’zgarmas sonlar) bo’lgan hol. Bu holda xususiy yechimini quyidagicha aniqlanadi:

a) βi xarakteristik tenglamaning ildizi bo’lmasa, bu holda xususiy yechimni

$$y^* = Acos\beta x + Bsin\beta x$$

ko’rinishda izlanadi. A va B noaniq o’zgarmas koeffitsiyentlar.

b) βi xarakteristik tenglamaning μ karrali ildizi bo’lsa, u holda xususiy yechimni

$$y^* = x^{\mu}(Acos\beta x + Bsin\beta x)$$

ko’rinishda izlanadi.

III. O’ng tomon

$$f(x) = P(x)e^{\alpha x}cos\beta x + Q(x)e^{\alpha x}sin\beta x$$

ko’rinishda bo’lgan hol. Bu yerda P(x) va Q(x) lar x ga nisbatan ko’phadlar.

a) agar $\alpha + \beta i$ xarakteristik tenglamaning ildizi bo’lmasa, xususiy yechimni

$$y^* = u(x)e^{\alpha x}cos\beta x + v(x)e^{\alpha x}sin\beta x$$

ko'rinishda izlanadi. Bu yerda $u(x)$ va $v(x)$ - darajasi $P(x)$ va $Q(x)$ ko'phadlarning eng yuqori darajasiga teng bo'lgan ko'phadlar.

b) agar $\alpha+\beta i$ xarakteristik tenglamaning μ karrali ildizi bo'lsa, xususiy yechimni

$$y^{*}=x^{\mu}[u(x)e^{\alpha x}cos\beta x+v(x)e^{\alpha\square}sin\beta x]$$

ko'rinishda izlanadi.

1. Quyidagi tenglamalar yechilsin:

1) $y''-\frac{y\prime}{x}=x;$

2) $y''-4y=x+3e^{x};$

3) $x^{2}y''-xy'+y=4x^{3};$

4) $y''-\frac{x}{x-1}y'+\frac{1}{x-1}y=x-1;$

5) $x^{2}y''-xy'+y=0;$

6) $(4x-1)^{2}y''-2(4x-1)y'+8y=0;$

Javob: 1) $y=\overline{c_1}x^{2}+\overline{c_2}+\frac{x^{3}}{3};$ 2) $y=c_1e^{2x}+c_2e^{-2x}+\frac{1}{4}x+\frac{3}{5}e^{x};$

3) $y=x^{3}+x(c_1+c_2\ln|x|);$ 4) $y=c_1e^{x}+c_2x-x^{2}-1;$

5) $y=(c_1+c_2\ln x)x;$ 6) $y=c_1(4x-1)+c_2\sqrt{4x-1}.$

2. Quyidagi o'zgarmas koeffitsientli chiziqli bir jinsli tenglamalar yechilsin:

1) $y''-7y'+6y=0;$

2) $y''-y'-2y=0;$

3) $y''-y'=0;$

4) $y''-4y'+4y=0;$

5) $y''-2y'+y=0;$

6) $4y''-20y'+25y=0;$

7) $y''-4y'+13y=0;$

8) $y''+25y=0;$

9) $y''+4y'+8y=0;$

10) $y^{IV}-13y^{II}+36y=0;$

11) $y^{V}-16y^{I}=0;$

12) $y^{VI} - 13y^{IV} + 36y^{II} = 0$;

13) $y^{IV} - 8y^{II} + 16y = 0$;

Javoblar: 1) $y = c_1 e^{6x} + c_2 e^x$;2) $y = c_1 e^{2x} + c_2 e^{-x}$;
3) $y = c_1 + c_2 e^x$; 4) $y = (c_1 + c_2 x)e^{2x}$; 5) $y = (c_1 + c_2 x)e^x$;6) $y = (c_1 + c_2 x)e^{2,5x}$;7) $y = e^{2x}(c_1 cos3x + c_2 sin3x)$;
8) $y = c_1 cos5x + +c_2 sin5x$;9) $y = e^{-2x}(c_1 cos2x + c_2 sin2x)$;
10) $y = c_1 e^{3x} + +c_2 e^{-3x} + c_3 e^{2x} + c_4 e^{-2x}$; 11) $y = c_1 + c_2 e^{2x} + +c_3 e^{-2x} + c_4 cos2x + c_5 sin2x$; 12) $y = c_1 e^{3x} + c_2 e^{-3x} + c_3 e^{2x} + +c_4 e^{-2x} + c_5 + c_6 x$; 13) $y = e^{2x}(c_1 + c_2 x) + e^{-2x}(c_3 + c_4 x)$.

3. $y'' - y' - 2y = 0$ tenglamaning $y(0) = 1,\ y'(0) = 3$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.Javob: $y = e^{2x} - e^{-x}$.

4. $y'' + 4y' + 29y = 0$ tenglamaning $y(0) = 0,\ y'(0) = 15$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $y = 3e^{-2x} sin5x$.

5. $y^V = y'$ tenglamaning $y(0) = 0$, $y'(0) = 1$, $y''(0) = 0$, $y'''(0) = 1$ va $y^{IV}(0) = 2$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.Javob: $y = e^x + cosx - 2$.

6. Quyidagi o'zgarmas koeffitsientli chiziqli bir jinslimas tenglamalar yechilsin:

1) $y'' + 4y' + 3y = x$;

2) $y'' - 4y = 8x^3$;

3) $y'' + 3y' = 9x$;

4) $y'' + 4y' + 5y = 5x^2 - 32x + 5$;

5) $y'' - 3y' + 2y = e^x$;

6) $y'' - 2y = xe^{-x}$;

7) $y^{III} + 8y = e^{-2x}$;

8) $y^{IV} - 81y = 27e^{-3x}$;

9) $y'' + 3y' + 2y = sin2x + 2cos2x$;

10) $y'' + y' + 2{,}5y = 25cos2x$;

11) $y'' - 5y' + 6y = 13sin3x$.

Javoblar:1) $y = c_1e^{-x} + c_2e^{-3x} + \frac{1}{3}x - \frac{4}{9}$; 2) $y = c_1e^{2x} + +c_2e^{-2x} - 2x^3 - 3x$;3) $y = c_1 + c_2e^{-3x} + \frac{3}{2}x^2 - x$;

4) y=$e^{-2x}(c_1cosx + c_2sinx) + x^2 - 8x + 7$;

5)$y = c_1e^{2x} + (c_2 - x)e^x$; 6) $y = c_1e^{x\sqrt{2}} + c_2e^{-x\sqrt{2}} - (x-2)e^{-x}$;

7) $y = \left(c_1 + \frac{x}{12}\right)e^{-2x} + \left(c_2\sqrt{3}cosx + c_3\sqrt{3}sinx\right)e^x$;

8) $y = c_1e^{3x} + \left(c_2 - \frac{x}{4}\right)e^{-3x} + c_3cos3x + c_4sin3x$;

9) $y = c_1e^{-x} + c_2e^{-2x} + 0{,}25\sqrt{2}\cos\left(\frac{\pi}{4} - 2x\right)$;

10) $y = e^{-\frac{x}{2}}\left(c_1cos\frac{3x}{2} + c_2sin\frac{3x}{2}\right) - 6cos2x + 8sin2x$;

11) $y = c_1e^{2x} + c_2e^{3x} + \frac{1}{6}(5cos3x - sin3x)$.

7. $y'' - 7y' + 6y = xe^x$tenglamani $y(0)$=1, $y'(0) = 3$boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $y = \frac{74}{125}e^x + \frac{51}{125}e^{6x} - e^x\left(\frac{x^2}{10} + \frac{x}{25}\right)$.

8. $y'' + y = -sin2x$ tenglamani $y(\pi) = y'(\pi) = 1$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $y = \frac{1}{3}sin2x - \frac{1}{3}sinx - cosx$.

9. $y'' - 3y' + 2y = e^{3x}(x^2 + x)$ tenglamani $y(0) = 1,\ y'(0) = -2$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $y = 4(e^x - e^{-2x}) + \frac{1}{2}(x^2 - 2x + 2)e^{3x}$.

10. $y'' + y' - 2y = cosx - 3sinx$ tenglamani $y(0) = 1,\ y'(0) = 2$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $y = e^x + sinx$.

11. $y''' - y' = -3x^2 + 6$ tenglamani $y(0) = y'(0) = y''(0) = 1$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $y = e^x + x^3$.

12. $y^{III} - y' = -2x$ tenglamani $y(0) = 0,\ y'(0) = y''(0) = 2$ boshlang'ich shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $y = e^x - e^{-x} + x^2$.

8-§. Differensial tenglamalar sistemasi

$$\begin{cases} \frac{dy_1}{dx} = f_1(x, y_1, y_2, \dots, y_n) \\ \frac{dy_2}{dx} = f_2(x, y_1, y_2, \dots, y_n) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{dy_n}{dx} = f_1(x, y_1, y_2, \dots, y_n) \end{cases} \qquad (1)$$

ga birinchi tartibli oddiy differensial tenglamalar sistemasi deyiladi. Bunda y_1,y_2, …, y_n izlanayotgan funksiyalar, x esa argument. Bunday sistemaga ***normal sistema*** deyiladi.

(1) sistemani integrallash uni va $(y_1)_{x=x_0} = y_{1_0}$, $(y_2)_{x=x_0} = y_{2_0}$, …, $(y_n)_{x=x_0} = y_{n_0}$ (2) boshlang'ich shartlarni qanoatlantiruvchi y_1 , y_2 , …, y_n funksiyalarni topish demakdir. (1) sistemani integrallash quyidagicha bajariladi:

(1) sistemaning tenglamalaridan birinchisini x bo'yicha differensiallaymiz:

$$\frac{d^2y_1}{dx^2} = \frac{\partial f_1}{\partial x} + \frac{\partial f_1}{\partial y_1}\frac{dy_1}{dx} + \dots + \frac{\partial f_1}{\partial y_n}\frac{dy_n}{dx}.$$

$\frac{dy_1}{dx}, \frac{dy_2}{dx}, \dots, \frac{dy_n}{dx}$ hosilalarni ularning tenglamalardagi $f_1, f_2, \dots, f_n$ ifodalari bilan almashtirib,

$$\frac{d^2y_1}{dx^2} = F_2(x, y_1, y_2, \dots, y_n)$$

tenglamani hosil qilamiz. Hosil bo'lgan tenglamani differensiallab

$$\frac{d^3y_1}{dx^3} = F_3(x, y_1, y_2, \dots, y_n)$$

tenglamani hosil qilamiz. Shu tarzda davom ettirib

$$\begin{cases} \frac{dy_1}{dx} = F_1(x, y_1, y_2, \dots, y_n) \\ \frac{d^2y_1}{dx^2} = F_2(x, y_1, y_2, \dots, y_n) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{d^ny_1}{dx^n} = F_n(x, y_1, y_2, \dots, y_n) \end{cases} \qquad (3)$$

sistemani hosil qilamiz.

Oldingi $n-1$ ta tenglamadan $y_2, y_3, \dots, y_n$ larni x, y_1 va $\frac{dy_1}{dx}$, $\frac{d^2y_1}{dx^2}$, …, $\frac{d^ny_1}{dx^n}$ hosilalar orqali ifodasini aniqlaymiz:

$$\begin{cases} y_2 = \varphi_2\left(x, y_1, y_1', \dots, y_1^{(n-1)}\right) \\ y_3 = \varphi_3\left(x, y_1, y_1', \dots, y_1^{(n-1)}\right) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ y_n = \varphi_n\left(x, y_1, y_1', \dots, y_1^{(n-1)}\right) \end{cases} \qquad (4)$$

Bu ifodalarni (3) sistemaning eng oxirgisiga qo'yib, y_1 ni aniqlash uchun n-tartibli tenglama hosil qilamiz:

$$\frac{d^n y_1}{dx^n} = \Phi(x, y_1, y_1', \dots, y_1^{(n-1)}) \qquad (5)$$

Bu tenglamani yechib, y_1 ni aniqlaymiz.

$$y_1 = \Psi_1(x, c_1, c_2, \dots, c_n) \qquad (6)$$

(6) ifodani $n-1$ marta differensiallab, $\frac{dy_1}{dx}$, $\frac{d^2y_1}{dx^2}$, …, $\frac{d^ny_1}{dx^n}$ hosilalarni $x, c_1, c_2, \dots, c_n$ larning funksiyasi kabi aniqlaymiz.

Bu funksiyalarni (4) tenglamalarga qo'yib, $y_2, y_3, \dots, y_n$ larni topamiz:

$$\begin{cases} y_2 = \Psi_2(x, c_1, c_2, \dots, c_n) \\ y_3 = \Psi_3(x, c_1, c_2, \dots, c_n) \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ y_n = \Psi_n(x, c_1, c_2, \dots, c_n) \end{cases} \qquad (7)$$

Hosil qilingan yechimlar berilgan (2) boshlang'ich shartlarni qanoatlantirishi uchun (6) va (7) tenglamalardan $c_1, c_2, \dots, c_n$ o'zgarmas miqdorlarning mos qiymatlari topiladi.

1. Quyidagi tenglamalar sistemasi yechilsin:

1) $\begin{cases} \frac{dx}{dt} + y = 0 \\ \frac{dx}{d\square} - \frac{dy}{dt} = 3x + y; \end{cases}$ 2) $\begin{cases} \frac{dx}{dt} = y - 7x \\ \frac{dy}{dt} + 2x + 5y = 0; \end{cases}$ 3) $\begin{cases} \frac{dx}{dt} + x - y = e^t \\ \frac{dy}{dt} - x + y = e^t; \end{cases}$

4) $\begin{cases} \frac{dx}{dt} = 2x + y \\ \frac{dy}{dt} = 3x + 4y; \end{cases}$ 5) $\begin{cases} \frac{dx}{dt} = x - 3y \\ \frac{dy}{dt} = 3x + y; \end{cases}$ 6) $\begin{cases} \frac{dx}{dt} = 4x + 6y \\ \frac{dy}{dt} = 2x + 3y + t. \end{cases}$

Javob: 1) $x = c_1e^t + c_2e^{-3t}, y = -\frac{dx}{dt} = c_1e^t - 3c_2e^{-3t}$;

2)$x = e^{-6t}(c_1cost + c_2sint)$,$y$=$e^{-6t}[(c_1 + c_2)cost$+$(c_2 - c_1)sint]$;

3) $x = e^t + c_1 + c_2e^{-2t}, y = e^t + c_1 - c_2e^{-2t}$;

4) $x = c_1e^t + c_2e^{5t}, y = -c_1e^t + 3c_2e^{5t}$;

5) $x = e^t(c_1cos3t + c_2sin3t)$, □ $= e^t(c_1sin3t - c_2cos3t)$;

6)$y = c_1 + c_2e^{7t} - \frac{3}{49}t(7t + 2)$,$y = -\frac{2}{3}c_1 + \frac{1}{2}c_2e^{7t} + \frac{1}{49}(14t^2 - 3t - 1)$

2. $\begin{cases} \frac{dx}{dt} = 2x + y \\ \frac{dy}{dt} = x + 2y \end{cases}$ sistemaning $x(0) = 1$, $y(0) = 3$ shartlarni qanoatlantiruvchi yechimi topilsin. Javob: $x = 2e^{3t} - e^t, y = 2e^{3t} + e^t$.

3. $\begin{cases} \frac{dx}{dt} = y + t \\ \frac{dy}{dt} = x + e^t; \end{cases}$ sistemaning $x(0) = 1,\ y(0) = 0$ shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $x = \frac{1}{4}(3e^t + 5e^{-t}) + \frac{1}{2}te^t - 1, y = \frac{5}{4}(e^t - e^{-t}) + \frac{1}{2}te^t - t.$

4. $\begin{cases} \frac{dx}{dt} = x + y \\ \frac{dy}{dt} = x - y; \end{cases}$ sistemaning $x(0) = 2,\ y(0) = 0$ shartlarni qanoatlantiruvchi yechimi topilsin.

Javob: $x = \left(\frac{\sqrt{2}}{2} + 1\right)e^{t\sqrt{2}} + \left(1 - \frac{\sqrt{2}}{2}\right)e^{-t\sqrt{2}}, y = \frac{\sqrt{2}}{2}e^{t\sqrt{2}} - \frac{\sqrt{2}}{2}e^{-t\sqrt{2}}.$

X BOB. QATORLAR

1-§. Sonli qatorlar

$u_1, u_2, u_3, \dots, u_n, \dots$ (1) sonli ketma-ketlikning hadlaridan hosil qilingan

$$u_1 + u_2 + u_3 + \dots + u_n + \dots = \sum_{n=1}^{\infty} u_n \qquad (2)$$

yig'indiga ***sonli qator*** deyiladi. $u_1, u_2, u_3, \dots, u_n, \dots$ lar sonli qatorning hadlari deyiladi. u_n – sonli qatorning ***n-hadi*** yoki ***umumiy hadi*** deyiladi.

Sonli qatorning dastlabki n ta hadining yig'indisi S_n bilan belgilanadi va u qatorning ***n-xususiy yig'indisi*** deyiladi. Demak,

$$S_n = u_1 + u_2 + u_3 + \dots + u_n \qquad (3)$$

Agar $\lim\limits_{n\to\infty} S_n = S$ –chekli limit mavjud bo'lsa, u holda qator ***yaqinlashuvchi*** va S –uning yig'indisi deyiladi.

Agar $\lim\limits_{n\to\infty} S_n = \infty$ yoki mavjud bo'lmasa, u holda qator uzoqlashuvchi deyiladi.

$$R_n = u_{n+1} + u_{n+2} + \dots + u_{n+k} + \dots \qquad (4)$$

ifodaga ***qatorning qoldig'i*** deyiladi.

Geometrik progressiyaning hadlaridan tuzilgan

$$b + bq + bq^2 + bq^3 + \dots + bq^{n-1} + \dots = \sum_{n=1}^{\infty} bq^{n-1}$$

qator geometrik qator deyiladi. U $|q| \geq 1$ da uzoqlashuvchi va $|q| < 1$ bo'lganda yaqinlashuvchidir.

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n}$$

qator ***garmonik qator*** deb ataladi. Bu qator uzoqlashuvchidir.

$$1 + \frac{1}{2^p} + \frac{1}{3^p} + \cdots + \frac{1}{n^p} + \cdots = \sum_{n=1}^{\infty} \frac{1}{n^p}$$

qator ***umumlashgan garmonik qator*** deb ataladi. Bu qator $p \leq 1$ da uzoqlashuvchi, $p > 1$ da yaqinlashuvchidir.

Teorema (qator yaqinlashishining zaruriy sharti). Agar $\sum_{n=1}^{\infty} u_n$ qator yaqinlashsa, u holda $\lim_{n\to\infty} u_n = 0$ bo’ladi.

Teorema (qator uzoqlashuvchi bo’lishining yetarli sharti). Agar $\lim_{n\to\infty} u_n \neq 0$ bo’lsa, $\sum_{n=1}^{\infty} u_n$ qator uzoqlashuvchi bo’ladi.

Yaqinlashuvchi qatorlar bir qator xossalarga ega:

1. Agar $\sum_{n=1}^{\infty} u_n$ qator yaqinlashuvchi va yig’indisi S ga teng bo’lsa, u holda $\sum_{n=1}^{\infty} Cu_n$ qator ham yaqinlashuvchi va yig’indisi CS ga teng bo’ladi.

2. Agar $\sum_{n=1}^{\infty} u_n$ va c qatorlar yaqinlashuvchi bo’lib, yig’indilari mos ravishda S_1va S_2 ga teng bo’lsa, u holda $\sum_{n=1}^{\infty} (u_n \pm v_n)$ qator ham yaqinlashuvchi bo’lib, yig’indisi $S_1 \pm S_2$ ga teng bo’ladi.

3. Agar qator yaqinlashuvchi bo’lsa, u holda undan istalgan chekli sondagi hadlarni tashlab yuborish yoki unga chekli sondagi hadlarni qo’shish natijasida hosil bo’lgan qator ham yaqinlashuvchi bo’ladi.

1. Quyidagi qatorlar uchun yaqinlashish ning zaruriy sharti bajariladimi?

1) $\frac{1}{2} + \frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \cdots$;

2) $\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots$;

3) $\frac{2}{3} + \frac{4}{9} + \frac{6}{27} + \frac{8}{81} + \cdots$.

Javoblar: 1) bajarilmaydi; 2) bajariladi; 3) bajariladi.

2. $u_n = \frac{n}{10^n+1}$ qatorning dastlabki 4 ta hadini yozing.

3. $\frac{1}{2} + \frac{3}{2^2} + \frac{5}{2^3} + \frac{7}{2^4} + \cdots$ qatorning umumiy hadini yozing.

4. $\frac{2}{3} + \left(\frac{3}{7}\right)^2 + \left(\frac{4}{11}\right)^3 + \left(\frac{5}{15}\right)^4 + \cdots$ qatorning umumiy hadini yozing.

5. $\frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \frac{1}{7\cdot9} + \cdots$ qatorning yig’indisini toping.

Javob: $\frac{1}{2}$.

6. $\frac{1}{1\cdot2}+\frac{1}{2\cdot3}+\frac{1}{3\cdot4}+\cdots$ qatorning yig'indisini toping.

Javob: 1.

7. $\frac{1}{1\cdot4}+\frac{1}{4\cdot7}+\frac{1}{7\cdot10}+\cdots$ qatorning yig'indisini toping.

Javob: $\frac{1}{3}$.

8. $\frac{1}{1\cdot2\cdot3}+\frac{1}{2\cdot3\cdot4}+\frac{1}{3\cdot4\cdot5}+\cdots$ qatorning yig'indisini toping.

Javob: $\frac{1}{4}$.

9. Quyidagi qatorlarning yaqinlashuvchi ekanini isbotlang va yig'indisini toping.

1) $\sum_{n=1}^{\infty}\frac{3^n+2^n}{6^n}$; 2) $\sum_{n=1}^{\infty}\frac{n^2}{(2n-1)^2(2n+1)^2}$; 3) $\sum_{n=1}^{\infty}\frac{1}{n(n+3)}$

4) $\sum_{n=1}^{\infty}\frac{1}{(3n-1)(3n+2)}$; 5) $\sum_{n=1}^{\infty}\frac{5^n+2^n}{10^n}$; 6) $\sum_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2}$

Javoblar: 1) S= $\frac{3}{2}$; 2) S= $\frac{1}{8}$; 3) S= $\frac{11}{18}$; 4) S= $\frac{1}{6}$; 5) S= $\frac{5}{4}$; 6) S= 1.

10. $\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}+\cdots$ qatorni yaqinlashuvchi ekanligi ko'rsatilsin.

11. $\frac{1}{11}+\frac{1}{12}+\frac{1}{13}+\frac{1}{14}+\cdots$ qator tekshirilsin.

12. $\frac{1}{2}+\frac{2}{5}+\frac{3}{8}+\frac{4}{11}+\cdots$ qator tekshirilsin.

13. $0{,}6+0{,}51+0{,}501+0{,}5001+\cdots$ qator tekshirilsin.

2-§. Musbat hadli qatorlarning yaqinlashish va uzoqlashish alomatlari

1. ***Taqqoslash alomati.*** Agar musbat hadli ikkita $\sum_{n=1}^{\infty}u_n$ va $\sum_{n=1}^{\infty}v_n$ qator berilgan bo'lib, biror N nomerdan boshlab $u_n \le v_n$ tengsizlik bajarilsa, u holda:

1) $\sum_{n=1}^{\infty}v_n$ qator yaqinlashuvchi bo'lsa, $\sum_{n=1}^{\infty}u_n$ qator ham yaqinlashuvchi bo'ladi;

2) $\sum_{n=1}^{\infty}u_n$ qator uzoqlashuvchi bo'lsa, $\sum_{n=1}^{\infty}v_n$ qator ham uzoqlashuvchi bo'ladi.

2. ***Dalamber alomati.*** Agar musbat hadli $\sum_{n=1}^{\infty} u_n$ qator uchun $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = l$ mavjud bo'lsa, u holda $l < 1$ da qator yaqinlashuvchi, $l > 1$ da uzoqlashuvchi bo'ladi. $l = 1$ bo'lsa qatorni yaqinlashish yoki uzoqlashishi aniq bo'lmaydi.

3. ***Koshi alomati.*** Agar musbat hadli $\sum_{n=1}^{\infty} u_n$ qator uchun $\lim_{n\to\infty} \sqrt[n]{u_n} = d$ mavjud bo'lsa, u holda $d < 1$ da qator yaqinlashuvchi, $d > 1$ da uzoqlashuvchi bo'ladi. $d = 1$ bo'lsa qatorni yaqinlashish yoki uzoqlashishi aniq bo'lmaydi.

4. ***Koshining integral alomati.*** Agar $\sum_{n=1}^{\infty} u_n$ qatorning hadlari musbat va o'smaydigan bo'lib, $x > 1$ da aniqlagan, uzluksiz, musbat va monoton kamayuvchi $f(x)$ funksiya uchun $f(1) = u_1$, $f(2) = u_2$, $f(3) = u_3$, …, $f(n) = u_n$, … tengliklar o'rinli bo'lsa, u holda $\int_1^{\infty} f(x)dx$ xosmas integral yaqinlashsa, berilgan qator ham yaqinlashuvchi va aksincha, xosmas integral uzoqlashsa, qator ham uzoqlashuvchi bo'ladi.

5. ***Taqqoslashning ikkinchi alomati.*** Agar $\lim_{n\to\infty} \frac{u_n}{v_n} = k$ chekli limit mavjud bo'lsa, u holda $\sum_{n=1}^{\infty} u_n$ va $\sum_{n=1}^{\infty} v_n$ qatorlar bir vaqtda uzoqlashuvchi yoki yaqinlashuvchi bo'ladi.

1. $\sum_{n=1}^{\infty} \frac{1}{n\cdot 2^n} = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 2^2} + \frac{1}{3\cdot 2^3} + \cdots + \frac{1}{n\cdot 2^n} + \cdots$ qatorning yaqinlashuvchi yoki uzoqlashuvchi ekanligi aniqlansin. Javob: Yaqinlashuvchi.

2. $\sum_{n=2}^{\infty} \frac{\ln n}{n} = \frac{ln2}{2} + \frac{ln3}{3} + \frac{ln4}{4} + \cdots + \frac{lnn}{n} + \cdots$ qatorning yaqinlashuvchi yoki uzoqlashuvchi ekanligi aniqlansin. Javob: Uzoqlashuvchi.

3. $\sum_{n=1}^{\infty} \frac{1}{2n-1} = \frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$ qatorning yaqinlashuvchi yoki uzoqlashuvchi ekanligi aniqlansin. Javob: Uzoqlashuvchi.

4. $\sum_{n=1}^{\infty} \frac{1}{2^n+1}$ qator tekshirilsin. Javob: Yaqinlashuvchi.

5. $1+\frac{1}{2^p}+\frac{1}{3^p}+\frac{1}{4^p}+\cdots+\frac{1}{n^p}+\cdots$ qator tekshirilsin.

Javob: Uzoqlashuvchi.

6. Umumiy hadi $u_n=\frac{1}{4\cdot 2^{n-3}}$ bo'lgan qator tekshirilsin.

Javob: Yaqinlashuvchi.

7. $\frac{1}{2}+\frac{1}{5}+\frac{1}{8}+\frac{1}{11}+\cdots$ qator tekshirilsin. Javob: Uzoqlashuvchi.

8. Dalamber alomatidan foydalanib quyidagi qatorlar tekshirilsin:

1) $\frac{2}{5}+\frac{4}{9}+\frac{6}{27}+\frac{8}{81}+\cdots$ (Yaqinlashadi);

2) $1+\frac{2}{2!}+\frac{4}{3!}+\frac{8}{4!}+\cdots$ (Yaqinlashadi);

3) $1+\frac{1\cdot 2}{1\cdot 3}+\frac{1\cdot 2\cdot 3}{1\cdot 3\cdot 5}+\cdots$ (Yaqinlashadi);

4) $1+\frac{3}{2\cdot 3}+\frac{3^2}{2^2\cdot 5}+\frac{3^3}{2^3\cdot 7}+\cdots$ (Uzoqlashadi);

5) $\sum_{n=1}^{\infty} \frac{2n-1}{2^n}=\frac{1}{2}+\frac{3}{2^2}+\frac{5}{2^3}+\cdots+\frac{2n-1}{2^n}+\cdots$ (Yaqinlashadi);

6) $\frac{2}{1}+\frac{2^2}{2^{10}}+\frac{2^3}{3^{10}}+\cdots+\frac{2^n}{n^{10}}+\cdots$ (Uzoqlashadi);

7) $\frac{1}{\sqrt{3}}+\frac{2}{3}+\frac{3}{3\sqrt{3}}+\frac{4}{9}+\frac{5}{9\sqrt{3}}+\cdots$ (Yaqinlashadi);

8) $\frac{10}{1!}+\frac{10^2}{2!}+\frac{10^3}{3!}+\cdots$ (Yaqinlashadi);

9) $\frac{10}{11}+\left(\frac{10}{11}\right)^2\cdot 2^5+\left(\frac{10}{11}\right)^3\cdot 3^5+\left(\frac{10}{11}\right)^4\cdot 4^5+\cdots$ (Yaqinlashadi);

10) $\frac{11}{10}+\left(\frac{11}{10}\right)^2\cdot\frac{1}{2^5}+\left(\frac{11}{10}\right)^3\cdot\frac{1}{3^5}+\left(\frac{11}{10}\right)^4\cdot\frac{1}{4^5}+\cdots$ (Uzoqlashadi);

9. Koshi alomatidan foydalanib quyidagi qatorlar tekshirilsin:

1) $\sum_{n=1}^{\infty} \frac{1}{2^n}\left(1+\frac{1}{n}\right)^{n^2}$; 2) $\sum_{n=1}^{\infty} \frac{3n-1}{\left(\sqrt{2}\right)^n}$;

3) $\sum_{n=1}^{\infty} \left(\frac{n}{2n+1}\right)^n$; 4) $\sum_{n=1}^{\infty} arctg^n \frac{1}{n}$;

5) $\frac{1}{ln2}+\frac{1}{ln^2 3}+\frac{1}{ln^3 4}+\cdots+\frac{1}{ln^n(n+1)}+\cdots$;

6) $\arcsin 1 + arcsin^2 \frac{1}{2} + \cdots + arcsin^n \frac{1}{n} + \cdots;$

7) $\frac{2}{3} + \frac{\left(\frac{3}{2}\right)^4}{9} + \cdots + \frac{\left(\frac{n+1}{n}\right)^{n^2}}{3^n} + \cdots;$

8) $\sum_{n=1}^{\infty} \left(\frac{2n^2+2n+1}{5n^2+2n+1} \right)^n;$

9) $3 + (2{,}1)^2 + (2{,}01)^3 + (2{,}001)^4 + \cdots;$

10) $\frac{1}{3} + \frac{3}{3^2} + \frac{5}{3^3} + \cdots + \frac{2\mathrm{n}-1}{3^{\mathrm{n}}} + \cdots .$

Javoblar: 1) Uzoqlashuvchi; 2) Yaqinlashuvchi; 3) Yaqinlashuvchi;

4) Yaqinlashuvchi; 5) Uzoqlashuvchi; 6) Yaqinlashuvchi;

7) Yaqinlashuvchi;8) Yaqinlashuvchi; 9) Uzoqlashuvchi;

10) Yaqinlashuvchi.

10. Koshining integral alomatidan foydalanib quyidagi qatorlar tekshirilsin:

1) $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots;$

2) $1 + \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{7}} + \frac{1}{\sqrt{10}} + \cdots;$

3) $\frac{1}{2^3} + \frac{2}{3^3} + \frac{3}{4^3} + \cdots;$

4) $\frac{1}{1+1^2} + \frac{1}{1+2^2} + \frac{1}{1+3^2} + \cdots;$

5) $\frac{1}{1+1^2} + \frac{2}{1+2^2} + \frac{3}{1+3^2} + \cdots;$

6) $\frac{1}{3^2-1} + \frac{1}{5^2-1} + \frac{1}{7^2-1} + \cdots;$

7) $\frac{1}{2ln^2 2} + \frac{1}{3ln^2 3} + \frac{1}{4ln^2 4} + \cdots;$

8) $\frac{1}{9ln9} + \frac{1}{19ln19} + \frac{1}{29ln29} + \cdots;$

9) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots + \frac{1}{n^2} + \cdots;$

10) $\sum_{n=1}^{\infty} \frac{1}{(n+1)\sqrt{n}}.$

Javoblar: 1) uzoqlashadi; 2) uzoqlashadi; 3) yaqinlashadi; 4) yaqinlashadi; 5) uzoqlashadi; 6) yaqinlashadi; 7) yaqinlashadi; 8) uzoqlashadi; 9) yaqinlashadi; 10) yaqinlashadi.

3-§. O'zgaruvchan ishorali qatorlar

Hadlarining ishoralari turlicha bo'lgan qator ***o'zgaruvchan ishorali qator*** deyiladi. Agar qator hadlarining ishoralari navbatlashuvchi bo'lsa, u holda qatorni ***ishoralari navbatlashuvchi qator*** deyiladi. Bu qatorni quyidagicha yoziladi:

$$\sum_{n=1}^{\infty}(-1)^{n+1}u_n = u_1 - u_2 + u_3 - u_4 + \cdots + (-1)^{n+1}u_n + \cdots \ (u_n > 0).$$

Leybnis teoremasi. Agar ishoralari navbatlashuvchi

$$u_1 - u_2 + u_3 - u_4 + \cdots \qquad (u_n > 0)$$

qatorning hadlari $u_1 > u_2 > u_3 > \cdots$ va $\lim\limits_{n\to\infty} u_n = 0$ bo'lsa, u holda berilgan qator yaqinlashadi, uning yig'indisi musbat va birinchi hadidan katta bo'lmaydi.

Ishoralari navbatlashuvchi qatorning qoldig'i $|R_n| < u_{n+1}$ tengsizlik bilan baholanadi.

O'zgaruvchan ishorali $u_1 + u_2 + u_3 + \cdots + u_n + \cdots$qator hadlarining absolut qiymatlaridan tuzilgan

$$|u_1| + |u_2| + |u_3| + \cdots + |u_n| + \cdots$$

qator yaqinlashuvchi bo'lsa, berilgan qator ***absolut yaqinlashuvchi*** qator deyiladi.

Agar o'zgaruvchan ishorali qator yaqinlashuvchi bo'lib, bu qator hadlarining absolut qiymatlaridan tuzilgan qator uzoqlashuvchi bo'lsa, u holda berilgan o'zgaruvchan ishorali qator ***shartli yaqinlashuvchi qator*** deyiladi.

Teorema. Agar qator absolut yaqinlashuvchi bo'lsa, uning hadlarini o'rinlarini ixtiyoriy ravishda almashtirilganda ham u absolut yaqinlashuvchiligicha qoladi. Bu holda qatorning yig'indisi qator hadlarining tartibiga bog'liq bo'lmaydi.

Quyidagi qatorlarning shartli yoki absolut yaqinlashishini tekshiring:

1) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots;$

2) $1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \cdots;$

3) $\frac{sin\alpha}{1^2} + \frac{sin2\alpha}{2^2} + \frac{sin3\alpha}{3^2} + \cdots + \frac{sinn\alpha}{n^2} + \cdots;$

4) $\frac{cos\frac{\pi}{4}}{3} + \frac{cos\frac{3\pi}{4}}{3^2} + \frac{cos\frac{5\pi}{4}}{3^3} + \cdots + \frac{\cos(2n-1)\frac{\pi}{4}}{3^n} + \cdots;$

5) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$;

6) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{3n-2}{3n-1}$;

7) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\ln(n+1)}$;

8) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n\cdot 2^n}$;

9) $\frac{1}{2}-\frac{4}{5}+\frac{7}{8}-\frac{10}{11}+\cdots$;

10) $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n-1}$.

Javoblar: 1) yaqinlashadi; 2) yaqinlashadi; 3) yaqinlashadi; 4) yaqinlashadi;5) shartli yaqinlashadi; 6) uzoqlashadi; 7) shartli yaqinlashadi; 8) absolut yaqinlashadi; 9) uzoqlashadi; 10) noabsolut yaqinlashadi.

4-§. Funksional qatorlar

Ta'rif. Agar qatorning hadlari x ning funksiyalaridan iborat bo'lsa, u holda unga ***funksional qator*** deyiladi.

U quyidagicha yoziladi:

$$u_1(x)+u_2(x)+\ u_3(x)+\cdots+u_n(x)+\cdots\ =\sum_{n=1}^{\infty}u_n\,(x)$$

Agar $\sum_{n=1}^{\infty}\ u_n(x_0)$ sonli qator yaqinlashsa, u holda funksional qator $x=x_0$ nuqtada yaqinlashuvchi deyiladi.

x ning $\sum_{n=1}^{\infty}\ u_n(x)$ qatorni yaqinlashuvchi qiladigan barcha qiymatlar to'plami funksional qatorning ***yaqinlashish sohasi*** deyiladi.

$S_n(x)=u_1(x)+u_2(x)+\ u_3(x)+\cdots+u_n(x)$ yig'indi funksional qatorning ***n-qismiy yig'indisi*** deyiladi. $\lim\limits_{n\to\infty}S_n(x)=S(x)$ ga funksional qatorning yig'indisi deyiladi. $R_n(x)=S(x)-S_n(x)$ ayirmaga ***qatorning qoldig'i*** deyiladi.

Agar yaqinlashuvchi $\sum_{n=1}^{\infty} u_n(x)$ funksional qator uchun har qanday $\varepsilon > 0$ berilganda ham shunday $N(\varepsilon)$ nomer topish mumkin bo'lsaki, $n \geq N$ bo'lganda $[a, b]$ kesmadagi istalgan x uchun $|R_n(x)| < \varepsilon$ tengsizlik bajarilsa, berilgan funksional qator $[a, b]$ da ***tekis yaqinlashuvchi*** deyiladi.

Qator tekis yaqinlashuvchi bo'lishining **Veyershtrass alomati**.

Agar $\sum_{n=1}^{\infty} u_n(x)$ funksional qator uchun hadlari musbat shunday $\sum_{n=1}^{\infty} c_n$ qator mavjud bo'lib, $x\epsilon[a, b]$ da $|u_n(x)| \leq C_n$ bo'lsa, u holda funksional qator $[a, b]$ kesmada tekis yaqinlashadi va bu holda $\sum_{n=1}^{\infty} u_n(x)$ qator ***kuchaytirilgan qator*** deyiladi.

Tekis yaqinlashuvchi funksional qatorlar bir qator xossalarga ega:

a) Tekis yaqinlashuvchi funksional qatorning hadlari $[a, b]$ kesmada uzluksiz bo'lsa, uning yig'indisi S(x) ham bu kesmada uzluksiz bo'ladi;

b) Agar $\sum_{n=1}^{\infty} u_n(x)$ funksional qatorning hadlari $[a, b]$ kesmada uzluksiz bo'lib, qator bu kesmada tekis yaqinlashuvchi bo'lsa, u holda

$$\int_a^b S(x)dx = \int_a^b u_1(x)dx + \int_a^b u_2(x)dx + \int_a^b u_3(x)dx + \ldots + \int_a^b u_n(x)dx + \ldots = \sum_{n=1}^{\infty} \int_a^b u_n(x)dx$$

bo'ladi. Bu yerda $S(x)$ $-$qator yig'indisi;

c) $\sum_{n=1}^{\infty} u_n(x)$ funksional qatorning hadlari $[a, b]$ kesmada aniqlangan vabu kesmada $u'_n(x)$ uzluksiz hosilalarga ega bo'lsin:

Agar bu kesmada berilgan qator yaqinlashuvchi va uning hadlari hosilalaridan tuzilgan

$$\sum_{n=1}^{\infty} u'_n(x) = u'_1(x) + u'_2(x) + \cdots + u'_n(x)$$

qator tekis yaqinlashuvchi bo'lsa, u holda funksional qatorning yig'indisi S(x) ham $[a, b]$ kesmada hosilaga ega bo'ladi va

$$S'(x) = \sum_{n=1}^{\infty} u_n'(x)$$

bo'ladi.

Quyidagi qatorlarni yaqinlashish sohalari topilsin:

1. $\sum_{n=1}^{\infty} \frac{1}{1+x^{2n}} = \frac{1}{1+x^2} + \frac{1}{1+x^4} + \cdots + \frac{1}{1+x^{2n}} + \cdots$;2. $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^{n-1}\sqrt{n}}$;

3. $\sum_{n=1}^{\infty} \frac{2^n n!}{(2n)!} x^{2n}$;   4. $\sum_{n=1}^{\infty} \frac{(x+8)^{3n}}{n^2}$;   5. $\sum_{n=1}^{\infty} 10^{2n}(2x-3)^{2n-1}$;

6. $\sum_{n=1}^{\infty} \frac{1}{n(x+2)^n}$;   7. $\sum_{n=1}^{\infty} x^{n-1}$;   8. $\sum_{n=1}^{\infty} \ln^n x$;

9. $\sum_{n=1}^{\infty} \frac{x^n}{1+x^{2n}}$;   10. $1 + \frac{1}{2^x} + \frac{1}{3^x} + \frac{1}{4^x} + \cdots$;   11. $\sum_{n=1}^{\infty} e^{-(n-1)x}$;

12. $xtg\frac{x}{2} + x^2 tg\frac{x}{4} + \cdots + x^n tg\frac{x}{2^n} + \cdots$.

Javoblar: 1) $(-\infty; -1) \cup (1; +\infty)$; 2) $(-3; 3]$; 3) $(-\infty; +\infty)$; 4) $[-9; -7]$; 5) $(1{,}45; 1{,}55)$; 6) $(-\infty; -3) \cup (-1; +\infty)$; 7) $(-1; 1)$; 8) $(\frac{1}{e}; e)$; 9) $x \neq \pm 1$;10) $(1; +\infty)$; 11) $(0; +\infty)$; 12) $(-2; 2)$.

13. Veyershtrass alomatiga ko'ra

$$sinx + \frac{1}{2^2}\sin^2 2x + \frac{1}{3^2}\sin^3 3x + \cdots$$

qatorni $(-\infty; +\infty)$ oraliqda tekis yaqinlashishi ko'rsatilsin.

14. $\frac{1}{x^2+1} - \frac{1}{x^4+2} + \frac{1}{x^6+3} - \frac{1}{x^2+4} + \cdots$

qator x ning $(-\infty; +\infty)$ oraliqdagi barcha qiymatlarida tekis yaqinlashuvchi ekanligi ko'rsatilsin.

15. $\sum_{n=1}^{\infty} x^n$ qator $(-1; 1)$ oraliqda tekis yaqinlashuvchi emasligi ko'rsatilsin.

16. $cosx + \frac{1}{2}cos\,2x + \frac{1}{2^2}cos\,3x + \frac{1}{2^3}cos\,4x + \cdots + \frac{1}{2^{n-1}}cos\,nx + \cdots$

qatorni $\left[\frac{\pi}{4}; \frac{\pi}{3}\right]$ kesmada hadma-had integrallash mumkinmi?

17. $arctgx + arctg\frac{x}{2\sqrt{2}} + arctg\frac{x}{3\sqrt{3}} + \cdots + arctg\frac{x}{n\sqrt{n}} + \cdots$

qatorga qator hadlarini hadma-had differensiallash haqidagi teoremani qo'llash mumkinmi?

18. $\frac{cosx}{1}+\frac{cos2x}{2^2}+\frac{cos3x}{3^2}+\cdots+\frac{cosnx}{n^2}+\cdots$

qator $(-\infty;+\infty)$ oraliqda kuchaytirilgan qator ekanligi ko'rsatilsin.

5-§. Darajali qatorlar

Ta'rif. $a_0+a_1x+a_2x^2+\cdots+a_nx^n+\cdots$ (1) ko'rinishdagi funksional qator darajali qator deb ataladi, bunda $a_0, a_1, a_2, \ldots, a_n, \ldots$ o'zgarmas sonlar bo'lib, ular qatorning koeffitsiyentlari deb ataladi.

Teorema (Abel teoremasi). 1) Agar darajali qator noldan farqli biror x_0 qiymatda yaqinlashsa, x ning $|x|<|x_0|$ tengsizlikni qanoatlantiruvchi har qanday qiymatlarida u absolut yaqinlashadi;

2) agar qator biror x_0 qiymatda uzoqlashsa, x ning $|x|>|x_0|$ tengsizlikni qanoatlantiruvchi har qanday qiymatlarida uzoqlashadi.

Teorema. Darajali qatorning yaqinlashish sohasi markazi koordinatalar boshida bo'lgan intervaldan iboratdir.

Ta'rif. Darajali qatorning yaqinlashish intervali deb, $-R$ dan $+R$ gacha bo'lgan shunday intervalga aytiladiki,bu interval ichida yotuvchi har qanday x nuuqtada qator yaqinlashadi, shu bilan birga absolut yaqinlashadi, uning tashqisidagi x nuqtalarida esa qator uzoqlashadi. R soni darajasi qatorning ***yaqinlashish radiusi*** deb ataladi.

$x=R$ va $x=-R$ da berilgan qatorning yaqinlashishi yoki uzoqlashishi har bir qator uchun alohida-alohida tekshiriladi. Ba'zi qatorlar uchun R=0 yoki $R=\infty$ bo'lishi mumkin.

Agar qatorning barcha $a_0, a_1, a_2, \ldots, a_n, \ldots$ koeffitsiyentlari nolga teng bo'lmasa, $\sum_{n=0}^{\infty} a_n x^n$ qatorning yaqinlashish radiusi

$$R=\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right| \text{ yoki } \quad R=\lim_{n\to\infty}\frac{1}{\sqrt[n]{|a_n|}}$$

formuladan topiladi.

Agar qator faqat juft yoki toq darajalarni o'z ichiga olsa yoki darajalari karrali bo'lsa, u holda yaqinlashish oralig'i Dalamber yoki Koshi alomatlaridan foydalanib topiladi.

$\sum_{n=0}^{\infty} a_n x^{np}$ qator uchun yaqinlashish radiusi:

$$R = \sqrt[p]{\lim_{n\to\infty}\left|\frac{a_n}{a_{n+1}}\right|} \quad \text{yoki} \quad R = \sqrt[p]{\lim_{n\to\infty}\frac{1}{\sqrt[n]{|a_n|}}}$$

formulalardan topiladi.

Darajali qatorlar quyidagi xossalarga ega:

a) yaqinlashish oralig'ining ichida yotuvchi har qanday $[a, b]$ kesmada darajali qator tekis yaqinlashadi. Uning yig'indisi yaqinlashish oralig'ida uzluksiz funksiya bo'ladi;

b) darajali qatorlarni ularning yaqinlashish oralig'ida hadma-had integrallash va differensiallash mumkin.

$$a_0 + a_1(x-a) + a_2(x-a)^2 + a_3(x-a)^3 + \cdots + a_n(x-a)^n + \cdots$$

ko'rinishdagi funksional qator ham darajali qator deyiladi. Bundagi $a_0, a_1, a_2, \dots, a_n$, ...lar o'zgarmas sonlar bo'lib, qatorning koeffitsiyentlari deyiladi.

$a = 0$ bo'lganda yuqorida ko'rib o'tgan qator hosil bo'ladi. Yuqoridagi qatorning yaqinlashish sohasini topish uchun $x - a = X$ deb olamiz. U holda

$$a_0 + a_1X + a_2X^2 + \cdots + a_nX^n + \cdots$$

qator hosil bo'ladi. Agar $-R < X < R$ bo'lsa, u holda berilgan darajali qatorni yaqinlashish sohasi $a - R < X < a + R$ bo'ladi.

1. Quyidagi qatorlarning yaqinlashish intervallari topilsin:

1) $1 + x + x^2 + x^3 + \cdots + x^n + \cdots;$ Javob: (-1;1);

2) $\frac{2x}{1} - \frac{(2x)^2}{2} + \frac{(2x)^3}{3} - \cdots;$ Javob: $\left(-\frac{1}{2}; \frac{1}{2}\right]$;

3) $x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots;$ Javob: (-∞;+∞);

4) $1 + x + (2x)^2 + (3x)^3 + \cdots + (nx)^n + \cdots;$ Javob: $R = 0$;

5) $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \cdots;$ Javob: $[-1;1)$;

6) $1+\frac{x^3}{10}+\frac{x^6}{10^2}+\frac{x^9}{10^3}+\cdots;$ Javob: $(-\sqrt[3]{10};\sqrt[3]{10});$

7) $1+\frac{x}{3\cdot 2}+\frac{x^2}{3^2\cdot 3}+\frac{x^3}{3^3\cdot 4}+\cdots;$ Javob: $[-3;3);$

8) $1+\frac{2x}{3^2\sqrt{3}}+\frac{4x^2}{5^2\sqrt{3^2}}+\frac{8x^3}{7^2\sqrt{3^3}}+\cdots.$ Javob: $\left[-\frac{\sqrt{3}}{2};\frac{\sqrt{3}}{2}\right].$

2. $(x-2)+\frac{1}{2^2}(x-2)^2+\frac{1}{3^2}(x-2)^3+\cdots$ qatorning yaqinlashish intervali topilsin. Javob: $1\le x\le 3.$

3. $\sum_{n=0}^{\infty}\left(\frac{k+1}{2k+1}\right)^k(x-2)^n$ qatorning yaqinlashish sohasi topilsin.

Javob: $2-\sqrt{2}<x<2+\sqrt{2}.$

4. $\frac{x+1}{1!}+\frac{(x+1)^2}{3!}+\frac{(x+1)^3}{5!}+\cdots$qator tekshirilsin. Javob:$-\infty<x<+\infty.$

5. $(x-4)+\frac{1}{\sqrt{2}}(x-4)^2+\frac{1}{\sqrt{3}}(x-4)^3+\cdots$ qator tekshirilsin.

Javob: $1\le x\le 3.$

6. $\frac{x-1}{2}+\frac{(x-1)^2}{2^2}+\frac{(x-1)^3}{2^3}+\cdots$ qator tekshirilsin. Javob: $1<x<3.$

6-§. Funksiyalarni Teylor va Makloren qatorlariga yoyish

Agar $y=f(x)$ funksiya $x=x_0$ nuqta atrofida $(n+1)-$ tartibgacha hosilalarga ega bo’lsa, u holda quyidagi Teylor formulasi o’rinlidir:

$$f(x)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+\cdots+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+R_n(x)$$

bu yerda $R_n(x)=\frac{f^{(n+1)}(x_0+\theta(x-x_0))}{(n+1)!}(x-x_0)^{n+1}$ $(0<\theta<1)$ bo’lib, unga Teylor formulasining Lagranch shaklidagi ***qoldiq hadi*** deyiladi. $x_0=0$ da Teylor formulasining xususiy holi – Makloren formulasi hosil bo’ladi.

$$f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\cdots+\frac{f^{(n)}(0)}{n!}x^n+R_n(x),$$

bu yerda $R_n(x)=\frac{f^{(n+1)}(\theta x)}{(n+1)!}x^{n+1}$ $(0<\theta<1).$

Agar $y=f(x)$ funksiya x_0 nuqta atrofida istalgan marta differensiallanuvchi va bu nuqtaning biror atrofida $\lim_{n\to\infty}R_n(x)=0$ bo’lsa, Teylor va Makloren formulalaridan quyidagi

$$f(x)=f(x_0)+\frac{f'(x_0)}{1!}(x-x_0)+\frac{f''(x_0)}{2!}(x-x_0)^2+\cdots+\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n+\ldots$$

va $f(x)=f(0)+\frac{f'(0)}{1!}x+\frac{f''(0)}{2!}x^2+\cdots+\frac{f^{(n)}(0)}{n!}x^n+\ldots$ qatorlar hosil bo'lib, ularni ***<u>Teylor va Makloren qatorlari</u>*** deb ataladi. Bu qatorlar x ning $\lim_{n\to\infty} R_n(x)=0$ bo'ladigan qiymatlarida $f(x)$ ga yaqinlashadi.

Ba'zi bir funksiyalarning Makloren qatoriga yoyilmalaridan amaliyotda ko'p qo'llaniladi. Quyida ularni keltiramiz:

$$e^x=1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots+\frac{x^n}{n!}+\cdots-\infty<x<+\infty;$$

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots+(-1)^{n+1}\frac{x^{2n-1}}{(2n-1)!}+\cdots-\infty<x<+\infty;$$

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\cdots+(-1)^n\frac{x^{2n}}{(2n)!}+\cdots-\infty<x<+\infty;$$

$$\ln(1+x)=1-\frac{x^2}{2!}+\frac{x^3}{3!}-\cdots+(-1)^{n-1}\frac{x^n}{n!}+\cdots-1<x\leq 1;$$

$(1+x)^m=1+\frac{m}{1!}x+\frac{m(m-1)}{2!}x^2+\quad\ldots\quad+\frac{m(m-1)\ldots(m-n+1)}{n!}x^n\quad+\ldots$ $(-1<x<1)$.

Oxirgi qator ***<u>binomial qator</u>*** deb ataladi.

Ba'zi hollarda funksiyaning taqribiy qiymatini berilgan aniqlikda hisoblash uchun uning darajali qatorga yoyilmasidan foydalaniladi.

Ba'zi bir integrallarni hisoblahsda integral ostidagi funksiyani darajali qatorga yoyib, darajali qatorni integrallash to'g'risidagi teoremadan foydalanib, $\int_0^x f(x)dx$ integralni darajali ko'rinishida tasvirlanadi va uning qiymatini bu qatorning yaqinlashish oralig'idagi x ning har qanday qiymatida berilgan aniqlik bilan hisoblash mumkin bo'ladi.

$(1+x)^m=1+\frac{m}{1!}x+\frac{m(m-1)x^2}{2!}x^2+\frac{m(m-1)(m-2)}{n!}x^3+\cdots$ qatordan m ning ba'zi bir qiymatlari uchun quyidagi qatorlarni hosil qilish mumkin:

$m=-1$ bo'lganda:

$$\frac{1}{1+x}=1-x+x^2-x^3+\cdots$$

$m=\frac{1}{2}$ bo'lganda:

$$\sqrt{1+x}=1+\frac{1}{2}x-\frac{1}{2\cdot 4}x^2+\frac{1\cdot 3}{2\cdot 4\cdot 6}x^3-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}x^4+\cdots$$

$m=-\frac{1}{2}$ bo'lganda:

$$\frac{1}{\sqrt{1+x}}=1-\frac{1}{2}x+\frac{1\cdot 3}{2\cdot 4}x^2-\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3+\frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}x^4-\cdots$$

Oxirgi tenglikdagi x ni o'rniga $-x^2$ ifodani qo'ysak:

$$\frac{1}{\sqrt{1-x^2}}=1+\frac{1}{2}x^2+\frac{1\cdot 3}{2\cdot 4}x^4+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^6+\cdots+\frac{1\cdot 3\cdot\ldots\cdot(2n-1)}{2\cdot 4\cdot\ldots\cdot 2n}x^{2n}+\cdots$$

$|x|<1$ bo'lganda darajali qaotorlarni integrallash haqidagi teoremaga asosan quyidagini hosil qilamiz:

$$\int_0^x\frac{dt}{\sqrt{1-t^2}}=\arcsin x=x+\frac{1}{2\cdot 3}x^3+\frac{1\cdot 3}{2\cdot 4\cdot 5}x^5+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7}x^7+\cdots+\frac{1\cdot 3\cdot\ldots\cdot(2n-1)x^{2n+1}}{2\cdot 4\cdot\ldots\cdot 2n\cdot(2n+1)}+\cdots$$

1. $y=x^4-3x^2+2x+2$ funksiyani $x-1$ ning darajalari bo'yicha yoying.J: $2+3(x-1)^2+4(x-1)^3+(x-1)^4$.

2. $y=x^4-5x^3+x^2-3x+4$ ko'phadni $x-4$ ning darajalari bo'yicha yoying.

3. $y=x^{10}-3x^5+1$ funksiyani $x-1$ ning darajalari bo'yicha yoying.

4. $f(x)=\ln x$ funksiyani $x_0=1$ nuqta atrofida Teylor qatoriga yoying

5. $f(x)=\frac{1}{x+1}$ funksiyani Makloren qatoriga yoying.

6. $f(x)=x^2e^x$ funksiyani Makloren qatoriga yoying.

7. $f(x)=\sec x$ va $f(x)=\ln(e^x+x)$ funksiyalarni Makloren qatoriga yoyilmasining dastlabki 3 ta hadini yozing.

8. $f(x)=e^x$ ni $x+2$ ning darajalari bo'yicha; 2) $f(x)=\sqrt{x}$ ni $x-4$ ning darajalari bo'yicha; 3) $f(x)=\cos\frac{x}{2}$ ni $x-\frac{\pi}{2}$ ning darajalari bo'yicha yoying.

9. $f(x)=e^x$, $f(x)=\sin x$, $f(x)=\cos x$, $f(x)=(1+x)^m$, $f(x)=\ln(1+x)$funksiyalarni Makloren qatoriga yoyilmasidan foydalanib: 1)$f(x)=(1+x)e^x$, 2) $f(x)=\sin^2 x$, 3) $f(x)=\frac{x-3}{(1+x)^2}$, 4) $f(x)=e^{-x}\sin x$, 5) $f(x)=\ln(1+3x+2x^2)$ funksiyalarni x ning darajalari bo'yicha qatorga yoying.

10. Integral ostidagi funksiyalarni Makloren qatoriga yoyilmasini hadlab integrallash natijasida quyidagi integrallarni qatorga yoyilmasini toping:

1) $\int \sin x^2\, dx$; 2) $\int \sqrt{x} e^x dx$; 3) $\int \sqrt{1-x^2}$ □x.

11. Quyidagi funksiyalarni darajali qatorga yoying.

1) $f(x) = x\cos 2x$; 2) $f(x) = \ln\frac{1+x}{1-x}$; 3) $f(x) = e^x \sin x$.

12. Mos kelgan qatorlardan foydalanib, $\ln 1{,}1$ va $\sqrt[4]{17}$ lari 0,0001 aniqlikda taqribiy hisoblang.

7-§. Furye qatorlari

$\frac{a_0}{2} + a_1\cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + \cdots$ yoki

$\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$ (1) funksional qatorga ***trigonometrik qator*** deyiladi. a_0, a_n, b_n o'zgarmas sonlar bo'lib, ularga trigonometrik qatorning koeffitsientlari deyiladi.

Agar (1) qator yaqinlashsa, u holda uning yig'indisi davri 2π bo'ldan $f(x)$ davriy funksiya bo'ladi.

(1) qatordagi a_0, a_n, b_n lar quyidagi

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx;\ \ a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx;\ b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx \quad (2)$$

formulalar bo'yicha aniqlanib, ularni ***Furye koeffitsientlari*** deb ataladi. Koeffitsientlari (2) formulalar bo'yicha topiladigan (1) trigonometrik qator ***Furye qatori*** deyiladi.

Agar $f(x)$ funksiya juft bo'lsa, u holda Furye qatorida faqat kosinuslar qatnashadi, chunki bu holda barcha $b_n = 0$ bo'lib, $a_n = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$ bo'ladi. Agar $f(x)$ toq bo'lsa, u holda Furye qatorida faqat sinuslar qatnashadi, chunki u holda barcha $a_n = 0$ bo'lib, $b_n = \frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$ bo'ladi.

1. Davri 2π bo'lgan

$$f(x)=\begin{cases}0, agar -\pi < x < 0\ bo'lsa,\\ x, \quad agar\ 0 < x < \pi\ bo'lsa\end{cases}$$

funksiyani Furye qatoriga yoying.

J: $f(x)=\frac{\pi}{4}+\sum_{n=1}^{\infty}\left(-\frac{2}{\pi(2n-1)^2}\cos(2n-1)x+\frac{(-1)^{n+1}}{n}\sin nx\right).$

2. $f(x)=\begin{cases}-x,\ agar -2 \le x < 0\ bo'lsa,\\ x, \quad agar\ 0 \le x \le 2\ bo'lsa\end{cases}$ funksiyani Furye qatoriga yoying.J: $f(x)=1-\frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\frac{\pi(2n-1)}{2}.$

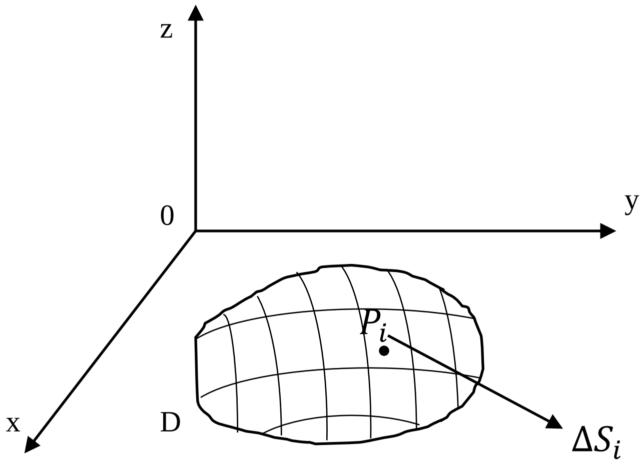
3. $-\pi \le x \le \pi$ oraliqda $f(x)=x$ funksiyani Furye qatoriga yoying.J: $f(x)=2\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{n}.$

4. $f(x)=\begin{cases}-2,\ agar -\pi < x \le 0\ bo'lsa,\\ 1, \quad agar\ 0 < x \le \pi\ bo'lsa\end{cases}$ funksiyani Furye qatoriga yoying.J: $f(x)=-1+\frac{6}{\pi}\sum_{n=1}^{\infty}\frac{\sin(2n-1)x}{2n-1}.$

XI BOB. KARRALI VA EGRI CHIZIQLI INTEGRALLAR

1-§. Ikki o’lchovli integral va uni hisoblash

$z = f(x, y) = f(p)$ funksiya L chiziq bilan chegaralangan yopiq D sohada aniqlangan va uzluksiz bo’lib, $\Delta S_1, \Delta S_2, \dots, \Delta S_n$ lar D sohani n ta elementar bo’laklarga bo’lish natijasida hosil bo’lgan yuzalar bo’lsin (1-chizma).



1-chizma

Har bir ΔS_i elementar sohada ixtiyoriy $P_i(x_i, y_i)$ nuqtani tanlaymiz va funksiyaning P_i nuqtadagi qiymatini hisoblab

$$f(P_i)\Delta S_i = f(x_i, y_i)\Delta S_i$$

ko’paytmani tuzamiz. So’ngra bu ko’paytmalarning barchasini yig’indisini tuzamiz:

$$\sum_{i=1}^{n} f(P_i)\Delta S_i = \sum_{i=1}^{n} f(x_i, y_i)\Delta S_i$$

Bu yig’indi $z = f(x, y) = f(p)$ funksiya uchun D sohadagi ***integral yig’indi*** deyiladi.

ΔS_i yuzalar soni cheksiz orttirilsa, u holda ular diametrlarining eng kattasi nolga intilgandagi integral yig’indining limiti $z = f(x, y)$ funksiyadan D soha bo’yicha olingan ikki o’lchovli integral deyiladi va quyidagicha yoziladi:

$$\iint_D f(P)ds \quad yoki \quad \iint_D f(x, y)ds$$

Demak, $\iint_D f(x,y)ds = \lim_{\max \Delta S_i \to 0} \sum_{i=1}^{n} f(x_i, y_i)\Delta S_i$,bu yerda D–integrallash sohasi, $f(x,y)$ integral ostidagi funksiya, ds – yuz elementi deyiladi. Dekart koordinatalarida $ds = dxdy$ bo'lganligi uchun ikki o'lchovli integral

$$\iint_D f(x,y)ds = \iint_D f(x,y)dxdy$$

Agar $f(x,y) \geq 0$ bo'lsa, $f(x,y)$ funksiyaning D soha bo'yicha olingan ikki o'lchovli integrali $z = f(x,y)$ sirt, $z = 0$ tekislik va yasovchisi OZ oqqa parallel, yo'naltiruvchisi esa D sohaning chegarasidan iborat bo'lgan silindrik sirt bilan chegaralangan jismning hajmi V ga teng bo'ladi.

Ikki o'lchovli integralni hisoblash ikkita aniq integralni ketma-ket hisoblashga keltiriladi.

Agar D soha $y_1 = f_1(x)$, $y_2 = f_2(x)$ funksiyalarning grafiklari hamda $x = a, x = b$ to'g'ri chiziqlar bilan chegaralangan bo'lsa, u holda ikki o'lchovli integral

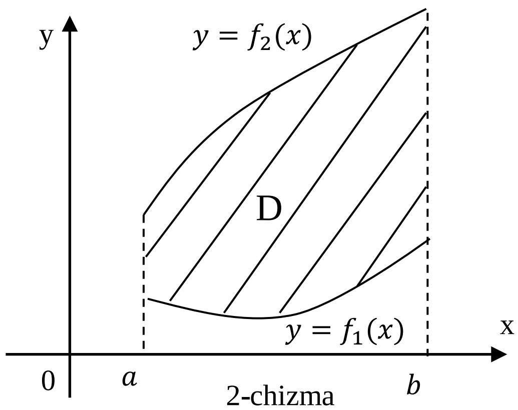
$$\iint_D f(x,y)dxdy = \int_a^b \left[\int_{f_1(x)}^{f_2(x)} f(x,y)dy \right] dx = \int_a^b dx \int_{f_1(x)}^{f_2(x)} f(x,y)dy$$

formula bilan hisoblanadi (2-chizma).

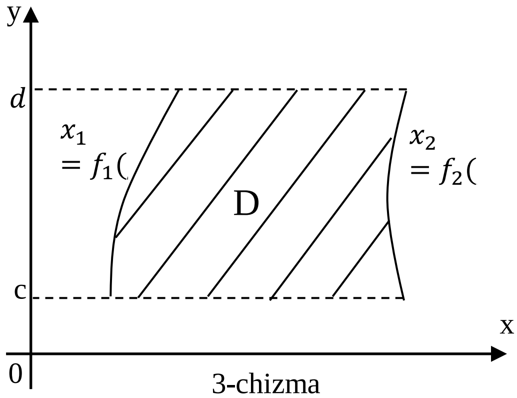
Agar D soha $y = c, y = d$ to'g'ri chiziqlar hamda $x_1(y)$ va $x_2(y)$ chiziqlar bilan chegaralangan bo'lsa, u holda ikki o'lchovli integral

$$\iint_D f(x,y)dxdy = \int_c^d \left[\int_{x_1(y)}^{x_2(y)} f(x,y)dx \right] dy = \int_c^d dy \int_{x_1(y)}^{x_2(y)} f(x,y)dx$$

formula bilan hisoblanadi (3-chizma).



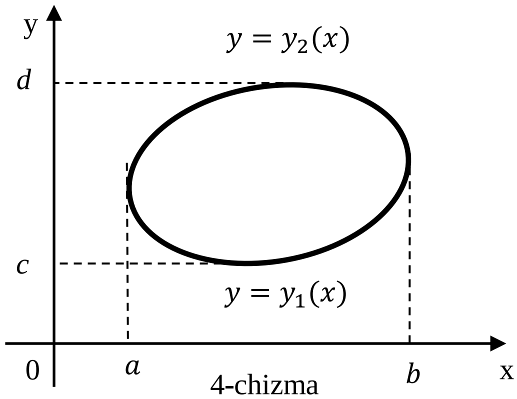
2-chizma



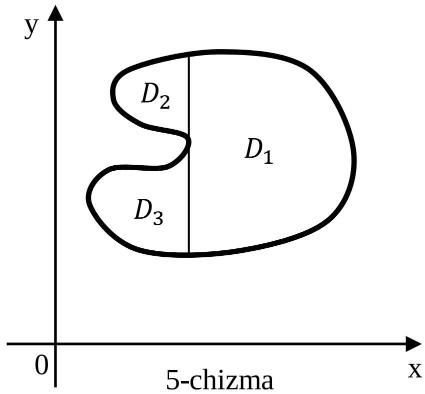
3-chizma

Agar integrallash sohasi 4-chizmadagidek bo'lsa, ya'ni u $x = a, x = b, y = c, y = d$ to'g'ri chiziqlar bilan faqat bitta nuqtada keshishsa, u holda ikki o'lchovli integral quyidagicha hisoblanadi:

$$\iint\limits_D f(x,y)dxdy = \int\limits_a^b dx \int\limits_{y_1(x)}^{y_2(x)} f(x,y)dy = \int\limits_c^d dy \int\limits_{x_1(y)}^{x_2(y)} f(x,y)dx$$



4-chizma



5-chizma

Agar integrallash sohasi 5-chizmadagidek bo'lsa, u holda ikki o'lchovli integralni hisoblash uchun D soha $x = x_0$ nuqta bilan bo'laklarga bo'linib, yuqoridagi formulalardan foydalanib hisoblanadi.

Oddiy aniq integralning barcha xossalari ikki o'lchovli integrallar uchun ham o'rinlidir.

Quyidagi ikki o'chovli integrallar hisoblansin.

1. $\iint\limits_D (x-y)dxdy$, bu yerda D soha $y = 2 - x^2$ va $y = 2x - 1$ chiziqlar bilan chegaralangan. J: $\frac{64}{15}$.

2. $\iint\limits_D e^{\frac{y}{x}}dxdy$, bu yerda D soha $y = x,\ y = 0,\ x = 1$ to'g'ri chiziqlar bilan chegaralangan. J: $\frac{e-1}{2}$.

3. $\iint\limits_D xydxdy$, bu yerda D soha: $a)\ x = 0, x = a, y = 0, y = b$ to'g'ri chiziqlar bilan chegaralangan to'g'ri to'rtburchak; b) $4x^2 + y^2 \leq 4$ ellips bilan chegaralangan; c) $y = x - 4$ to'g'ri chiziq va $y^2 = 2x$ parabola bilan chegaralangan. J:$a)\ \frac{a^2b^2}{2}$; b) 0; c) 90.

4. $\iint\limits_D (x+y)dxdy$, bu yerda D soha $x = 0, y = 0, x + y = 3$ to'g'ri chiziqlar bilan chegaralangan. J: 9.

5. $\iint_D \frac{xdxdy}{x^2+y^2}$, bu yerda D soha $x = 2, y = x,\ x = 2y$ to'g'ri chiziqlar bilan chegaralangan. J: $\frac{\pi}{2} - 2arctg\frac{1}{2}$.

6. Quyidagi ikki karrali integrallar hisoblansin:

1) $\int\limits_0^1 dx \int\limits_x^{2x} (x - y + 1)dy$, J: $\frac{1}{3}$;

2) $\int\limits_{-2}^4 dy \int\limits_0^y \left(\frac{y^3}{x^2 + y^2} \right) dy$, J: 6π;

3) $\int\limits_1^3 dx \int\limits_{x^3}^x (x - y)dy$, J: $112\frac{8}{105}$;

4) $\int\limits_0^4 dx \int\limits_1^e x \ln y dy$, J: 8;

5) $\int\limits_{-3}^8 dy \int\limits_{y^2-4}^5 (x + 2y)dx$, J: $50\frac{2}{5}$;

6) $\int\limits_0^2 dx \int\limits_0^3 (x^2 + 2xy)dy$, J: 26;

7) $\int\limits_2^0 dy \int\limits_0^{y^2} (x + 2y)dx$, J: $-11{,}2$;

8) $\int\limits_0^1 dv \int\limits_0^v e^{\frac{u}{v}}du$, J: $\frac{e-1}{2}$;

9) $\int_0^5 dx \int_0^{5-x} \sqrt{4+x+y}\,dy$, J: $\frac{506}{15}$;

10) $\int_1^2 dx \int_x^{x^2} (2x-y)dy$, J: 0,9;

11) $\int_0^{2\pi} \cos^2 x dx \int_0^a y dy$, J: $\frac{a^2\pi}{2}$;

12) $\int_0^a dy \int_{\sqrt{ay}}^{\sqrt{2a^2-y^2}} dx$, J: $\frac{a^2(3\pi-2)}{12}$.

7. Quyidagi ikki o’lchovli integrallarda integrallash tartibi o’zgartirilsin:

1) $\int_0^1 \left[\int_x^{\sqrt{x}} f(x,y)dy \right] dx$; J: $\int_0^1 \left[\int_{y^2}^{y} f(x,y)dx \right] dy$;

2) $\int_1^e dx \int_0^{\ln x} f(x,y)dy$; J: $\int_0^1 dy \int_{e^y}^{e} f(x,y)dx$;

3) $\int_0^1 dy \int_{2-y}^{\sqrt{1-y^2}} f(x,y)dx$; J: $\int_1^2 dy \int_{2-x}^{\sqrt{2x-x^2}} f(x,y)dy$;

4) $\int_0^1 dx \int_0^x f(x,y)dy$; J: $\int_0^1 dy \int_y^1 f(x,y)dx$.

2-§. Ikki o’lchovli integrallar yordamida yuzlar va hajmlarni hisoblash

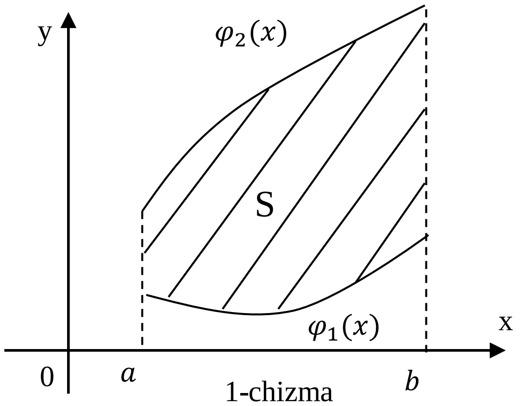
Agar $\iint\limits_D f(x,y)dxdy$ integralda $f(x,y) \equiv 1$ bo’lsa, u holda bu integralning qiymati D sohaning yuzini beradi. Demak,

$$S = \iint\limits_D dxdy$$

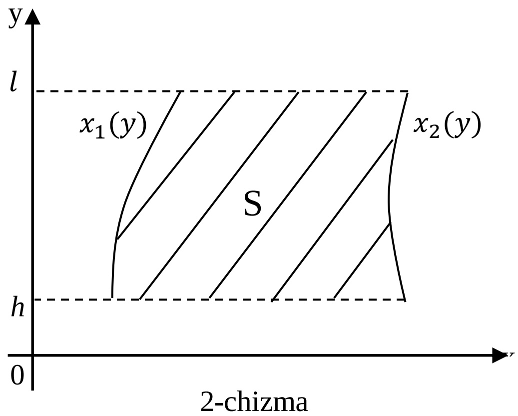
Agar D soha to’g’ri bo’lsa (1-chizma), u holda yuz ushbu

$$S = \int_a^b \left(\int_{\varphi_1(x)}^{\varphi_2(x)} dy \right) dx$$

ikki karrali integral bilan hisoblanadi.



1-chizma



2-chizma

Agar D soha $h \le y \le l,\ x_1(y) \le x \le x_2(y)$ tengsizliklar bilan aniqlangan bo'lsa (2-chizma), u holda bu sohaning yuzi ushbu

$$S = \iint\limits_D dxdy = \int\limits_h^l dy \int\limits_{x_1(y)}^{x_2(y)} dx$$

ikki karrali integral bilan hisoblanadi.

Agar D soha qutb koordinatalarida $\varphi_1 \le \varphi \le \varphi_2$, $r_1(\varphi) \le r \le r_2(\varphi)$ tengsizliklar bilan aniqlansa, u holda bu sohaning yuzi

$$S = \iint\limits_D rdrd\varphi = \int\limits_{\varphi_1}^{\varphi_2} d\varphi \int\limits_{r_1(\varphi)}^{r_2(\varphi)} rdr$$

formula bilan hisoblanadi.

$z = f(x, y)$ sirt, $z = 0$ tekislik va yo'naltiruvchisi D sohaning chegarasidan iborat bo'lgan to'g'ri chiziq, yasovchisi esa Oz o'qqa parallel silindrik sirt bilan chegaralangan jismning V hajmi, D soha bo'yicha $f(x, y)$ funksiyadan olingan ikki o'lchovli integralga teng, ya'ni

$$V = \iint\limits_D f(x, y)dxdy$$

Sirt zichligi birga teng bo'lgan yassi shakl og'irlik markazining koordinatalari:

$$x_c = \frac{\iint\limits_D xdxdy}{\iint\limits_D dxdy}; \qquad y_c = \frac{\iint\limits_D ydxdy}{\iint\limits_D dxdy}$$

formulalar bilan hisoblanadi. Agar sirt zichligi o'zgaruvchan, ya'ni $\gamma = \gamma(x, y)$ bo'lsa, u holda yuqoridagi formulalar quyidagi ko'rinishda bo'ladi:

$$x_c = \frac{\iint_D x\gamma(x,y)dxdy}{\iint_D \gamma(x,y)dxdy}; \qquad y_c = \frac{\iint_D y\gamma(x,y)dxdy}{\iint_D \gamma(x,y)dxdy}$$

D yassi shaklning Oy va Ox o'qlarga nisbatan statik momentlari

$$M_y = \iint_D \gamma(x,y)xdxdy; \qquad M_x = \iint_D \gamma(x,y)ydxdy$$

formulalar yordamida hisoblanadi.

$\iint_D \gamma(x,y)dxdy$ integral qaralayotgan shaklning massasini ifodalaydi.

1. Quyidagi chiziqlar bilan chegaralangan yuzlar ikki o'lchovli integrallar bilan yozilsin va hisoblansin:

1) $xy = 4, y = x, x = 4;$ J: $6 - 4\ln 2;$

2) $y = x^2, 4y = x^2, y = 4;$ J: $10\frac{2}{3};$

3) $y = x^2, 4y = x^2,\ x = \pm 2;$ J: 4;

4) $y^2 = 4 + x, x + 3y = 0;$ J: $20\frac{5}{6};$

5) $ay = x^2 - 2ax, y = x;$ J: $\frac{9a^2}{2};$

6) $y = \ln x, x - y = 1, y = -1;$ J: $\frac{1}{2} - \frac{1}{e};$

7) $y = x^2, y = x + 2;$ J: 4,5;

8) $y = \sin x, y = \cos x, x = 0;$ J: $\sqrt{2} - 1;$

9) $x = 4y - y^2,\ x + y = 6;$ J: $\frac{1}{6};$

10) $y = 2 - x, y^2 = 4x + 4.$ J: $\frac{64}{3}.$

2. Quyidagi sirtlar bilan chegaralangan jismning hajmi hisoblansin:

1) $z = x^2 + y^2, x + y = 4,\ x = 0, y = 0, z = 0;$ J: $42\frac{2}{3};$

2) $z = x^2 + y^2, y = x^2, y = 1, z = 0;$ J: $\frac{88}{105};$

3) $y = x^2, y = 1, x + y + z = 4, z = 0;$ J: $\frac{68}{15};$

4) $z = y^2 - x^2, z = 0, y = \pm 2;$ J: $\frac{32}{3}$.

3. $y^2 = 4x + 4$ va $y^2 = -2x + 4$ chiziqlar bilan chegaralangan figura og'irlik markazining koordinatalari topilsin:J: $x_c = \frac{2}{5},\ y_c = 0$.

4. $\frac{x^2}{25} + \frac{y^2}{9} = 1$ va uning $\frac{x}{5} + \frac{y}{3} = 1$ vatari bilan chegaralangan jismning og'irlik mazkazi koordinatalari topilsin.

J:$x_c = \frac{10}{3(\pi-2)},\ y_c = \frac{2}{\pi-2}$.

3-§. Uch o'lchovli integral va uni hisoblash

Fazo S yopiq sirt bilan chegaralangan biror V soha va uning chegarasida biror $f(x,y,z)$ uzluksiz funksiya aniqlangan bo'lsin. $f(x,y,z) \geq 0$ bo'lgan holda bu funksiyani qandaydir bir moddaning □ sohaga taqsimlanish zichligi deb olishimiz mumkin. V sohani ixtiyoriy ravishda ΔV_i sohalarga bo'lamiz. (ΔV_i belgi sohaning o'zgarishi emas, balki unng hajmi ham deb qaraymiz). Har bir ΔV_i sohada ixtiyoriy P_i nuqtani tanlab olamiz va f funksiyaning bu nuqtadagi qiymatini $f(P_i)$ bilan belgilab,

$$\sum_{i=1}^{n} f(P_i)\Delta V_i \qquad (1)$$

integral yig'indini tuzamiz. Agar bunda ΔV_i ning eng katta diametrini nolga intiladigan qilib ΔV_i larning sonini cheksiz orttirib boramiz. Agar $f(x,y,z)$ funksiya uzluksiz bo'lsa, (1) integral yig'indining limiti mavjud bo'ladi. V sohani bo'lish usuliga ham P_i nuqtani tanlash usuliga ham bo'liq bo'lmagan bu limit

$$\iiint_V f(P)dV$$

ko'rinishda belgilanadi va uch o'lchovli integral deyiladi. Demak,

$$\lim_{dia□\Delta V_i \to 0} \sum_{i=1}^{n} f(P_i)\Delta V_i = \iiint_V f(P)dV \text{ yoki}$$

$$\iiint\limits_V f(P)dV = \iiint\limits_V f(x,y,z)dxdydz \qquad (2)$$

Agar $f(x,y,z)$ funksiya V sohadagi modda taqsimlanishining hajm zichligi deb xisoblansa, (2) integral V hajmga kirgan barcha moddaning massasini beradi.

V soha pastdan $z = \varphi(x,y)$ sirt bilan, yuqoridan $z = \psi(x,y)$ sirt bilan chegaralangan bo'lsin. V sohaning XOY tekislikdagi proyeksiyasi D soha bo'lib, u $y = \varphi_1(x),\ y = \varphi_2(x),\ x = a,\ x = b$ chiziqlar bilan chegaralangan deylik. U holda V soha bo'yicha olingan uch karrali integral quyidagicha aniqlanadi:

$$\int_a^b \left\{ \int_{\varphi_1(x)}^{\varphi_2(x)} \left[\int_{\varphi(x,y)}^{\psi(x,y)} f(x,y,z) \right] dy \right\} dx \qquad (3)$$

Uch o'lchovli integralni xisoblash uchta aniq integralni yoki bitta ikki o'lchovli va bitta aniq integralni ketma-ket hisoblashga keltiriladi.

Agar V soha ushbu

$$\begin{cases} a \le x \le b \\ y_1(x) \le y \le y_2(x) \\ z_1(x,y) \le z \le z_2(x,y) \end{cases}$$

tengsizliklar sistemasi bilan aniqlangan bo'lsa, u holda uch o'lchovli integral quyidagi formula bilan hisoblanadi:

$$\iiint\limits_V f(x,y,z)dxdydz = \int_a^b dx \int_{y_1(x)}^{y_2(x)} dy \int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)\,dz \quad \text{yoki}$$

$$\iiint\limits_V f(x,y,z)dxdydz = \iint\limits_D dxdy \int_{z_1(x,y)}^{z_2(x,y)} f(x,y,z)\,dz$$

Agar $f(x,y,z) = 1$ bo'lsa, V soha bo'yicha olingan uch o'lchovli integral V sohaning hajmini ifodalaydi.

$$V = \iiint\limits_V dxdydz$$

Quyidagi uch o'lchovli integrallar hisoblansin:

1. $\iiint\limits_V zdxdydz$, bu yerda V soha $x+y+z=1, z=0, y=0, x=0$ tekisliklar bilan chegaralangan. J: $\frac{1}{24}$.

2. $\iiint\limits_V xydxdydz$, bu yerda V soha $z=xy$ giperbolik paraboloid hamda $x+y=1, z=0$ tekisliklar bilan chegaralangan. J: $\frac{1}{180}$.

3. $\iiint\limits_V y\cos(z+x)dxdydz$, bu yerda V soha $y=\sqrt{x}$ silindr va $y=0$, $z=0$, $x+z=\frac{\pi}{2}$ tekisliklar bilan chegaralangan. J: $\frac{\pi^2}{16}-\frac{1}{2}$.

4. $\iiint\limits_V xyzdxdydz$, bu yerda V soha $y=x^2, x=y^2, z=xy$ va $z=0$ sirtlar bilan chegaralangan. J: $\frac{1}{96}$.

5. $\iiint\limits_V (2x+y)dxdydz$, bu yerda V soha $y=x$, $x=1$, $z=1$ va $z=1+x^2+y^2$ sirtlar bilan chegaralangan. J: $\frac{41}{60}$.

6. Quyidagi uch karrali integrallar hisoblansin:

1) $\int\limits_0^1 dx\int\limits_0^x dy\int\limits_0^{xy} x^3y^3zdz$, J: $\frac{1}{110}$;

2) $\int\limits_0^3 dx\int\limits_0^{2x} dy\int\limits_0^{\sqrt{xy}} zdz$, J: $\frac{81}{4}$;

3) $\int\limits_{-1}^1 dx\int\limits_{x^2}^1 dy\int\limits_0^2 (4+z)dz$, J: $\frac{40}{3}$;

4) $\int\limits_0^1 dx\int\limits_0^{\sqrt{x}} dy\int\limits_{1-x}^{2-2x} dz$, J: $\frac{1}{12}$;

5) $\int\limits_0^c dz\int\limits_0^b dy\int\limits_0^a (x^2+y^2+z^2)dx$, J: $\frac{abc}{3}(a^2+b^2+c^2)$;

6) $\int\limits_0^a ydy\int\limits_0^h dx\int\limits_0^{a-y} dz$, J: $\frac{a^3h}{6}$;

7. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoidning hajmi hisoblansin. J: $\frac{4}{3}\pi a^3$.

8. $x^2+y^2+z^2=64$ sferaning hajmi topilsin. J: $\frac{256\pi}{3}$.

4-§. Birinchi va ikkinchi tur egri chiziqli integrallar

$f(x,y) = f(P)$ funksiya AB yassi silliq egri chiziqning barcha nuqtalarida aniqlangan va uzluksiz bo'lsin. Bu yoyni uzunliklari $\Delta l_1, \Delta l_2, \dots, \Delta l_n$ bo'lgan n ta elementar yoychalarga bo'lamiz. Har bir$i -$ bo'lakda ixtiyoriy $P_i(x_i, y_i)$ nuqtani tanlab olib, funksiyaning bu P_i nuqtadagi qiymatini mos elementar yoycha uzunligiga ko'paytiramiz. Bu ko'paytmalardan

$$\sum_{i=1}^{n} f(P_i)\Delta l_i \text{ yoki } \quad \sum_{i=1}^{n} f(x_i, y_i)\Delta l_i \qquad (1)$$

yig'indini tuzamiz. Bu yig'indi $f(x,y)$ funksiya uchun AB yoy bo'yicha integral yig'indi deyiladi.

(1) integral yig'indining elementar yoychalar uzunliklarining eng kattasi nolga intilgandagi limiti birinchi tur egri chiziqli integral deyiladi.

$$\lim_{diam\Delta l_i \to 0} \sum_{i=1}^{n} f(x_i, y_i)\Delta l_i = \int_{\breve{AB}} f(x,y)dl$$

Agar AB fazodagi egri chiziq yoyi va bu egri chiziq bo'ylab uzluksiz $f(x,y,z)$ funksiya berilgan bo'lsa, u holda egri chiziqli integral quyidagicha bo'ladi:

$$\lim_{max\Delta l_i \to 0} \sum_{i=1}^{n} f(x_i, y_i, z_i)\Delta l_i = \int_{\breve{AB}} f(x,y,z)dl$$

Birinchi tur egri chiziqli integral uchun quyidagi tenglik o'rinlidir:

$$\int_{\breve{AB}} f(x,y)dl = \int_{\breve{BA}} f(x,y)dl$$

Birinchi tur egri chiziqli integralni hisoblash aniq integralni hisoblahsga keltiriladi:

1) agar yassi AB egri chiziq $x = x(t), y = y(t), \alpha \le t \le \beta$ parametrik tenglamalar bilan berilgan bo'lsa, egri chiziqli integral quyidagi formula bilan hisoblanadi:

$$\int_{\breve{AB}} f(x,y)dl = \int_{\alpha}^{\beta} f[x(t),y(t)]\sqrt{x'^2 + y'^2}dt$$

Agar AB egri chiziq fazoda $x = x(t)$, $y = y(t)$, $z = z(t)$, $\alpha \leq t \leq \beta$ parametrik tenglamalar bilan berilgan bo'lsa, u holda egri chiziqli integral quyidagi formula bilan hisoblanadi:

$$\int_{\breve{AB}} f(x,y,z)dl = \int_{\alpha}^{\beta} f[x(t),y(t),z(t)]\sqrt{x'^2 + y'^2 + z'^2}dt$$

2) agar AB egri chiziq $y = y(x)(a \leq x \leq b)$ tenglama bilan berilgan bo'lsa, egri chiziqli integral quyidagicha hisoblanadi:

$$\int_{\breve{AB}} f(x,y)dl = \int_{a}^{b} f[x,y(x)]\sqrt{1 + y'^2}dx$$

3) agar AB yassi egri chiziq $x = x(y)(c \leq y \leq d)$ tenglama bilan berilgan bo'lsa, egri chiziqli integral quyidagicha hisoblanadi:

$$\int_{\breve{AB}} f(x,y)dl = \int_{c}^{d} f[x(y),y]\sqrt{1 + x'^2}dy$$

$P(x,y)$ va $Q(x,y)$ funksiyalar biror yassi silliq AB egri chiziqning barcha nuqtalarida aniqlangan va uzluksiz, hamada$\Delta x_1, \Delta x_2, \dots, \Delta x_n$ va $\Delta y_1, \Delta y_2, \dots, \Delta y_n$ lar elementar yoychalarning Ox va Oy o'qlarga proyeksiyalari bo'lsa, u holda

$$\sum_{i=1}^{n}[P(x_i,y_i)\Delta x_i + Q(x_i,y_i)\Delta y_i] \qquad (2)$$

yig'indi $P(x,y)$ va $Q(x,y)$ funksiyalar uchun koordinatalar bo'yicha integral yig'indi deyiladi.

(2) integral yig'indining $max\Delta x_i \to 0$ va $max\Delta y_i \to 0$ dagi limiti $\breve{AB}$ yoy yo'nalishi bo'yicha ikkinchi tur egri chiziqli integral deyiladi.

$$\lim_{\substack{max\Delta x_i \to 0 \\ max\Delta y_i \to 0}} \sum_{i=1}^{n} P(x_i,y_i)\Delta x_i + Q(x_i,y_i)\Delta y_i = \int_{\breve{AB}} P(x,y)dx + Q(x,y)dy.$$

Ikkinchi tur egri chiziqli integral uchun

$$\int\limits_{\breve{AB}} P(x,y)dx + Q(x,y)dy = -\int\limits_{\breve{BA}} P(x,y)dx + Q(x,y)dy$$

Agar integrallash yo'li yopiq egri chiziq bo'lsa, u holda egri chiziqli integral quyidagicha yoziladi:

$$\oint P(x,y)dx + Q(x,y)dy$$

Ikkinchi tur egri chiziqli integralni hisoblash ham aniq integralni hisoblashga keltiriladi:

1) agar yassi AB egri chiziq $x = x(t), y = y(t)$ parametrik tenglama bilan berilgan bo'lsa, u holda ikkinchi tur egri chiziqli integral quyidagicha hisoblanadi:

$$\int\limits_{\breve{AB}} P(x,y)dx + Q(x,y)dy = \int\limits_{t_A}^{t_B} [P(x(t),y(t))x'(t) + Q(x(t),y(t))y'(t)]dt$$

Agar AB egri chiziq fazoda $x = x(t)$, $y = y(t)$, $z = z(t)$ parametrik tenglama bilan berilgan bo'lsa, u holda ikkinchi tur egri chiziqli integral quyidagicha hisoblanadi:

$$\int\limits_{\breve{AB}} P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = \int\limits_{t_A}^{t_B} [P\big(x(t),y(t),z(t)\big)x'(t) +$$

$$+Q\big(x(t),y(t),z(t)\big)y'(t) + R\big(x(t),y(t),z(t)\big)z'(t)]dt.$$

2) agar yassi AB egri chiziq $y = y(x)$ tenglama bilan berilgan bo'lsa, u holda ikkinchi tur egri chiziqli integral quyidagicha hisoblanadi:

$$\int\limits_{\breve{AB}} P(x,y)dx + Q(x,y)dy = \int\limits_{a}^{b} [P(x,y(x)) + Q(x,y(x))y'(x)]dx$$

3) agar yassi AB egri chiziq $x = x(y)$ tenglama bilan berilgan bo'lsa, u holda ikkinchi tur egri chiziqli integral quyidagicha hisoblanadi:

$$\int\limits_{\breve{AB}} P(x,y)dx + Q(x,y)dy = \int\limits_{c}^{d} [P(x(y),y)x'(y) + Q(x(y),y)]dy.$$

1. $\int\limits_L (x-y)dl$ egri chiziqli integral hisoblansin. Bu yerda L – to’g’ri chiziqning $A(0,0)$ dan $B(4,3)$ gacha bo’lagi. J: $\frac{5}{2}$.

2. $\int\limits_L xdl$ ni hisoblang. Bu yerda $LO(0,0)$ va $A(1,2)$ nuqtalarini tutashtiruvchi to’g’ri chiziq kesmasi. J: $\frac{\sqrt{5}}{2}$.

3. $\int\limits_L x^2 ydl$ ni hisoblang. Bu yerda $Lx^2+y^2=9$ aylananing birinchi kvadrantda yotuvchi qismi. J: 27.

4. $\int\limits_L \frac{dl}{x+y}$ ni hisoblang. Bu yerda $Ly=x+2$ to’g’ri chiziqning $A(2,3)$ dan $\mathrm{B}(3,5)$ gacha bo’lgan qismi. J: $\frac{\sqrt{2}}{2}$.

5. $\int\limits_L (4\sqrt[3]{x}-3\sqrt{y})dl$ ni hisoblang. Bu yerda $L\ E(-1,0)$ va $H(0,1)$ nuqtalardan o’tuvchi to’g’ri chiziq kesmasi. J: $-5\sqrt{2}$.

6. $A(4,2)$ va $B(2,0)$ nuqtalar berilgan: 1) OA to’g’ri chiziq; 2) OBA siniq chiziq bo’yicha $\int\limits_L [(x-y)dx-xdy]$ ni hisoblang.J: 1) 8; 2) 4.

7. $A(0{,}1)$, $B(2{,}5)$ va $C(0{,}5)$ nuqtalar berilgan. $\int\limits_L [(x+y)dx-2ydy]$ integral: 1) AB to’g’ri chiziq bo’yicha; 2) $y=x^2+1$ parabolaning $\breve{AB}$ yoyi bo’yicha; 3) ABC siniq chiziq bo’yicha hisoblansin.

J: 1) -16; 2) $-\frac{52}{3}$; 3) -12.

8. Agar $x=\sqrt{\cos t}$, $y=\sqrt{\sin t}$, $0\le t\le\frac{\pi}{2}$ bo’lsa, $\int\limits_L x^2ydy-y^2xdx$ ni hisoblang. J: $\frac{\pi}{4}$.

9. $\int\limits_L y(x-y)dx+xdy$ ni hisoblang: 1) $y{=}2x$ chiziq bo’yicha; 2) $y{=}2x^2$ chiziq bo’yicha. J: 1) $\frac{1}{3}$; 2) $\frac{31}{30}$.

10. $\int\limits_L y^2dx+x^2dy$ ni hisoblang. Bu yerda $L: x = a\cos t\,, y = b\sin t$ ellipsning soat mili harakati bo'yicha aylanib o'tiladigan yuqori yarmi. J: $\frac{4}{3}ab^2$.

GLOSSARIY

№	O‘zbek tilida	Рус тилида	Ingliz tilida
1.	To‘plam	Множество	Set
2.	To‘plam elementi	Элементы множества	The element of a set
3.	Bo‘sh to‘plam	Пустое множество	Empty set
4.	To‘plam qismi	Подмножество	Part of the set
5.	To‘plamlar tengligi	Равенства множеств	Equality of sets
6.	To‘plamlar birlashmasi	Объединение множеств	The combination of sets
7.	To‘plamlar kesishmasi	Пересечения множества	Intersection of sets
8.	To‘plamlar ayirmasi	Разность множества	Diversity of sets
9.	To‘plam to‘ldiruvchisi	Дополнение к данному множеству	The complement of a set
10.	Dekart ko‘paytmasi	Декартовые произведения	Dekart’s product
11.	Chekli to‘plam	Конечные множества	Restricted set
12.	Cheksiz to‘plam	Бесконечные множества	Unrestricted set
13.	O‘zaro bir qiymatli moslik	Взаимно однозначные соответствия	One valued mutual correspondence
14.	Ekvivalent to‘plamlar	Эквивалентные множества	Equivalent sets
15.	To‘plam quvvati	Мощность множества	Power of the set
16.	Sanoqli to‘plam	Счетное множество	Countable set
17.	Sanoqsiz to‘plam	Несчетное множество	Uncountable set
18.	Matritsa	Матрицы	Matrix
19.	Matritsa tartibi	Порядок матрицы	The order of matrix
20.	Matritsa elementi	Элементы матрицы	The element of matrix
21.	To‘rtburchakli	Прямоугольная матрица	Square matrix

	matritsa		
22.	Kvadrat matritsa	Квадратная матрица	Quadratic matrix
23.	Ustun matritsa	Матрица столбец	Column matrix
24.	Satr matritsa	Матрица строка	Line matrix
25.	Teng matritsa	Равные матрицы	Equal matrix
26.	Dioganal element	Диагональный элемент	Diagonal element
27.	Dioganal matritsa	Диагональная матрица	Diagonal matrix
28.	Birlik matritsa	Единичная матрица	Single matrix
29.	Nol matritsa	Нулевая матрица	Zero matrix
30.	Matritsalar yig‘indisi	Сумма матриц	Sum of matrixes
31.	Matritsalar ayirmasi	Разность матриц	Diversity of matrixes
32.	Matritsalar ko‘paytmasi	Произведение матриц	Product of matrixes
33.	Matritsaning transponirlangani	Транспонированные матрицы	Transponed matrix
34.	Teskari matritsa	Обратная матрица	Inverse matrix
35.	Matritsaning rangi	Ранг матрицы	Rang of matrix
36.	Determinant (aniqlovchi)	Детерминант (определитель)	Determinant
37.	Determinantning elementi	Элементы определителя	The element of determinant
38.	Determinantning satri	Строка определителя	Line of determinant
39.	Determinantning ustuni	Столбцы определителя	Column of determinant
40.	Algebraik to‘ldiruvchi	Алгебраические дополнение	Algebraic complement
41.	Determinantning	Миноры определителя	Minors of determinant

	minori		
42.	Chiziqli tenglamalar	Системы линейных уравнений	Linear equation
43.	Sistema koeffitsentlari	Коэффициенты системы	Quotients of a system
44.	Sistema ozod xodlari	Свободные члены системы	Free parts of a system
45.	Sistema yechimi	Решение системы	Decision of a system
46.	Birgalikda bo‘lgan sistema	Совместная система	Joint system
47.	Birgalikda bo‘lmagan sistema	Несовместная система	Disjoined system
48.	Aniq sistema	Определенная система	Definite system
49.	Aniqmas (noaniq) sistema	Неопределенная система	Indefinite system
50.	Kengaytirilgan matritsa	Расширенная матрица	Broad matrix
51.	Matritsalar usuli	Способ матриц	Method of matrixes
52.	Kramer usuli	Способ Крамера	Kramer’s method
53.	Asosiy determinant	Основной определитель	The main determinant
54.	Yordamchi determinantlar	Вспомогательные определители	Secondary determinants
55.	Kramer formulalari	Формулы Крамера	Kramer’s formulas
56.	Gauss usuli	Способ Гаусса	Method of Gauss
57.	Umumiy yechim	Общее решение	General decision
58.	Bir jinsli sistema	Однородная система	Similar system
59.	Skalyar	Скаляр	Scalar
60.	Vektor	Вектор	Vector
61.	Vektorning moduli	Модуль вектора	Module of Vector

62.	Vektorning geometrik talqini	Геометрическое столькование вектора	Geometric interpretation of Vector
63.	Vektorning boshi	Начало вектора	The beginning of vector
64.	Vektorning uchi	Вершина вектора	Apex of vector
65.	Vektorning oxiri	Конец вектора	The end of vector
66.	Nol vektor	Нулевой вектор	Zero vector
67.	Kolliniar vektorlar	Коллинеарные векторы	Co-linear vectors
68.	Komplanar vektorlar	Компланарные векторы	Compiled vectors
69.	Vektorning tengligi	Равенство векторов	The equality of the vector
70.	Vektorni songa ko‘paytmasi	Произведение число на вектора	Product numbers to vector
71.	Qarama-qarshi vektorlar	Противоположные векторы	Contrast vectors
72.	Vektorlarni qo‘shish	Сложение векторов	Adding of vectors
73.	Parallelogramm qoidasi	Правила параллелограмма	The rule of parallelogram
74.	Uchburchak qoidasi	Правила треугольника	The rule of triangle
75.	Ko‘pburchak qoidasi	Правила многоугольника	The rule of polygon
76.	Vektorlarning ayirmasi	Разность векторов	Diversity of vectors
77.	Vektorlarning o‘qdagi proyektsiyasi	Проекция вектора на ось	Projection of vectors on axix

78.	Vektorning yoyilmasi	Разложения вектора	Expansion of vector
79.	Vektorning koordinatalari	Координаты вектора	Coordinates of vector
80.	Birlik vektorlar	Единичный вектор	Single vectors
81.	Skalyar ko‘paytma	Скалярное произведения	Scalar product
82.	Skalyar ko‘paytmaning mexanik ma’nosi	Механический смысл скалярного произведения	Mechanic meaning of Scalar product
83.	Vektor ko‘paytma	Векториальное произведения	Vector product
84.	Aralash ko‘paytma	Смешанные произведения	Mixed product
85.	Aralash ko‘paytmaning geometrik ma’nosi	Геометрический смысл смешанного произведения	Geometric meaning of mixed product
86.	Aylana tenglamasi	Уравнение окружности	Equation of a circle
87.	To‘g‘ri chiziqning umumiy tenglamasi	Общее уравнение прямой	General equation of straight line
88.	To‘g‘ri chiziqning burchak koeffitsientli tenglamasi	Уравнение прямой с угловым коэффициентом	Equation of angled quotient of a straight line
89.	To‘g‘ri chiziqning burchak koeffitsienti	Угловой коэффициент прямой	Angled quotient of a straight line
90.	Normal tenglama	Нормальное уравнение	Normal equation
91.	Kanonik tenglama	Каноническое уравнение	Canonic equation
92.	Parametrik tenglama	Параметрическое уравнение	Parametric equation

93.	To‘g‘ri chiziqlar dastasi	Кучка прямых линий	Group of straight line
94.	Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq	Уравнение прямой проходящий через две данной точки	Straight line crossing two points
95.	Ikki to‘g‘ri chiziq orasidagi burchak	Угол между двумя прямыми	The angle between two straight lines
96.	Parallellik sharti	Условие параллельности	Condition of parallelism
97.	Perpendikulyarlik sharti	Условие перпендикулярности	Condition of perpendicularity
98.	Nuqtadan to‘g‘ri chiziqgacha masofa	Расстояние от точки до прямой	Distance from the point to the line
99.	Ikki o‘zgaruvchi 2 -tartibli tenglamalar	Уравнение второго порядка с двумя неизвестными	Equation with two unknown quantities
100.	Ikkinchi tartibli egri chiziqlar	Кривые второго порядка	Curve lines of the second order
101.	Aylana	Окружность	Circle
102.	Aylana markazi	Центр окружности	The centre of a circle
103.	Aylana radiusi	Радиус окружности	Radius of a circle
104.	Aylananing kanonik tenglamasi	Каноническое уравнение окружности	Canonical equation of a circle
105.	Ellips	Эллипс	Ellipse
106.	Ellipsning fokuslari	Фокусы эллипса	Focuses of the ellipse
107.	Ellipsning kanonik tenglamasi	Каноническое уравнение эллипса	Canonical equation of an ellipse
108.	Ellipsning uchlari	Вершины эллипса	The tops of an ellipse
109.	Ellipsning o‘qlari	Оси эллипса	The axis of an ellipse

110.	Fonal radiuslar	Фокальные радиусы	Fonal radiuses
111.	Ellips ekssentrisiteti	Эксцентриситет эллипса	Eccentricity of an ellipse
112.	Ellips direktrisalari	Директрисы эллипса	Directrices of an ellipse
113.	Giperbola	Гипербола	Hyperbola
114.	Fokus	Фокус	Focus
115.	Giperbolaning noaniq tenglamasi	Каноническое уравнение гиперболы	Unknown equation of a hyperbola
116.	Giperbolaning uchlari	Вершины гиперболы	The tops of a hyperbola
117.	Giperbolaning o‘qlari	Оси гиперболы	The axis of a hyperbola
118.	Asimptotalar	Асимптоты	Asimitators
119.	Giperbolaning ekstsentrisiteti	Эксцентриситет гиперболы	Eccentricity of a hyperbola
120.	Direktrisa	Директриса	Directrix
121.	Parabola	Парабола	Parabola
122.	Parabolaning kanonik tenglamasi	Каноническое уравнения параболы	Canonical equation of a parabola
123.	Parallel ko‘chirish	Параллельный перенос	Parallel transportation
124.	Burish	Поворот	Turning
125.	Koordinatalar sistemasini almashtirish	Преобразование системи координат	Substitution of systems of coordinates
126.	Fazodagi nuqta koordinatalari	Координаты точки на пространстве	Coordinates on space points
127.	Tekislikning umumiy tenglamasi	Общее уравнение плоскости	General equation of flatness

128.	Tekislikning normal vektori	Нормальное вектора плоскости	Normal vector of flatness
129.	Tekislikning kesmalar bo‘yicha tenglamasi	Уравнения плоскости в отрезах	Equation of flatness on segments
130.	Normallovchi ko‘paytiruvchi	Нормирующий множитель	Normalizing multiplier
131.	Berilgan nuqtadan o‘tuvchi tekisliklar	Плоскости проходящей через данной точки	Flatnesses crossing the given points
132.	Berilgan uchta nuqtadan o‘tuvchi tekislik	Плоскости проходящей через три данные точки	Flatness crossing the three given points
133.	Ikki tekislik orasidagi burchak	Уголь между двумя плоскостями	The angle between two flatnesses
134.	Ikki tekislikning parallellik sharti	Условия параллельности двух плоскости	Parallel conditions of two flatnesses
135.	Ikki tekislikning perpendikulyar sharti	Условия перпендикулярности двух плоскости	Perpendicular conditions of two flatnesses
136.	Nuqtadan tekislikacha bo‘lgan masofa	Расстояние от точки до прямой	Distance from the point to the flatness
137.	Yo‘naltiruvchi vektor	Направляющий вектор	Guide vector
138.	Fazodagi ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasi	Уравнения прямой проходящий через две точки на пространстве	Straight line equation going through two points on space
139.	Fazodagi to‘g‘ri	Уголь между прямыми на	The angle between the

	chiziqlar orasidagi burchak	пространстве	straight lines on space
140.	Fazodagi ikki to‘g‘ri chiziqning parallellik sharti	Условие параллельности двух прямых на пространстве	The condition of parallelism of two straight lines on space
141.	Fazodagi ikki to‘g‘ri chiziqning perpendikulyarlik sharti	Условие перпендикулярности двух прямых на пространстве	The condition of perpendicularity of two straight lines on space
142.	Fazodagi to‘g‘ri chiziq va tekislik sharti orasidagi burchak	Уголь между прямой и плоскости в пространстве	The angle between straight lines and flatness on space
143.	To‘g‘ri chiziq va tekislikning parallellik sharti	Условие параллельности прямой и плоскости	The condition of parallelism of a straight line and flatness
144.	To‘g‘ri chiziq va tekislikning perpendikulyarlik sharti	Условие перпендикулярности прямой и плоскости	The condition of perpendicularity of a straight line and flatness
145.	To‘g‘ri chiziq va tekislikning kesishish nuqtasi	Точка пересечения прямой и плоскости	The point of crossing a straight line and flatness
146.	Sonli to‘plamlar	Числовые множества	Numerical sets
147.	Natural sonlar to‘plami	Множества натуральных чисел	Set of natural numbers
148.	Butun sonlar to‘plami	Множества целых чисел	Set of whole numbers

149.	Ratsional sonlar to‘plami	Множества рациональных чисел	Set of rational quantities
150.	Irratsional sonlar to‘plami	Множества иррациональных чисел	Set of irrational quantities
151.	Haqiqiy sonlar to‘plami	Множества действительных чисел	Set of real numbers
152.	Sonlar o‘qi	Числовая ос	Numerical axis
153.	Oraliq	Интервал	Interval
154.	Kesma	Отрезок	Segment
155.	Yarim oraliq	Полуинтервал	Half-interval
156.	Yarim cheksiz oraliq	Полубесконечный интервал	Half infinite interval
157.	Cheksiz oraliq	Бесконечный интервал	Infinite interval
158.	Ochiq to‘plamyopiq to‘plam	Открытые множество	Open set
159.	Yopiq to‘plam	Замкнутое множество	Reserved set
160.	Nuqta atrofi	Окресность точки	Environs of the point
161.	Yuqori chegaralangan to‘plam	Множество ограниченную сверху	Limited set from the top
162.	Quyidan chegaralangan to‘plam	Множество, ограниченное снизу	Limited set from below
163.	Chegaralangan to‘plam	Ограниченное множество	Limited set
164.	Sonning absolyut qiymati	Абсолютное значение числа	Absolute meaningful quantity
165.	Sonli ketma-ketlik	Числовая последовательность	Quantity succession

166.	Quyidan chegaralangan ketma-ketlik	Числовая последовательность, ограниченная снизу	Quantity succession from below
167.	Yuqoridan chegaralangan ketma-ketlik	Числовая последовательность, ограниченная сверху	Quantity succession from the top
168.	Chegaralangan ketma-ketlik	Ограниченная последовательность	Limited succession
169.	Sonli ketma-ketlik limiti	Передел числовой последовательности	Limit of quantity succession
170.	O‘zgarmas ketma-ketlik	Постоянная последовательность	Constant succession
171.	Yaqinlashuvchi ketma-ketlik	Сходящая последовательность	Intimate succession
172.	Uzoqlashuvchi ketma-ketlik	Расходящая последовательность	Disperse succession
173.	Monoton ketma-ketlik	Монотонная последовательность	Monotonous succession
174.	Muxim ketma-ketlik	Замечательный предел	Substantial limit
175.	O‘zgarmas miqdorlar	Постоянные величины	Constant quantities
176.	O‘zgaruvchi miqdorlar	Переменные величины	Variable quantities
177.	Funksiya	Функция	Function
178.	Aniqlash sohasi	Область определения	Field of definition
179.	Qiymatlar sohasi	Область значений	Field of value
180.	Funksiya grafigi	График функции	Diagram of function
181.	O‘suvchi funksiya	Возрастающая функция	Increasing function

182.	Kamayuvchi funksiya	Убивающая функция	Decreasing function
183.	Monoton funksiyalar	Монотонные функции	Monotonous functions
184.	Juft funksiya	Четная функция	Even functions
185.	Toq funksiya	Нечетная функция	Odd functions
186.	Davriy funksiya	Периодичная функция	Periodical function
187.	Chegarlangan funksiya	Ограниченная функция	Limited function
188.	Chegaralanmagan funksiya	Неограниченная функция	Unlimited function
189.	O‘zgarmas funksiya	Постоянная функция	Constant function
190.	Murakkab funksiya	Сложная функция	Complex function
191.	Teskari funksiya	Обратная функция	Inverse function
192.	Oshkormas funksiya	Неявная функция	Non – evident function
193.	Asosiy elementar funksiyalar	Основные элементарные функции	Main elementary functions
194.	Funksiyaning limiti	Предел функции	Limit of function
195.	Chap limit	Левый предел	Left limit
196.	O‘ng limit	Правый предел	Right limit
197.	Cheksiz kichik limit	Бесконечно малые величины	Unlimited small quantity
198.	Cheksiz katta limit	Бесконечно большие величины	Unlimited large quantity
199.	Yig‘indining limiti	Предел суммы	Limit of sum
200.	Ko‘paytmaning limiti	Предел произведения	Limit of derivative
201.	Bo‘linmaning limiti	Предел частного	Limit of quotient

202.	Funksiyaning nuqtadagi uzluksizligi	Непрерывность функции в точке	Continuity of function on the point
203.	Argument orttirmasi	Приращение аргумента	Increase of argument
204.	Funksiya orttirmasi	Приращение функции	Increase of function
205.	Oraliqda uzluksizlik	Непрерывность в интервале	Continuity in the interval
206.	Kesmada uzluksizlik	Непрерывность в отрезке	Continuity on segment
207.	Kesmadagi eng katta qiymat	Наибольшее значениена отрезке	The largest value on segment
208.	Kesmadagi eng kichik qiymat	Наименьшее значениена отрезке	The least value on segment
209.	Uzulish nuqtalari	Точки разрыва	Point of break
210.	Funksiyaning hosilasi	Производная функция	Derivative of function
211.	Hosilaning geometrik ma’nosi	Геометрический смысл производной	Geometric significance of a derivative
212.	Hosilaning mexanik ma’nosi	Механический смысл производной	Mechanic significance of a derivative
213.	Differentsiallashuvchi funksiya	Дифференцируемые функции	Differentiated functions
214.	Differentsiallash amali	Действия дифференциала	Operation of differential
215.	Hosilani hisoblash algaritmi	Алгоритм вычисления производной	Algorithm of calculation of a derivative
216.	O‘zgarmas son	Производная постоянная	Derivative of a

	hosilasi	числа	constant number
217.	Yig‘indini hosilasi	Производная суммы	Sum of derivative
218.	Ko‘paytmani hosilasi	Производная произведения	Derivative of product
219.	Bo‘linmaning hosilasi	Производная частного	Derivative of quotient
220.	Teskari funksiya hosilasi	Производная обратной функции	Derivative of inverse function
221.	Murakkab funksiya hosilasi	Производная сложной функции	Derivative of complex function
222.	Oshkormas funksiya hosilasi	Производная неявной функции	Derivative of non-evident function
223.	Darajali-ko‘rsatkichli funksiya	Степенно показательная функция	Degree model function
224.	Hosilalar jadvali	Таблицы производных	Schedule of derivatives
225.	Parametrik shaklda berilgan funksiyaning hosilasi	Производная функции заданной в параметрической форме	Derivative of function set in parametric form
226.	Funksiya differentsiali	Дифференциал функции	Function of differential
227.	Ko‘paytmaning differentsiali	Дифференциал суммы	Differential of sum
228.	Yig‘indini differentsiali	Дифференциал произведения	Differential of a derivative
229.	Bo‘linmaning differentsiali	Дифференциал частного	Differential of quotient

230.	Yuqori tartibli hosilalar	Производные высшего порядка	High order derivatives
231.	Ikkinchi tartibli hosilaning mexanik ma’nosi	Механический смысл производная второго порядка	Mechanic significance of a second order derivative
232.	Funktsiyuaning o‘sish oralig‘i	Интервал возрастания функции	Interval of the increase of function
233.	Funktsiyuaning kamayish oralig‘i	Интервал убывания функции	Interval of the decrease of function
234.	Funktsiyuaning maksimumi	Максимум функции	Maximum of a function
235.	Funktsiyuaning minimumi	Минимум функции	Minimum of a function
236.	Funktsiyuaning ekstremumlari	Экстремумы функции	Extremuims of function
237.	Kritik nuqta	Стационарные точки	Stationary point
238.	Botiqlik oralig‘i	Интервал вогнутости	Interval of conicavity
239.	Qavarinlik oralig‘i	Интервал выпуклости	Point of bending
240.	Burilish nuqta	Точки перегиба	Turning point
241.	Og‘ma asimtota	Наклонная асимптота	Inclined asymptote
242.	Gorizontal asimtota	Горизонтальная асимптота	Horizontal asymptote
243.	Vertical asimtota	Вертикальная асимптота	Vertical asymptote
244.	$\frac{0}{0}$ ko ‘rinishdagi aniqmaslik	Неопределенность вида $\frac{0}{0}$	Vagueness in the form of
245.	$\frac{\infty}{\infty}$ ko ‘rinishdagi aniqmaslik	Неопределенность вида $\frac{\infty}{\infty}$	Vagueness in the form of
246.	Aniqmasliklarni	Раскрытие	Opening of vagueness

	ochish	неопределенности	
247.	Lopitalning I-qoidasi	Первое правило Лопиталя	Lopital’s first rule
248.	Lopitalning II-qoidasi	Второе правило Лопиталя	Lopital’s second rule
249.	$0 \cdot \infty$ ko‘rinishdagi aniqmaslik	Неопределенность вида $0 \cdot \infty$	Vagueness in the form of
250.	1^{∞} ko‘rinishdagi aniqmaslik	Неопределенность вида 1^{∞}	Vagueness in the form of
251.	∞^{0} ko‘rinishdagi aniqmaslik	Неопределенность вида ∞^{0}	Vagueness in the form of
252.	$\infty \cdot \infty$ ko‘rinishdagi aniqmaslik	Неопределенность вида $\infty \cdot \infty$	Vagueness in the form of
253.	Boshlang‘ich funksiya	Первообразная функция	Prototype function
254.	Aniqmas interval	Неопределенный интеграл	Indefinite integral
255.	Integral ostidagi ifoda	Подинтегральная выражения	Under integral expression
256.	Integral ostidagi funksiya	Подинтегральная функция	Under integral function
257.	Integrallash o‘zgaruvchisi	Переменная интегрирования	Variable integration
258.	Integrallash amali	Действия интегрирования	Operation of integration
259.	Integrallash jadvali	Таблицы интегралов	Schedule of integration
260.	Aniqmas integralli	Непосредственное	Immediate calculation

	bevosita xisoblash	вычисления неопределенного интеграла	of an indefinite integral
261.	O‘zgaruvchilarni almashtirish usuli	Метод замены переменных	Method of substitution of variables
262.	Bo‘laklab integrallash usuli	Метод интегрирования по частям	Method of integration on parts
263.	Ko‘phad	Многочлен	Multinominal
264.	Ratsional funksiya	Рациональная функция	Rational function
265.	I – tur eng sodda rational kasr	Самый простой рациональный дробь I - типа	The most simple rational fraction of the I st type
266.	II – tur eng sodda rational kasr	Самый простой рациональный дробь II – типа	The most simple rational fraction of the II nd type
267.	III – tur eng sodda rational kasr	Самый простой рациональный дробь III – типа	The most simple rational fraction of the III rd type
268.	IV – tur eng sodda rational kasr	Самый простой рациональный дробь IV – типа	The most simple rational fraction of the IV th type
269.	Noma’lum koeffissientlar usuli	Метод неизвестных коэффициентов	Method of unknown coefficient
270.	Irrational funksiya	Иррациональная функция	Irrational function
271.	Universal almashtirish	Универсальная подстановка	Universal substitution
272.	Integral yig‘indi	Интегральная сумма	Integral sum
273.	Aniq integral	Определенный интеграл	Concrete integral
274.	Quyi chegara	Нижняя граница	Lower limit

275.	Yuqori chegara	Верхняя граница	Upper limit
276.	Aniq integralning geometrik ma’nosi	Геометрический смысл определенного интеграла	Geometrical meaning of a definite integral
277.	Nyuton – Leybnits formulasi	Формула Ньютона-Лейбница	Formula of Newton – Laybnits
278.	To‘g‘ri to‘rtburchaklar formulasi	Формула прямоугольника	Formula of right-angled quadrangle
279.	Egri chiziqli trapetsiya yuzasi	Площадь криволинейной трапеции	Area of curvilinear trapezium
280.	Egri chiziq yoyi uzunligi	Длина дуги кривая линии	The length of curvilinear arc
281.	Aylanma jism hajmi	Объем тела вращения	Volume of rotation of a circle
282.	O‘zgaruvchan kuch bajargan ish	Работа выполненные переменной силы	The work done by variable power
283.	Og‘irlik markazining koordinatalari	Координаты центра тяжести	Coordinates of centre of gravity
284.	Xosmas inregral	Несобственный интеграл	Improper integral

ADABIYOTLAR

1. Matematika ta’limi va fanlarini yanada rivojlantirishni davlat tomonidan qo’llab quvvatlash, shuningdek, O’zbekiston Respublikasi fanlar akademiyasining V.I. Ramanovskiy nomidagi matematika instituti faoliyatini tubdan takomillashtirish chora-tadbirlari to’g’risida”gi farmoni. 2019 yil 9-iyul.

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