O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

TOSHKENT ARXITEKTURA QURILISH INSTITUTI

OLIY MATEMATIKA

Sirtdan o‘qiyotgan talabalar uchun uslubiy ko‘rsatmalar va nazorat ishlari

2-QISM

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Oliy matematika. Sirtdan o‘qiyotgan talabalar uchun uslubiy ko‘rsatmalar va nazorat ishlari/ Sh.R.Xurramov, A.Abduraximov, F.S.Xolto‘rayev, N.U.o‘.Annayev–Toshkent: 2019.– 38 bet.

Uslubiy qo‘llanma oliy ta’lim muassasalarining 5340200 - Bino va inshootlar qurilishi, 5340300 – Shahar qurilishi va xo‘jaligi, 5340400 - Muhandislik kommunikatsiyalari qurilishi va montaji, 5340500 -Qurilish materiallari, buyumlarini va konstruksiyalarini ishlab chiqarish, 340700 -Gidrotexnika qurilishi, 5341100 – Qiymat injenering, 5311500 - Geodeziya, kartografiya va kadastr ta’lim yo‘nalishlari bo‘yicha sirtdan ta’lim olayotgan talabalari uchun mo‘ljallangan.

Qo‘llanmada «Oliy matematika» kursining «Bir o‘zgaruvchi funksiyasining differensial hisobi», «Bir o‘zgaruvchi funksiyasining integral hisobi», va «Oddiy differensial tenglamalar» nazorat ishlari uchun topshiriiqlar variantlari keltirilganNazorat ishlarining har bir topshirig‘iga oid na’munaviy misol-masalalar yechib ko‘rsatilgan. Sirtdan o‘qiyotgan talabalarning nazorat ishlarini bajarishlari bo‘yicha umumiy tavsiyalar berilgan.

SO‘Z BOSHI

Ushbu o‘quv-uslubiy materiallar qurilish ta’lim yo‘nalishlarida sirtdan o‘qiyotgan talabalar uchun mo‘ljallangan va «Oliy matematika» fanini o‘rganishda ular uchun ko‘rsatma vazifasini o‘taydi. U sirtqi talabalar uchun nazorat ishlarini bajarishlariga oid asosiy tavsiyalarni va shuningdek fanning «Bir o‘zgaruvchi funksiyasining differensial hisobi», «Bir o‘zgaruvchi funksiyasining integral hisobi» va «Oddiy differensial tenglamalar» bo‘limlarini o‘rganish bo‘yicha uslubiy ko‘rsatmalarni o‘z ichiga oladi.

Uslubiy ko‘rsatmada «Oliy matematika» fanidan savollar, tavsiya qilinayotgan adabiyotlar ro‘yxati va nazorat ishlari uchun yigirma besh variantdan iborat topshiriiqlar keltirilgan. Nazorat ishlarining har bir topshirig‘iga oid namunaviy misol-masalalar yechib ko‘rsatilgan.

Materiallarda nazorat topshiriqlari yigirma besh variant uchun berilgan bo‘lib ular uchta qismga ajratilgan.

Ushbu qo‘llanma «Matematika va tabiiy fanlar» kafedrasi tomonidan qurilish ta’lim yo‘nalashlari sirtqi talabalarini o‘quv-uslubiy ta’minlashning tarkibiy qismlaridan biri hisoblanadi.

SIRTDAN O‘QIYOTGAN TALABALAR USHUN NAZORAT ISHLARINI BAJARISH BO‘YICHA UMUMIY TAVSIYALAR

1. Sirtqi talaba fanni o‘rganish jarayonida oily matematikaning turli bo‘limlaridan nazorar ishlaruini bajarishi lozim. Bu nazorat ishlari o‘qituvchi tomonidan taqriz qilinadi. Bajarilgan ishga yozilgan taqriz talabaga uning materialni o‘zlashtirganligi bo‘yicha baho berish imkonini beradi, mavjud kamchiliklarini ko‘rsatadi va keyingi ishlarini muvofiqlashtiradi va o‘qituvchining qo‘yiladigan savollarni tizimlashtirishida yordam beradi.

2. O‘rganilayotgan material bo‘yicha yetarli sondagi misol va masala yechmasdan talaba nazorat ishini bajarishga kirishmasligi lozim.

3. Har bir nazorat ishi mustaqil bajarilishi kerak. Mustaqil bajarilmagan nazorat ishi taqrizchi - o‘qituvchiga uning ishida materialni o‘zlashtirish bo‘yicha kamchiliklarni ko‘rsatishi uchun imkon bermaydi, natijada talaba kerakli bilimga ega bo‘lmasdan yakuniy nazoratni topshirish uchun tayyor bo‘lmasligi mumkin.

4. Nazorat ishi o‘z vaqtida topshirilishi lozim. Bu talabning bajarilmasligi taqrizchi - o‘qituvchiga talabaning kamchiliklarini o‘z vaqtida ko‘rsatish imkonini bermaydi va ishning taqriz qilinishi vaqtini cho‘zilishiga olib keladi.

5. Nazarat ishini bajarish va rasmiylashlashtirishda talaba quyidagi qoidalarga qat’iy amal qilishi lozim:

a) nazorat ishi alohida daftarga taqrizchi - o‘qituvchining qaydlari uchun xoshiya qoldirilgan holda bajarilshi kerak;

b) daftarning muqavasida quyidagilar qayd etilishi lozom:

- oily matematikadan nazaorat ishi va uning tartib raqami;

- talabaniing familiyasi va ismi-sharifi, reyting daftarchasining nomeri;
- fakultet, kurs, guruh;
- ishning oily o‘quv yurtiga jo‘natilgan sanasi va talabaning manzili.

v) masalalarning yechimi uning keltirilgan tartibida joylashtirilishi kerak;

g) har bir masalani yechishdan oldin uning sharti zarur joylarda harfli ifodalar o‘zining variantiga mos qiymatlar bilan almashtirilgan holda to‘liq ko‘chirilishi kerak;

d) masala yechimining asosiy bosqichlari qisqa va lo‘nda izohlar bilan berilishi lozim;

e) nazorat ishining oxirida foydalanilgan adabiyotlar ro‘yxati berilishi kerak.

6. Talaba reyting daftarchasi nomerining oxirgi ikki raqamiga mos variantni bajaradi. Bunda bu ikki raqam 25 ga bo‘linadi va qoldiq talaba bajarishi kerak bo‘lgan variant nomerini bildiradi. Agar bu ikki raqam 75,50,25,00 dan iborat bo‘lsa, talaba 25- variantni bajaradi.

7. Taqriz qilingan ishni olgandan so‘ng talaba taqrizchi tomonidan ko‘rsatilgan kamchiliklarni tuzatishi va ishni qayta taqrizga jo‘natishi lozim.

8. Belgilangan tartibda taqrizdan o‘tgan va inobatga olingan (zachet qilingan) nazorat ishlarini topshirmagan talaba yakuniy nazoratga kiritilmaydi.

«OLIY MATEMATIKA» KURSIDAN SAVOLLAR RO‘YXATI.

Bir o‘zgaruvchi funksiyasining differensial hisobi

Hosila tushunchasiga olib keluvchi masalalar. Hosilaning ta’rifi, geometrik va mexanik ma’nolari. Funksiyaning differensiallanuvchanligi. Funksiyaning differensiali.

Yig’indi, ayirma, ko’paytma va bo’linmani differensiallash. Teskari funksiyani differensiallash. Murakkab funksiyani differensiallash. Asosiy elementar funksiyalarning hosilalari. Differensiallash qoidalari va hosilalar jadvali. Logarifmik differensiallash. Parametrik va oshkormas ko‘rinishda berilgan funksiyalarni differensiallash. Yuqori tartibli hosila va differensiallar.

Ferma, Roll, Lagranj va Koshi teoremalari. Lopital qoidasi. Teylor formulasi

Funksiyaning monotonlik shartlari. Funksiyaning ekstremumlari. Kesmada uzluksiz funkiyaning eng katta va eng kichik qiymatlari. Funksiya grafigining qavariqligi, botiqligi va egilish nuqtalari. Funksiya grafigining asimptotalari. Funksiyani tekshirish va grafigini chizisning umumiy sxemasi.

Bir o‘zgaruvchi funksiyasining integral hisobi

Boshlang’ich funksiya va aniqmas integral. Aniqmas integralning xossalari. Asosiy integrallar jadvali. Integrallash usullari.

Sodda kasrlarni integrallash. Ratsional kasr funksiyalarini integrallash.

Trigonometrik funksiyalarni integrallash.

Irratsional ifodalarni integrallash. Elementar funksiyalarda ifodalanmaydigan integrallar

Aniq integral tushunchasiga olib keluvchi masalalar. Integral yig'indi va aniq integral. Aniq integralning geometrik va mexanik ma'nolari. Aniq integralning xossalari.

Yuqori chegarasi o'zgaruvchi aniq integral. Nyuton-Leybnits formulasi. Aniq integralda o'zgaruvchini almashtirish. Aniq integralni bo'laklab integrallash.

Cheksiz chegarali xosmas integrallar. Uzilishga ega bo'lgan funksiyalarning xosmas integrallari. Xosmas integrallarning yaqinlashish alomatlari.

Aniq integralning geometrik va fizik tatbiqlari.

Oddiy differensial tenglamalar

Differensial tenglamalarga keltiruluvchi masalalar. Birinchi tartibli differensial tenglamalar. Koshi masalasi. O'zgaruvchilari ajraladigan differensial tenglamalar. Bir jinsli tenglamalar. Birinchi tartibli chiziqli differensial tenglamalar. Bernulli tenglamasi. To'liq differensialli tenglamalar.

Yuqori tartibli differensial tenglamalar. Koshi masalasi. Tartibini pasaytirish mumkin bo'lgan differensial tenglamalar

Chiziqli bir jinsli tenglamalar. O'zgarmas koeffitsiyentli ikkinchi tartibli chiziqli bir jinsli differensial tenglamalar. Bir jinsli bo'lmagan yuqori tartibli va ikkinchi tartibli chiziqli differensial tenglamalar. Lagranjning ixtijoriy o'zgarmasni variatsiyalash usuli. O'ng tomoni maxsus ko'rinishdagi tenglamalar.

Differensial tenglamalarni matematik paketlarda yechish.

Differensial tenglamalarning normal sistemasi. Normal sistemani yechish usullari. O'zgarmas koeffitsientli birinchi tartibli chiziqli differensial tenglamalar sistemasini yechish.

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NAZORAT ISHINI BAJARISH BO‘YICHA USLUBIY KO‘RSATMALAR

Uslubiy ko‘rsatmaning ushbu bandida nazorat ishlarining namunaviy masalalari yechib ko‘rsatilgan. Masalalarning yechimi talaba nazorat ishini bajarishi jarayonida o‘rganishi kerak bo‘lgan mavzular bo‘yicha keltirilgan. Masalalarning yechimi talaba o‘zining variantini bajarishida faodalanilishi mumkin bo‘lgan formula va tushunchalarni o‘z ichiga olgan. Ta’kidlash joizki, bu formula va nazariy tushunchalar faqat amaliy mashg‘ulotlarda va nazorat ishlarini bajarilishida qo‘llanilishi mumkin. Ular yakuniy nazoratni topshirish uchun yetarli emas.

1-MAVZU. BIR O‘ZGARUVCHI FUNKSIYASINING DIFFERENSIAL HISOBI

1-masala. Hosilalarni toping:

1) $y = \sqrt[5]{3-7x-x^2} + \dfrac{4}{(x-7)^5}$; 2) $y = arctg^3 4x \cdot 3^{\sin x}$;

3) $y = \dfrac{\sqrt[3]{2x^2-3x+1}}{e^{\frac{x}{3}}}$; 4) $y = x^{3\sin x}$.

Yechish. Berilgan hosilalarni differensiallash qoidalari va differensiallar jadvalidan foydalanib hisoblaymiz.

$$1)\ y' = \left(\sqrt[5]{3-7x-x^2}\right)' + \left(\frac{4}{(x-7)^5}\right)' = \left((3-7x-x^2)^{\frac{1}{5}}\right)' + \left(4(x-7)^{-5}\right)' =$$

$$= \frac{1}{5}(3-7x-x^2)^{-\frac{4}{5}}(3-7x-x^2)' + 4(-5)(x-7)^{-6}(x-7)' =$$

$$= \frac{1}{5\sqrt[5]{(3-7x-x^2)^4}} \cdot (-7-2x) - \frac{20}{(x-7)^6} \cdot 1 = -\frac{7+2x}{5\sqrt[5]{(3-7x-x^2)^4}} - \frac{20}{(x-7)^6}.$$

$$2)\ y' = (arctg^3 4x \cdot 3^{\sin x})' = (arctg 4x)' \cdot 3^{\sin x} + arctg^3 4x \cdot (3^{\sin x})' =$$

$$= 3arctg^2 4x (arctg 4x)' \cdot 3^{\sin x} + arctg^3 4x \cdot 3^{\sin x} \ln 3 \cdot (\sin x)' =$$

$$= 3arctg^2 4x \cdot \frac{1}{1+16x^2} \cdot (4x)' \cdot 3^{\sin x} + arctg^3 4x \cdot 3^{\sin x} \ln 3 \cdot \cos x =$$

$$= 3arctg^2 4x \cdot \frac{4}{1+16x^2} \cdot 3^{\sin x} + arctg^3 4x \cdot 3^{\sin x} \ln 3 \cdot \cos x =$$

$$= 3^{\sin x} arctg^2 4x \cdot \left(\frac{12}{1+16x^2} + \ln 3 \cdot arctgx \cdot \cos x \right).$$

$$3)\ \ y' = \left(\frac{\sqrt[3]{2x^2 - 3x + 1}}{e^{\frac{x}{3}}} \right)' = \frac{\left((2x^2 - 3x + 1)^{\frac{1}{3}} \right)' e^{\frac{x}{3}} - (2x^2 - 3x + 1)^{\frac{1}{3}} \left(e^{\frac{x}{3}} \right)'}{e^{\frac{2x}{3}}} =$$

$$= \frac{\frac{1}{3}(2x^2 - 3x + 1)^{-\frac{2}{3}} (2x^2 - 3x + 1)' e^{\frac{x}{3}} - (2x^2 - 3x + 1)^{\frac{1}{3}} e^{\frac{x}{3}} \left(\frac{x}{3} \right)'}{e^{\frac{2x}{3}}} =$$

$$= \frac{e^{\frac{x}{3}} \left(\frac{4x - 3}{3\sqrt[3]{(2x^2 - 2x + 1)^2}} - \frac{1}{3}\sqrt[3]{2x^2 - 3x + 1} \right)}{e^{\frac{2x}{3}}} =$$

$$= \frac{4x - 3 - 2x^2 + 3x - 1}{3e^{\frac{x}{3}} \sqrt[3]{(2x^2 - 3x + 1)^2}} = \frac{-2x^2 + 7x - 4}{3e^{\frac{x}{3}} \sqrt[3]{(2x^2 - 3x + 1)^2}}.$$

4) Logarifmik differensiallash formulasidan foydalanamiz:

$$(u^v)' = u^v \left(v' \ln u + \frac{vu'}{u} \right).$$

Shartga ko'ra $u = x,\ v = 3\sin x$. Bundan $u' = 1,\ v' = 3\cos x$.

U holda

$$y' = (x^{3\sin x})' = x^{3\sin x} \left(3\cos x \ln x + \frac{3\sin x \cdot 1}{x} \right) = x^{\sin x} \left(3\cos x \ln x + \frac{3\sin x}{x} \right).$$

2-masala. Parametrik ko'rinishida berilgan y funksiyalarning x bo'yicha ikkinchi tartibli hosilasini toping:

$$\begin{cases} x = t^2 + t + 1, \\ y = t^3 + t. \end{cases}$$

Yechish. Avval birinchi tartibli hosilani topamiz:

$$y'_x = \frac{y'_t}{x'_t} = \frac{(t^3 + t)'_t}{(t^2 + t + 1)'_t} = \frac{3t^2 + 1}{2t + 1}.$$

U holda

$$y''_{xx}=\frac{(y'_x)'_t}{x'_t}=\frac{\left(\dfrac{3t^2+1}{2t+1}\right)'_t}{2t+1}=\frac{(3t^2+1)'(2t+1)-(2t+1)'(3t^2+1)}{(2t+1)^3}=$$

$$=\frac{6t(2t+1)-2(3t^2+1)}{(2t+1)^3}=\frac{6t^2+6t-2}{(2t+1)^3}.$$

3-masala. limitni Lopital qoidasidan foydalanib toping:

$$\lim_{x\to\frac{\pi}{2}}(2-2x)^{tg\pi x}.$$

Yechish. $\lim\limits_{x\to\frac{1}{2}}(2-2x)^{tg\pi x}=(1^\infty)=e^{\lim\limits_{x\to\frac{\pi}{2}} tg\pi x\ln(2-2x)}$.

Bunda

$$\lim_{x\to\frac{1}{2}} tg\pi x\ln(2-2x)=(\infty\cdot 0)=\lim_{x\to\frac{1}{2}}\frac{\ln(2-2x)}{ctg\pi x}=\left(\frac{0}{0}\right).$$

Oxirgi limitga Lopital qoidasini qo‘llaymiz:

$$\lim_{x\to\frac{1}{2}}\frac{\ln(2-2x)}{ctg\pi x}=\lim_{x\to\frac{1}{2}}\frac{(\ln(2-2x))'}{(ctg\pi x)'}=\lim_{x\to\frac{\pi}{2}}\frac{\dfrac{-2}{2-2x}}{-\dfrac{\pi}{\sin^2\pi x}}=\frac{2}{\pi}.$$

Demak,

$$\lim_{x\to\frac{\pi}{2}}(2-2x)^{tg\pi x}=e^{\frac{2}{\pi}}.$$

4-masala. Funksiyani to‘la tekshiring va grafigini chizing:

$$y=\frac{x^2+1}{x-1}.$$

Yechish. 1^o. Funksiyaning aniqlanish sohasi: $D(f)=(-\infty;1)\cup(1;\infty)$;

2^o. $x=0$da $y=-1$ bo‘ladi. Funksiya Oy o‘qini $(0;-1)$nuqtada kesadi. $y\neq 0$ bo‘lgani uchun funksiya Ox o‘qini kesmaydi.

3^o. Funksiya $(1;+\infty)$intervalda musbat ishorali va $(-\infty;1)$intervalda manfiy ishorali.

4^o. Funksiya uchun $f(-x)=f(x)$va $f(-x)=-f(x)$tengliklar bajarilmaydi. Demak, u umumiy ko‘rinishdagi funksiya.

5^o. $\lim\limits_{x\to 1+0}\dfrac{x^2+1}{x-1}=+\infty$ va $\lim\limits_{x\to 1-0}\dfrac{x^2+1}{x-1}=-\infty$.

Demak, $x=1$ to‘g‘ri chiziq vertikal asimptota bo‘ladi.

$$k=\lim_{x\to\pm\infty}\frac{x^2+1}{x(x-1)}=1, \quad b=\lim_{x\to\pm\infty}\left(\frac{x^2+1}{x-1}-1\cdot x\right)=\lim_{x\to\pm\infty}\frac{x+1}{x-1}=1.$$

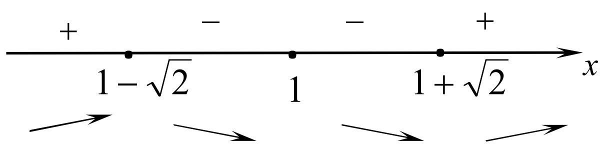
Demak, $y=x+1$ to‘g‘ri chiziq $x\to+\infty$ da ham $x\to-\infty$ da ham gorizontal asimptota bo‘ladi.

6^o. Funksiyaning o‘sish va kamayish oraliqlarini topamiz.

$f'(x)=\dfrac{2x(x-1)-x^2-1}{(x-1)^2}=\dfrac{x^2-2x-1}{(x-1)^2}$, $f'(x)=0$ dan $x_1=1-\sqrt{2}, \quad x_2=1+\sqrt{2}$.

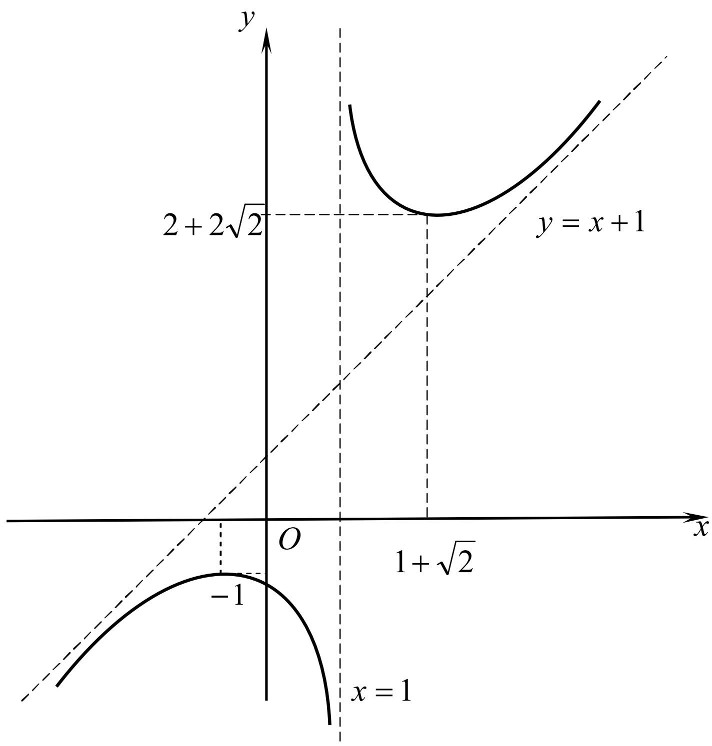
Hosila $x=1$ nuqtada mavjud emas va $x_1=1-\sqrt{2}, \quad x_2=1+\sqrt{2}$ $x=0$ nuqtalarda nolga teng. Bu nuqtalar berilgan funksiyaning aniqlanish sohasini to‘rtta $(-\infty;1-\sqrt{2})$, $(1-\sqrt{2};1)$, $(1;1+\sqrt{2})$, $(1+\sqrt{2};+\infty)$ intervallarga ajratadi. Funksiya $(-\infty;1-\sqrt{2})$, $(1+\sqrt{2};+\infty)$ intervallarda o‘sadi va $\left(1-\sqrt{2};1\right)$, $(1-\sqrt{2};1)$, $(1;1+\sqrt{2})$ intervallarda kamayadi.

7^o. Funksiyani ekstremumga tekshiramiz. Hosilaning har bir kritik nuqtadan chapdan o‘ngga o‘tgandagi ishoralarini chizmada belgilaymiz:



Demak, $x=1-\sqrt{2}$ maksimum nuqta, $x=1+\sqrt{2}$ minimum nuqta.

$y_{\max}=f(1-\sqrt{2})=2-2\sqrt{2}, \ y_{\min}=f(1+\sqrt{2})=2+2\sqrt{2}$.



1-shakl

8^o. Funksiyani qavariqlikka va botiqlikka tekshiramiz va egilish nuqtalarini topamiz.

$$f''(x)=\frac{(2x-2)(x-1)^2-2(x-1)(x^2-2x-1)}{(x-1)^4}==\frac{4}{(x-1)^3},\ f''(x)\neq 0$$

Ikkinchi tartibli hosila $x_3=1$ nuqtada mavjud emas. y'' hosilaning ishorasi bu nuqtadan chapda manfiy va o‘ngda musbat.

Demak, funksiyaning grafigi $(-\infty;1)$ intervalda qavariq, $(1;+\infty)$ intervalda botiq bo‘ladi. Funksiya grafigining egilish nuqtasi yo‘q.

1^o-8^o bandlardagi tekshirishlar asosida funksiya grafigini chizamiz (1-shakl).

2-MAVZU. BIR O‘ZGARUVCHI FUNKSIYASINING INTEGRAL HISOBI

5-masala. Aniqmas integrallarni toping:

1) $\displaystyle\int\frac{4x^2+7x+5}{(x-1)(x^2+2x+5)}dx$; 2) $\displaystyle\int\frac{2-\sin x+3\cos x}{1+\cos x}dx$;

3) $\displaystyle\int\frac{\sqrt[3]{(x+3)^2}+\sqrt[6]{x+3}}{\sqrt{x+3}+\sqrt[3]{x+3}.}dx$; 4) $\displaystyle\int\frac{\sqrt[3]{(1+\sqrt[4]{x})^2}}{x\cdot\sqrt[12]{x^5}}dx.$

Yechish. 1) Integral ostidgi funksiya to‘g‘ri kasrdan iborat. Kasrning maxrajidagi x^2+2x+5 kvadrat uchhad ko‘paytuvchilarga ajralmaydi, chunki

$$\frac{p^2}{4}-q=-4<0.$$

U holda kasrni

$$\frac{4x^2+7x+5}{(x-1)(x^2+2x+5)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+2x+5}$$

ko’rinishda yozib olamiz.

Tenglikning chap va o‘ng tomonlarini umumiy maxrajga keltiramiz va suratlarni tenglashtiramiz:

$$4x^2+7x+5=A(x^2+2x+5)+(Bx+C)(x-1).$$

$A,\ B,\ C$ koeffitsiyentlarni topamiz:

$$\begin{cases} x=1:16=8A,\\ x^2:\ 4=A+B,\\ x^0:\ 5=5A-C.\end{cases}$$

Bundan $A=2,\ B=2,\ C=5.$ Shunday qilib,

$$\int\frac{4x^2+7x+5}{(x-1)(x^2+2x+5)}dx=2\int\frac{dx}{x-1}+\int\frac{2x+5}{x^2+2x+5}dx=2\ln|x-1|+\int\frac{d(x^2+2x+5)}{x^2+2x+5}+$$

$$+3\int\frac{d(x+1)}{(x+1)^2+2^2}=2\ln|x-1|+\ln|x^2+2x+5|+\frac{3}{2}arctg\frac{x+1}{2}+C.$$

2) Integralda almashtirishlar bajaramiz:

$$\int\frac{2-\sin x+3\cos x}{1+\cos x}dx=\int\frac{3+3\cos x-1-\sin x}{1+\cos x}dx=3\int dx-\int\frac{1+\sin x}{1+\cos x}dx=3x-I_1+C.$$

I_1 integralni universal trigonometrik o‘rniga qo‘yish orqali ratsionallashtiramiz:

$$I_1=\int\frac{1+\sin x}{1+\cos x}dx=\left|\begin{array}{c} t=tg\frac{x}{2},\ \sin x=\frac{2t}{1+t^2},\ \cos x=\frac{1-t^2}{1+t^2},\\ dx=\frac{2dt}{1+t^2},\ x=arctgt\end{array}\right|=$$

$$=\int\frac{1+\frac{2t}{1+t^2}}{1+\frac{1-t^2}{1+t^2}}\cdot\frac{2dt}{1+t^2}=\int\frac{1+t^2+2t}{1+t^2}dt=\int dt+\int\frac{2tdt}{1+t^2}=t+\int\frac{d(1+t^2)}{1+t^2}=$$

$$=t+\ln|1+t^2|=tg\frac{x}{2}+\ln\left|1+tg^2\frac{x}{2}\right|=tg\frac{x}{2}-2\ln\left|\cos\frac{x}{2}.\right|$$

Demak,

$$\int\frac{2-\sin x+3\cos x}{1+\cos x}dx=3x-tg\frac{x}{2}+2\ln\left|\cos\frac{x}{2}\right|+C.$$

3) $x+3=t^6$ belgilash kiritamiz, chunki $EKUK(2,3,6)=6$.

Bundan $x=t^6-3,\ dx=6t^5dt.$

U holda

$$\int\frac{\sqrt[3]{(x+3)^2}+\sqrt[6]{x+3}}{\sqrt{x+3}+\sqrt[3]{x+3.}}dx=\int\frac{t^4+t}{t^3+t^2}\cdot 6t^5dt=$$

$$=6\int\frac{t^3+1}{t+1}\cdot t^4dt=6\int t^4(t^2-t+1)dt=$$

$$=\frac{6}{7}t^7-t^6+\frac{6}{5}t^5+C=\frac{6}{7}\sqrt[6]{(x+3)^7}+\frac{6}{5}\sqrt[6]{(x+3)^5}-x+C.$$

4). Integral ostidagi funksiyani standart shaklda yozib olamiz:

$$x^{-\frac{17}{12}}\left(1+x^{\frac{1}{4}}\right)^{\frac{2}{3}}.$$

Demak, $m=-\dfrac{17}{12},\ n=\dfrac{1}{4},\ p=\dfrac{2}{3}$. Bundan $\dfrac{m+1}{n}+p=-1$.

Chebishevning uchinchi o‘rniga qo‘yishidan foydalanamiz:

$$1+x^{\frac{1}{4}}=x^{\frac{1}{4}}t^3 \text{ yoki } x^{\frac{1}{4}}(t^3-1)=1.$$

Bundan

$$t=\left(\frac{1+\sqrt[4]{x}}{\sqrt[4]{x}}\right)^{\frac{1}{3}},\quad x=(t^3-1)^{-4},\quad dx=-12t^2(t^3-1)^{-5}dt.$$

U holda

$$\int\frac{\sqrt[3]{(1+\sqrt[4]{x})^2}}{x\cdot\sqrt[12]{x^5}}dx=-12\int(t^2-1)^{\frac{17}{3}}\cdot(t^3\cdot(t^3-1)^{-1})^{\frac{2}{3}}\cdot t^2(t^3-1)^{-5}dt=$$

$$=-12\int(t^2-1)^{\frac{17}{3}-\frac{2}{3}-5}t^{2+2}dt=-12\int t^4dt=$$

$$=-\frac{12}{5}t^5+C=-\frac{12}{5}\sqrt[3]{\left(\frac{1+\sqrt[4]{x}}{\sqrt[4]{x}}\right)^5}+C.$$

6-masala. Aniq integrallarni hisoblang:

1) $\displaystyle\int_0^{\frac{\pi}{9}}\frac{xdx}{\cos^2 3x}$; 2) $\displaystyle\int_{\frac{\pi}{2}}^{\pi}2^8\sin^6 x\cos^2 xdx.$

Yechish. 1) Aniq integralni bo‘laklab integrallash usuli bilan hisoblaymiz:

$$\int_0^{\frac{\pi}{9}}\frac{xdx}{\cos^2 3x}=\left|\begin{array}{c} u=x,\ du=dx,\\ dv=\dfrac{dx}{\cos^2 3x},\ v=\dfrac{1}{3}tg3x\end{array}\right|=\frac{1}{3}xtg3x\Big|_0^{\frac{\pi}{9}}-\frac{1}{3}\int_0^{\frac{\pi}{9}}tg3xdx=$$

$$=\frac{1}{3}\left(\frac{\pi}{9}tg\frac{\pi}{3}-0\right)+\frac{1}{9}\ln|\cos 3x|\Big|_0^{\frac{\pi}{9}}=\frac{\pi\sqrt{3}}{27}+\frac{1}{9}\left(\ln\left|\cos\frac{\pi}{3}\right|-\ln|\cos 0|\right)=$$

$$=\frac{\pi\sqrt{3}}{27}+\frac{1}{9}\left(\ln\frac{1}{2}-\ln 1\right)=\frac{1}{27}\left(\pi\sqrt{3}-3\ln 2\right).$$

2) Integral ostidagi funksiyaning darajasini pasaytiramiz:

$$2^8\sin^6 x\cos^2 x = 2^4(2^2\sin^4 x)(2^2\sin^2 x\cos^2 x) = 16(2\sin^2 x)^2(2\sin x\cos x)^2 =$$

$$= 16(1-\cos 2x)^2\sin^2 2x = 16(1-2\cos 2x+\cos^2 2x)\sin^2 2x =$$

$$= 16\sin^2 2x - 32\cos 2x\sin^2 2x + 16\sin^2 2x\cos^2 2x =$$

$$= 8(2\sin^2 2x) - 32\cos 2x\sin^2 2x + 4(2\sin 2x\cos 2x)^2 =$$

$$= 8 - 8\cos 4x - 32\cos 2x\sin^2 2x + 2(1-\cos 8x) =$$

$$= 10 - 8\cos 4x - 2\cos 8x - 32\sin^2 2x\cos 2x.$$

Integralni hisoblaymiz:

$$\int_{\frac{\pi}{2}}^{\pi} 2^8\sin^6 x\cos^2 x dx = 10\int_{\frac{\pi}{2}}^{\pi} dx - 8\int_{\frac{\pi}{2}}^{\pi}\cos 4x dx - 2\int_{\frac{\pi}{2}}^{\pi}\cos 8x dx - 32\int_{\frac{\pi}{2}}^{\pi}\sin^2 2x\cos 2x dx =$$

$$= 10x\Big|_{\frac{\pi}{2}}^{\pi} - 8\cdot\frac{\sin 4x}{4}\Big|_{\frac{\pi}{2}}^{\pi} - 2\cdot\frac{\sin 8x}{8}\Big|_{\frac{\pi}{2}}^{\pi} - 16\int_{\frac{\pi}{2}}^{\pi}\sin^2 2x d(\sin 2x) =$$

$$= 10\left(\pi - \frac{\pi}{2}\right) - 0 - 0 - 16\cdot\frac{\sin^3 2x}{3}\Big|_{\frac{\pi}{2}}^{\pi} = 5\pi.$$

7-masala. Berilgan l egri chiziqning ko‘rsatilgan o‘q atrofida aylanishidan hosil bo‘lgan sirt yuzasini hisoblang:

$l: x = 5\cos^3 t,\ y = 5\sin^3 t$ astroidaning $t=0$ dan $t=\dfrac{\pi}{2}$ gacha qismi, Oy.

Yechish. $x=\varphi(t),\ y=\psi(t),\ \alpha\le t\le\beta$ parametrik tenglamalar bilan berilgan egri chiziqning Oy o‘q atrofida aylanishidan hosil bo‘lgan jism sirti yuzasi

$$\sigma = 2\pi\int_{\alpha}^{\beta}\varphi(t)\sqrt{\varphi'^2(t)+\psi'^2(t)}dt$$

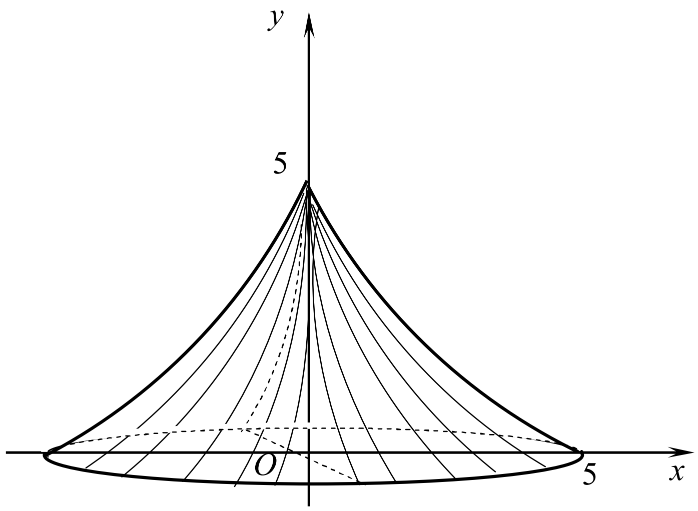
formula bilan hisoblanadi.

$x=5\cos^3 t,\ y=5\sin^3 t$ astroidaning $\left(0\le t\le\dfrac{\pi}{2}\right)$ Oy o‘q atrofida aylanishidan hosil bo‘lgan sirt yuazini hisoblaymiz: (2-shakl).

$$\sigma = 2\pi\int_0^{\frac{\pi}{2}} 5\cos^3 t\sqrt{(-15\cos^2 t\sin t)^2 + (15\sin^2 t\cos t)^2}dt =$$

$$= 150\pi\int_0^{\frac{\pi}{2}}\cos^3 t\sqrt{(\cos t\sin t)^2(\cos^2 t+\sin^2 t)}dt = 150\pi\int_0^{\frac{\pi}{2}}\cos^3 t\cos t\sin t dt =$$

$$=150\pi\int_0^{\frac{\pi}{2}}\cos^4 t\sin t dt=-150\pi\int_0^{\frac{\pi}{2}}\cos^4 t d(\cos t)=-150\pi\cdot\frac{\cos^5 t}{5}\Bigg|_0^{\frac{\pi}{2}}=30\pi.$$



2-shakl

8-masala. (8.1-8.15). Bir jinsli l egri chiziq og‘irlik markazining koordinatalarini toping:

$$l:\ x=a(t-\sin t),\ \ y=a(1-\cos t)\ \text{ sikloidaning bir arkasi.}$$

Yechish. Sikloidaning birinchi arkasi $x=\pi a$ to‘g‘ri chiziqqa nisbatan simmetrik bo‘ladi. Shu sababli sikloida og‘irlik markazining abssissasi $x_c=\pi a$ bo‘ladi.

Sikloida og‘irlik markazining ordinatasini

$$y_c=\frac{\int_a^b \gamma y dl}{m},\quad m=\int_a^b \gamma\cdot dl$$

formula bilan topamiz.
Bunda

$$dl=\sqrt{\left(a(t-\sin t)'\right)^2+\left(a(1-\cos t)'\right)}dt=\sqrt{a^2\left((1-\cos t)^2+\sin^2 t\right)}dt=$$

$$=a\sqrt{2-2\cos t}dt=2a\sin\frac{t}{2}dt.$$

Egri chiziq bir jinsli bo‘lgani uchun uning zichligi $\gamma=const$ bo‘ladi.
U holda

$$m=\gamma\int_0^{2\pi}dl=\ 2\gamma a\int_0^{2\pi}\sin\frac{t}{2}dt=-4\gamma a\cos\frac{t}{2}\Bigg|_0^{2\pi}=8\gamma a;$$

$$2\gamma a\int_0^{2\pi} a(1-\cos t)\sin\frac{t}{2}dt = 2\gamma a^2\int_0^{2\pi} 2\sin^2\frac{t}{2}\cdot\sin\frac{t}{2}dt =$$

$$= -8\gamma a^2\int_0^{2\pi}\left(1-\cos^2\frac{t}{2}\right)\cdot d\left(\cos\frac{t}{2}\right) = -8\gamma a^2\left(\cos\frac{t}{2}-\frac{1}{3}\cos^3\frac{t}{2}\right)\Bigg|_0^{2\pi} =$$

$$= -8\gamma a^2\left(-1-1+\frac{1}{3}+\frac{1}{3}\right) = \frac{32}{3}\gamma a^2;$$

$$y_c = \frac{32\gamma a^2}{3\cdot 8\gamma a} = \frac{4}{3}a.$$

Demak, $C\left(\pi a;\dfrac{4a}{3}\right)$.

8-masala. (8.16-8.25). Berilgan chiziqlar bilan chegaralangan bir jinsli D yassi figura og‘irlik markazining koordinatalarini toping:

$D:\ \dfrac{x}{a}+\dfrac{y}{b}=1$ to‘g‘ri chiziq va koordinata o‘qlari bilan chegaralangan.

Yechish. To‘g‘ri chiziq tenglamasidan topamiz: $y=-\dfrac{b}{a}x+b.$

Quyidagi formulalarni qo‘llaymiz:

$$x_c = \frac{\int_a^b \gamma xy dx}{m},\quad y_c = \frac{\frac{1}{2}\int_a^b \gamma y^2 dx}{m},\ m=\int_a^b \gamma y dx.$$

U holda

$$m=\gamma\int_0^a\left(-\frac{b}{a}x+b\right)dx = \gamma\left(-\frac{b}{a}\cdot\frac{x^2}{2}+bx\right)\Bigg|_0^a = \gamma\left(-\frac{ba}{2}+ba\right) = \frac{ba\gamma}{2};$$

$$\gamma\int_0^a x\left(-\frac{b}{a}x+b\right)dx = \gamma\left(-\frac{b}{a}\cdot\frac{x^3}{3}+b\frac{x^2}{2}\right)\Bigg|_0^a = \gamma\left(-\frac{ba^2}{3}+\frac{ba^2}{2}\right) = \frac{ba^2\gamma}{6};$$

$$\frac{\gamma}{2}\int_0^a\left(-\frac{b}{a}x+b\right)^2dx = \frac{\gamma}{2}\int_0^a\left(b^2-\frac{2b^2}{a}x+\frac{b^2}{a^2}x^2\right)dx =$$

$$= \frac{\gamma}{2}\left(b^2x-\frac{2b^2}{a}\cdot\frac{x^2}{2}+\frac{b^2}{a^2}\cdot\frac{x^3}{3}\right)\Bigg|_0^a = \frac{ab^2\gamma}{6};$$

$$x_c = \frac{ba^2\gamma\cdot 2}{6\cdot ba\gamma} = \frac{a}{3};\quad y_c = \frac{ab^2\gamma\cdot 2}{6\cdot ba\gamma} = \frac{b}{3}.$$

Demak, $C\left(\dfrac{a}{3};\dfrac{b}{3}\right)$.

3-MAVZU. ODDIY DIFFERENSIAL TENGLAMALAR

9-masala. Differensial tenglamaning umumiy yechimini toping:

1) $x\sqrt{4+y^2}dx + y\sqrt{3+x^2}dy = 0$; 2) $(3xy + x^2)y' - 3y^2 = 0$.

Yechish. 1) O‘zgaruvchilari ajraladigan differensial tenglama berilgan. Uning har ikkala tomonini $\sqrt{4+y^2}\cdot\sqrt{3+x^2} \neq 0$ ga bo‘lib, o‘zgaruvchilarni ajratamiz:

$$\frac{xdx}{\sqrt{3+x^2}} + \frac{ydy}{\sqrt{4+y^2}} = 0.$$

Bu tenglikni integrallaymiz:

$$\sqrt{3+x^2} + \sqrt{4+y^2} = C.$$

Bundan

$$\sqrt{4+y^2} = C - \sqrt{3+x^2}$$

yoki

$$y = \sqrt{(C - \sqrt{3+x^2})^2 - 4}.$$

2) Berilgan tenglamani

$$y' = \frac{3y^2}{3xy + x^2}$$

ko‘rinishga keltiramiz. Bu ifodada $f(x,y) = \dfrac{3y^2}{3xy + x^2}$

bir jinsli funksiya. Demak, berilgan tenglama bir jinsli tenglama.

Tenglamada $y = ux$, $y' = u'x + x$ o‘rniga qo‘yish bajaramiz:

$$u'x + u = \frac{3x^2u^2}{3x^2u + x^2} \quad \text{yoki} \quad u'x + u = \frac{3u^2}{3u+1}.$$

Bundan

$$u'x = \frac{3u^2 - 3u^2 - u}{3u+1} \quad \text{yoki} \quad u'x = -\frac{u}{3u+1}.$$

O‘zgaruvchilarni ajratamiz:

$$\frac{3u+1}{u}du = -\frac{dx}{x}.$$

Tenglamani integrallaymiz:

$$\int \frac{3u+1}{u}du = \ln C - \int \frac{dx}{x} \quad \text{yoki} \quad \ln|u| + 3u = \ln C - \ln|x|.$$

Bundan $3u = \ln \frac{C}{xu}$. $u = \frac{y}{x}$ o‘rniga qo‘yish bajaramiz:

$$3\frac{y}{x} = \ln\frac{C}{y} \quad \text{yoki} \quad y = Ce^{-\frac{3y}{x}}.$$

10-masala. Koshi masalasini yeching:

$$y' - ytgx + y^2 \cos x = 0, \;\; y(0) = \frac{1}{2}.$$

Yechish. Tenglamani $y' - ytgx = -y^2 \cos x$ ko‘rinishda yozamiz. Bu tenglama Bernulli tenglamasi. Bunda $n = 2$.

$z = y^{1-2} = y^{-1}$ belgilash kiritamiz va chiziqli

$$z' + ztgx = \cos x$$

tenglamani hosil qilamiz.

$z = uv, \;\; z' = u'v + v'u$ o‘rniga qo‘yish bajaramiz:

$$u'v + u(v' + vtgx) = \cos x.$$

$u, \; v$ funksiyalarni topish uchun

$$\begin{cases} v' + vtgx = 0, \\ u'v = \cos x \end{cases}$$

sistemani tuzamiz.

Sistemaning birinchi tenglamasidan $v = \cos x$ xususiy yechimni topamiz va uni sistemaning ikkinchi tenglamasiga qo‘yamiz:

$$u'\cos x = \cos x, \quad u' = 1, \quad u = x + C.$$

Berilgan tenglamaning umumiy yechimini topamiz:

$$z = uv, \quad z = (x + C)\cos x.$$

Bundan

$$y^{-1} = (x + C)\cos x \quad \text{yoki} \quad y = \frac{1}{(x + C)\cos x}.$$

Tenglamaning xususiy yechimni topish uchun ixtiyoriy o‘zgarmasning qiymatini boshlang‘ich shartdan topamiz:

$$\frac{1}{2}=\frac{1}{C} \quad \text{yoki} \quad C=2.$$

Demak, tenglamaning izlanayotgan xususiy yechimi

$$y=\frac{1}{(x+2)\cos x}.$$

11-masala. Differensial tenglamani ixtiyoriy o‘zgarmasni variatsiyalash usuli bilan yeching: $y''+9y=\dfrac{1}{\sin 3x}$.

Yechish. $k^2+9=0$ xarakteristik tenglama $k_{1,2}=\pm 3i$ ildizlarga ega. U holda mos bir jinsli tenglamaning umumiy yechimi $y_1=C_1\cos 3x+C_2\sin 3x$ ko‘rinishda bo‘ladi.

Berilgan tenglamaning xususiy yechimini

$$\bar{y}=C_1(x)\cos 3x+C_2(x)\sin 3x$$

ko‘rinishda izlaymiz.

$C_1(x)$ va $C_2(x)$ funksiyalarni topish uchun

$$\begin{cases} C_1'(x)\cos 3x+C_2'(x)\sin 3x=0, \\ -3C_1'(x)\sin 3x+3C_2'(x)\cos 3x=\dfrac{1}{\sin 3x} \end{cases}$$

sistemani tuzamiz va yechamiz:

$$C_1'(x)=-\frac{1}{3},\ C_2'(x)=\frac{1}{3}ctg3x.$$

Bundan

$$C_1(x)=-\frac{1}{3}x,\ C_2(x)=\frac{1}{9}\ln|\sin 3x|.$$

Demak, berilgan tenglamaning xususiy yechimini

$$\bar{y}=-\frac{1}{3}x\cos 3x+\frac{1}{9}\ln|\sin 3x|\sin 3x$$

va umumiy yechimi

$$y=C_1\cos 3x+C_2\sin 3x-\frac{1}{3}x\cos 3x+\frac{1}{9}\ln|\sin 3x|\sin 3x$$

yoki

$$y=\left(C_1-\frac{1}{3}x\right)\cos 3x+\left(C_2+\frac{1}{9}\ln|\sin 3x|\right)\sin 3x.$$

12-masala. Differensial tenglamalar sistemasining umumiy yechimini toping:

$$\begin{cases} y_1'= \ y_1+y_2+\sin x, \\ y_2'=3y_1-y_2-\cos x. \end{cases}$$

Yechish. 1) Sistemaga mos bir jinsli tenglamani tuzamiz:

$$\begin{cases} y_1'= \ y_1+y_2, \\ y_2'=3y_1-y_2. \end{cases}$$

Sistemaning xarakteristik tenglamasini tuzamiz va yechamiz:

$$\begin{vmatrix} 1-\lambda & 1 \\ 3 & -1-\lambda \end{vmatrix}=0, \ \lambda_1=-2, \ \lambda_2=2.$$

$\lambda_1=-2$ da $3\alpha_{11}+\alpha_{21}=0$ tenglikdan $\alpha_{21}=-3\alpha_{11}$ yoki $\alpha_{11}=1$ desak, $\alpha_{21}=-3$ kelib chiqadi.

$\lambda_2=2$ da shu kabi topamiz: $\alpha_{12}=1, \ \alpha_{22}=1.$

U holda bir jinsli sistemaning yechimi

$$\begin{cases} y_1= \ C_1e^{-2x}+C_2e^{2x}, \\ y_2=-3C_1e^{-2x}+C_2e^{2x} \end{cases}$$

bo‘ladi. Berilgan sistemaning xususiy yechimini

$$\begin{cases} \bar{y}_1=A_1\cos x+B_1\sin x, \\ \bar{y}_2=A_2\cos x+B_2\sin x \end{cases}$$

ko‘rinishda izlaymiz.

Bundan

$$\begin{cases} \bar{y}_1'=-A_1\sin x+B_1\cos x, \\ \bar{y}_2'=-A_2\sin x+B_2\cos x. \end{cases}$$

$\bar{y}_1, \ \bar{y}_2, \ \bar{y}_1, \ \bar{y}_2'$ larni berilgan sistemaga qo‘yamiz $\cos x$ va $\sin x$ lar oldidagi koeffitsiyentlarni tenglab, topamiz:

$$A_1=0, \ B_1=-\frac{1}{5}, \ A_2=-\frac{1}{5}, \ B_2=-\frac{4}{5}.$$

Demak, berilgan sistemaning xususiy yechimi va umumiy yechimi:

$$\begin{cases} \overline{y}_1 = -\frac{1}{5}\sin x, \\ \overline{y}_2 = -\frac{1}{5}\cos x - \frac{4}{5}\sin x \end{cases}, \qquad \begin{cases} y_1 = \quad C_1 e^{-2x} + C_2 e^{2x} - \frac{1}{5}\sin x, \\ y_2 = -3C_1 e^{-2x} + C_2 e^{2x} - \frac{1}{5}\cos x - \frac{4}{5}\sin x. \end{cases}$$

NAZORAT ISHINI BAJARISH UCHUN TOPSHIRIQLAR

1-masala. Hosilalarni toping:

1.1. 1) $y = \sqrt[3]{5x^4 - 2x - 1} + \frac{8}{(x-5)^2}$. 2) $y = ctg\frac{1}{x} \cdot \arccos x^4$.

3) $y = \frac{(2x+5)^3}{e^{tgx}}$. 4) $y = (\cos x)^{x^2-4}$.

1.2. 1) $y = \frac{3}{(x+2)^5} - \sqrt[7]{5x - 7x^2 - 3}$. 2) $y = tg\sqrt{x} \cdot arcctg 3x^5$.

3) $y = \frac{e^{tg 3x}}{4x^2 - 3x + 5}$. 4) $y = (x^3 + 1)^{\cos x}$.

1.3. 1) $y = \sqrt[3]{(x-7)^5} + \frac{5}{4x^2 + 3x - 5}$. 2) $y = tg^3 2x \cdot \arccos 2x^3$.

3) $y = \frac{e^{\sin 2x}}{(x+5)^4}$. 4) $y = (arctgx)^{5x-1}$.

1.4. 1) $y = \sqrt[5]{(x+4)^6} - \frac{2}{2x^2 - 3x + 7}$. 2) $y = 2^{tgx} \cdot arctg^5 3x$.

3) $y = \frac{e^{\cos 5x}}{\sqrt{x^2 - 5x - 2}}$. 4) $y = (arctgx)^{x-1}$.

1.5. 1) $y = \frac{3}{4x - 3x^2 + 1} - \sqrt{(x+5)^5}$. 2) $y = tg^3 2x \cdot \arcsin x^5$.

3) $y = \frac{\sqrt{x^2 - 3x - 7}}{e^{x^3}}$. 4) $y = x^{\cos 2x}$.

1.6. 1) $y=\dfrac{3}{(x-4)^2}+\sqrt[6]{2x^2-3x+1}$. 2) $y=ctg^7x\cdot\arccos 2x^3$.

3) $y=\dfrac{e^x-tgx}{4x^2+7x-5}$. 4) $y=x^{x+3}$.

1.7) 1) $y=\dfrac{3}{(x+4)^2}-\sqrt[3]{4-3x-x^4}$. 2) $y=e^{-\sin x}tg7x^6$.

3) $y=\dfrac{\cos^3 x}{(2x+4)^5}$. 4) $y=(\sin x)^{3x}$.

1.8. 1) $y=\dfrac{2}{(x-1)^3}-\dfrac{8}{6x^2+3x-7}$. 2) $y=e^{\cos x}\cdot ctg8x^3$.

3) $y=\sqrt{5x^2-x+1}\cdot e^{-3x}$. 4) $y=(\cos x)^{x^2}$.

1.9. 1) $y=\dfrac{7}{(x-1)^3}+\sqrt{8x-3x^2}$. 2) $y=\cos^5 x\cdot\arccos 4x$.

3) $y=\dfrac{2^{x^2}}{(2x-5)^7}$. 4) $y=(tgx)^{\sin x}$.

1.10. 1) $y=\sqrt[5]{3x^2+4x-5}+\dfrac{4}{(x-4)^4}$. 2) $y=\sin^3 7x\cdot arcctg5x^2$.

3) $y=\dfrac{e^{\sin 5x}}{(3x-2)^2}$. 4) $y=x^{3x}2^x$.

1.11. 1) $y=\sqrt[3]{3x^2-4x+5}+\dfrac{4}{(x-3)^5}$. 2) $y=\sin^2 3x\cdot arcctg3x^5$.

3) $y=(3x+1)^4\cdot e^{-4x}$. 4) $y=x^{\sin 3x}$.

1.12. 1) $y=\sqrt{3x^4-2x^3+x}-\dfrac{4}{(x+2)^3}$. 2) $y=\cos\sqrt[5]{x}\cdot arctgx^4$.

3) $y=(5x^2+4x-2)^2\cdot e^{-3x}$. 4) $y=(x^2+1)^{\sin x}$.

1.13. 1) $y=\dfrac{3}{(x+4)^2}-\sqrt[3]{(3x^2-x+1)^4}$. 2) $y=tg^6 2x\cdot\cos 7x^2$.

3) $y=\dfrac{e^{ctg 5x}}{(3x-5)^4}$. 4) $y=(\sin 2x)^{x+1}$.

1.14. 1) $y=\sqrt[3]{(x-4)^7}-\dfrac{10}{(3x^2-5x+1)}$.

2) $y=ctg^3 4x\cdot\arcsin\sqrt{x}$.

3) $y=\dfrac{(2x-3)^7}{e^{2x}}$.

4) $y=(x+1)^{tg\,2x}$

1.15. 1) $y=\sqrt[3]{3x^4+2x-5}+\dfrac{4}{(x-2)^5}$.

2) $y=2^{\cos x}\cdot arctg\,5x^3$.

3) $y=\dfrac{3^{x^2}}{(2x^2-x+4)^2}$.

4) $y=(\sin x)^{x^2-1}$

1.16. 1) $y=\sqrt[3]{(x-3)^4}-\dfrac{3}{2x^3-3x+1}$.

2) $y=4^{-x}\cdot\ln^5(x+2)$.

3) $y=\dfrac{e^{4x}}{(3x+5)^3}$.

4) $y=(3x^2-1)^{\arcsin x}$.

1.17. 1) $y=\dfrac{7}{(x+2)^5}-\sqrt{8-5x+2x^2}$.

2) $y=3^{tgx}\cdot\arcsin 7x^4$.

3) $y=\dfrac{e^{\sin 4x}}{(2x-5)^6}$.

4) $y=(e^x)^{x+4}$.

1.18. 1) $y=\sqrt[3]{(x-1)^5}+\dfrac{5}{2x^2-4x+7}$.

2) $y=5^{x^2}\cdot\arccos 2x^5$.

3) $y=\dfrac{3x^2-5x+10}{e^{x^4}}$.

4) $y=(x^3-1)^{x^2-1}$.

1.19. 1) $y=\sqrt{(x-4)^5}+\dfrac{5}{(2x^2+4x-1)^2}$.

2) $y=\sin^4 3x\cdot arctg\,2x^3$.

3) $y=\dfrac{\sqrt{7x^3-5x+2}}{e^{\cos x}}$.

4) $y=(tgx)^{x^3+1}$.

1.20. 1) $y=\sqrt[5]{7x^2-3x^3+5}-\dfrac{5}{(x-1)^3}$.

2) $y=tg^3 2x\cdot\arcsin\sqrt{x}$

3) $y=\dfrac{e^{tg\,3x}}{\sqrt{3x^2-x+4}}$.

4) $y=(e^{3x})^{\sin x}$.

1.21. 1) $y=\sqrt{(x-3)^7}+\dfrac{9}{7x^2-5x-8}$. 2) $y=\sin^5 3x\cdot arctg\sqrt{x}$.

3) $y=\dfrac{e^{x^3}}{\sqrt{x^2+5x-1}}$. 4) $y=x^{\arcsin x}$.

1.22. 1) $y=\sqrt[3]{x-8}-\dfrac{2}{1-3x-4x^2}$. 2) $y=\cos^4 3x\cdot\arcsin 3x^2$.

3) $y=\dfrac{e^{ctg5x}}{(3x^2-4x+2)}$. 4) $y=(\arcsin x)^x$.

1.23. 1) $y=\sqrt[4]{(x-1)^5}-\dfrac{4}{7x^2-3x+2}$. 2) $y=\sin^3 2x\cdot\cos 8x^5$.

3) $y=\dfrac{e^{\arccos^3 x}}{\sqrt{x+5}}$. 4) $y=(tgx)^{3e^x}$.

1.24. 1) $y=\sqrt[5]{(x-2)^6}+\dfrac{3}{6x^2+3x-7}$. 2) $y=\cos^5 3x\cdot tg(4x+1)^3$.

3) $y=\dfrac{e^{\sin 5x}}{(3x-2)^2}$ 4) $y=(\sin x)^{x+6}$.

1.25. 1) $y=\sqrt{1+5x-2x^2}+\dfrac{3}{(x-3)^4}$. 2) $y=tg^4 x\cdot\arcsin 4x^2$.

3) $y=\dfrac{\sqrt{3+2x-x^2}}{e^x}$. 4) $y=x^{\sin 5x-1}$.

2-masala. Parametrik ko‘rinishida berilgan y funksiyalarning x bo‘yicha ikkinchi tartibli hosilasini toping:

2.1. $\begin{cases} x=t+\sin t, \\ y=t-\cos t. \end{cases}$ **2.2.** $\begin{cases} x=t^5+2t, \\ y=t^3+8t-1. \end{cases}$

2.3. $\begin{cases} x=e^{2t}, \\ y=\cos t. \end{cases}$ **2.4.** $\begin{cases} x=ctgt, \\ y=\dfrac{1}{\cos^2 t}. \end{cases}$

2.5. $\begin{cases} x=\ln\cos 2t, \\ y=\sin^2 2t. \end{cases}$ **2.6.** $\begin{cases} x=\dfrac{1}{3}t^3+t, \\ y=\ln(t^2+1). \end{cases}$

2.7. $\begin{cases} x = 1 - e^{3t}, \\ y = \frac{1}{3}(e^{3t} + e^{-3t}). \end{cases}$

2.8. $\begin{cases} x = \ln(1+t^2), \\ y = t - arctg\, t. \end{cases}$

2.9. $\begin{cases} x = 2(t - \sin t), \\ y = 2(1 - \cos t). \end{cases}$

2.10. $\begin{cases} x = tgt, \\ y = \frac{1}{\sin^2 t}. \end{cases}$

2.11. $\begin{cases} x = \sin^3 4t, \\ y = \frac{1}{2}\cos^3 4t. \end{cases}$

2.12. $\begin{cases} x = tgt + ctgt, \\ y = 2\ln ctg\ t. \end{cases}$

2.13. $\begin{cases} x = 4 - e^{2t}, \\ y = \frac{3}{e^{2t}+1}. \end{cases}$

2.14. $\begin{cases} x = 3\cos^2 t, \\ y = 2\sin^3 t. \end{cases}$

2.15. $\begin{cases} x = 2 - \cos t, \\ y = t - \sin t. \end{cases}$

2.16. $\begin{cases} x = t + \ln\cos t, \\ y = t - \ln\sin t. \end{cases}$

2.17. $\begin{cases} x = 3 + \cos t, \\ y = t + \sin t. \end{cases}$

2.18. $\begin{cases} x = t\cos t, \\ y = t\sin t. \end{cases}$

2.19. $\begin{cases} x = 2t - \sin 2t, \\ y = \sin^3 t. \end{cases}$

2.20. $\begin{cases} x = \arcsin(t^2 - 1), \\ y = \arccos\ 2t. \end{cases}$

2.21. $\begin{cases} x = \cos\frac{t}{2}, \\ y = t - \sin t. \end{cases}$

2.22. $\begin{cases} x = t^2, \\ y = 1 - \cos t. \end{cases}$

2.23. $\begin{cases} x = t^3 + t^2 + t, \\ y = t^2 + \frac{1}{t}. \end{cases}$

2.24. $\begin{cases} x = t + \frac{1}{2}\sin 2t, \\ y = \cos 2t. \end{cases}$

2.25. $\begin{cases} x = t + \frac{1}{2}\sin 2t, \\ y = \cos 2t. \end{cases}$

3-masala. limitni Lopital qoidasidan foydalanib toping:

3.1. $\lim\limits_{x \to 0}\left(\frac{1}{x}\right)^{tgx}$.

3.2. $\lim\limits_{x \to \frac{\pi}{2}} \frac{tg3x}{tg5x}$.

3.3. $\lim\limits_{x \to 0}(x\ln x)$.

3.4. $\lim\limits_{x \to 0} \frac{\arcsin 4x}{5 - 5e^{-x}}$.

3.5. $\lim\limits_{x\to 0}\dfrac{tgx-\sin x}{4x-\sin x}.$

3.6. $\lim\limits_{x\to 0}(1-\cos 2x)ctg2x.$

3.7. $\lim\limits_{x\to 0}(1+x)^{\frac{1}{\sin x}}.$

3.8. $\lim\limits_{x\to 0}\sqrt{x}\ln^2 x.$

3.9. $\lim\limits_{x\to 0}\left(\dfrac{1}{x-1}-\dfrac{1}{\ln x}\right).$

3.10. $\lim\limits_{x\to 0}\left(\dfrac{1}{e^x-1}-\dfrac{1}{x}\right).$

3.11. $\lim\limits_{x\to 0}\dfrac{a^x-a^{\sin x}}{x^2}.$

3.12. $\lim\limits_{x\to 0}\dfrac{\ln(\cos ax)}{\ln(\cos bx)}.$

3.13. $\lim\limits_{x\to 0}\dfrac{e^x-1}{\ln(1+2x)}.$

3.14. $\lim\limits_{x\to 0}\dfrac{e^x-1}{\sin 2x}.$

3.15. $\lim\limits_{x\to 0}(\cos x)^{\frac{1}{x}}.$

3.16. $\lim\limits_{x\to 0}(1+\sin x)^{\frac{1}{x}}.$

3.17. $\lim\limits_{x\to \frac{\pi}{4}}(tgx)^{tg2x}.$

3.18. $\lim\limits_{x\to \infty}(x^3e^{-x}).$

3.19. $\lim\limits_{x\to 0}\dfrac{a^x-b^x}{x\sqrt{1-x^2}}.$

3.20. $\lim\limits_{x\to \pi}(\pi-x)tg\dfrac{x}{2}.$

3.21. $\lim\limits_{x\to \infty}\dfrac{\ln x}{\sqrt[3]{x}}.$

3.22. $\lim\limits_{x\to 1}\left(\dfrac{1}{\ln x}-\dfrac{x}{\ln x}\right).$

3.23. $\lim\limits_{x\to 0}(1+\sin x)^{ctgx}.$

3.24. $\lim\limits_{x\to 0}\dfrac{\ln\cos x}{x}.$

3.25. $\lim\limits_{x\to 0}\dfrac{x-arctgx}{x^3}.$

4-masala. Funksiyani to‘la tekshiring va grafigini chizing:

4.1. $y=\dfrac{x^2-x-1}{x^2-2x}$

4.2. $y=\dfrac{1}{1-x^2}.$

4.3. $y=\dfrac{(x-3)^2}{4(x-1)}.$

4.4. $y=\dfrac{2}{x^2+x+1}.$

4.5. $y=\dfrac{x-1}{x^2-2x}.$

4.6. $y=\dfrac{(x-1)^2}{x^2+1}.$

4.7. $y=\dfrac{2-4x^2}{1-4x^2}.$

4.8. $y=\dfrac{x^3+1}{x^2}.$

4.9. $y=\dfrac{2x^2}{4x^2-1}$.

4.10. $y=\dfrac{x}{3-x^2}$.

4.11. $y=\dfrac{2x+1}{x^2}$.

4.12. $y=\dfrac{(x+1)^2}{(x-1)^2}$.

4.13. $y=\dfrac{1}{x^2-9}$.

4.14. $y=\dfrac{x}{(x-1)^2}$.

4.15. $y=\dfrac{8(x-1)}{(x+1)^2}$.

4.16. $y=\dfrac{2x-1}{(x-1)^2}$.

4.17. $y=\dfrac{x^4}{x^3-1}$.

4.18. $y=\dfrac{x^3}{2(x+1)^2}$.

4.19. $y=\dfrac{3-x^2}{x+2}$.

4.20. $y=\dfrac{4x}{(x+1)^2}$.

4.21. $y=\dfrac{5x^2}{x^2-25}$.

4.22. $y=\dfrac{x^2-3x+3}{x-1}$.

4.23. $y=\dfrac{x^2+1}{x}$.

4.24. $y=\dfrac{x^3+16}{x}$.

4.25. $y=\dfrac{x^2+4x+1}{x^2}$.

5-masala. Aniqmas integrallarni toping:

5.1. 1) $\int\dfrac{7x-7}{(x+1)(x^2-4x+13)}dx.$ 2) $\int\dfrac{dx}{2+4\sin x+3\cos x}.$

3) $\int\dfrac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx.$ 4) $\int\dfrac{\sqrt{1+\sqrt[3]{x^2}}}{x^2}dx.$

5.2. 1) $\int\dfrac{x^2+3x-6}{(x+1)(x^2+6x+13)}dx.$ 2) $\int\dfrac{dx}{4\cos x+3\sin x}.$

3) $\int\dfrac{\sqrt{x+3}}{1+\sqrt[3]{x+3}}dx.$ 4) $\int\dfrac{\sqrt[3]{1+\sqrt[5]{x}}}{x\cdot\sqrt[15]{x^4}}dx.$

5.3. 1) $\int\dfrac{x^2-3x+1}{(x+2)(x^2+4)}dx.$ 2) $\int\dfrac{\sin xdx}{5+3\sin x}.$

3) $\int\dfrac{\sqrt{1+x}}{x^2\sqrt{x}}dx.$ 4) $\int\dfrac{\sqrt[3]{1+\sqrt[3]{x}}}{x\cdot\sqrt[9]{x^4}}dx.$

5.4. 1) $\int \frac{x^2-4x+12}{x^3+8}dx.$ 2) $\int \frac{\cos x dx}{1+\sin x+\cos x}.$

3) $\int \frac{1+\sqrt[3]{x^2}}{\sqrt{x}+\sqrt[3]{x}}dx.$ 4) $\int \frac{\sqrt[3]{1+\sqrt[5]{x^4}}}{x^2\cdot\sqrt[15]{x}}dx.$

5.5. 1) $\int \frac{3x+13}{(x-1)(x^2+2x+5)}dx.$ 2) $\int \frac{6\sin x-5\cos x+7}{1+\cos x}dx.$

3) $\int \frac{\sqrt{x-1}}{\sqrt[3]{x-1}+1}dx.$ 4) $\int \frac{\sqrt[3]{1+\sqrt[3]{x^2}}}{x\cdot\sqrt[9]{x^8}}dx.$

5.6. 1) $\int \frac{3x^2+5x-1}{(x+1)(x^2+2)}.$ 2) $\int \frac{dx}{3\cos x-5}.$

3) $\int \frac{\sqrt{x}dx}{3x+\sqrt[3]{x^2}}.$ 4) $\int \frac{\sqrt[4]{(1+\sqrt{x})^3}}{x\cdot\sqrt[8]{x^7}}dx.$

5.7. 1) $\int \frac{2x^3+1}{(x+2)(x^2+2x+3)}dx.$ 2) $\int \frac{dx}{5\cos x+3}.$

3) $\int \frac{\sqrt{x+1}+\sqrt[3]{x+1}}{\sqrt{x+1}}dx.$ 4) $\int \frac{\sqrt[3]{1+\sqrt[3]{x}}}{x\cdot\sqrt[9]{x^4}}dx.$

5.8. 1) $\int \frac{3x-5}{(x+1)(x^2+1)}dx.$ 2) $\int \frac{dx}{\sin x+\cos x+3}.$

3) $\int \frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt{x}+\sqrt[6]{x}}dx.$ 4) $\int \frac{\sqrt[5]{(1+\sqrt[4]{x^3})^4}}{x^2\cdot\sqrt[20]{x^7}}dx.$

5.9. 1) $\int \frac{5x+6}{(x-2)(x^2-x+1)}dx.$ 2) $\int \frac{1+\sin x}{\sin x+\cos x+1}dx.$

3) $\int \frac{\sqrt{x}}{x-4\sqrt[3]{x^2}}dx.$ 4) $\int \frac{\sqrt[4]{1+\sqrt[3]{x}}}{x\cdot\sqrt[12]{x^5}}dx.$

5.10. 1) $\int \frac{x^2+2x-1}{(x+2)(x^2+x+1)}dx.$ 2) $\int \frac{dx}{\cos x(1+\cos x)}.$

3) $\int \frac{x+\sqrt{x}+\sqrt[3]{x^2}}{x(1+\sqrt[3]{x})}dx.$ 4) $\int \frac{\sqrt[3]{(1+\sqrt[5]{x^4})^2}}{x^2\cdot\sqrt[3]{x}}dx.$

5.11. 1) $\int \frac{x^2+3x+2}{x^3-1}dx.$ 2) $\int \frac{dx}{\sin x+3\cos x+5}.$

3) $\int \frac{(\sqrt[3]{x}+1)(\sqrt{x}+1)}{\sqrt[6]{x^5}}dx.$ 4) $\int \frac{\sqrt[5]{1+\sqrt[3]{x}}}{x\cdot\sqrt[5]{x^2}}dx.$

5.12. 1) $\int \frac{36dx}{(x+2)(x^2-2x+10)}.$ 2) $\int \frac{dx}{2\cos x-\sin x+3}.$

3) $\int \frac{\sqrt[6]{x}dx}{1+\sqrt[3]{x}}.$ 4) $\int \frac{\sqrt[3]{(1+\sqrt{x})^2}}{x\cdot\sqrt[6]{x^5}}dx.$

5.13. 1) $\int \frac{x^2+3x+1}{(x+1)(x^2-x+1)}dx.$ 2) $\int \frac{dx}{2\sin x+\cos x}.$

3) $\int \frac{dx}{\sqrt[3]{x}+\sqrt{x}}.$ 4) $\int \frac{\sqrt[3]{1+\sqrt[4]{x}}}{x\cdot\sqrt[3]{x}}dx.$

5.14. 1) $\int \frac{3x+2}{(x+1)(x^2+2x+2)}dx.$ 2) $\int \frac{dx}{\cos x-3\sin x}.$

3) $\int \frac{1+\sqrt[3]{x-1}}{\sqrt{x-1}}dx.$ 4). $\int \frac{\sqrt[4]{1+\sqrt[3]{x^2}}}{x\cdot\sqrt[6]{x^5}}dx.$

5.15. 1) $\int \frac{5x+2}{(x+3)(x^2+2x+2)}dx.$ 2) $\int \frac{\sin x dx}{1+\sin x+\cos x}.$

3) $\int \frac{\sqrt{x+1}-1}{\sqrt[3]{x+1}+1}dx.$ 4) $\int \frac{\sqrt[5]{1+\sqrt[5]{x^6}}}{x^2\cdot\sqrt[25]{x^{11}}}dx.$

5.16. 1) $\int \frac{5x-3}{(x+1)(x^2+1)}dx.$ 2) $\int \frac{dx}{3\sin x-\cos x};$

3) $\int \frac{1+\sqrt[3]{x}}{x(\sqrt{x}+\sqrt[6]{x})}dx.$ 4) $\int \frac{\sqrt{1+\sqrt[3]{x}}}{x\cdot\sqrt{x}}dx.$

5.17. 1) $\int \frac{12-6x}{(x+2)(x^2-4x+13)}dx.$ 2) $\int \frac{dx}{3\cos x+5}.$

3) $\int \frac{1+\sqrt{x}}{x(1+\sqrt[3]{x})}dx.$ 4) $\int \frac{\sqrt[3]{(1+\sqrt[3]{x})^2}}{x\cdot\sqrt[9]{x^5}}dx.$

5.18. 1) $\int \frac{2x^2+2x+10}{(x-1)(x^2+2x+5)}dx.$ 2) $\int \frac{dx}{3\sin x-4\cos x}.$

3) $\int \frac{\sqrt[6]{x}dx}{\sqrt{x}+\sqrt[3]{x}}.$ 4) $\int \frac{\sqrt[4]{(1+\sqrt[5]{x^4})^3}}{x^2\cdot\sqrt[5]{x^2}}dx.$

5.19. 1) $\int \frac{3x+7}{(x+2)(x^2+2x+3)}dx.$ 2) $\int \frac{dx}{8+4\cos x}.$

3) $\int \frac{\sqrt{x}}{1-\sqrt[4]{x}}dx.$ 4) $\int \frac{\sqrt[5]{(1+\sqrt[3]{x^2})^4}}{x^2\cdot\sqrt[5]{x}}dx.$

5.20. 1) $\int \frac{4x+3}{(x-2)(x^2+x+1)}dx.$ 2) $\int \frac{dx}{3\cos x-4\sin x+4}.$

3) $\int \frac{\sqrt[3]{x}}{1+\sqrt{x}}dx.$ 4) $\int \frac{\sqrt{1+\sqrt[4]{x^3}}}{x^2\cdot\sqrt[8]{x}}dx.$

5.21. 1) $\int \frac{5x^2+17x+36}{(x+1)(x^2+6x+13)}dx.$ 2) $\int \frac{\cos dx}{2+\cos x}.$

3) $\int \frac{\sqrt{x}dx}{1+\sqrt[3]{x^2}}.$ 4) $\int \frac{\sqrt[3]{1+\sqrt{x}}}{x\cdot\sqrt[3]{x^2}}dx.$

5.22. 1) $\int \frac{2x+22}{(x+2)(x^2-2x+10)}dx.$ 2) $\int \frac{dx}{\sin x-3\cos x+2}.$

3) $\int \frac{\sqrt[6]{x+3}}{\sqrt{x+3}+\sqrt[3]{x+3}}dx.$ 4) $\int \frac{\sqrt[4]{(1+\sqrt[3]{x})^3}}{x\cdot\sqrt[12]{x^7}}dx.$

5.23. 1) $\int \frac{2x^2+7x+7}{(x-1)(x^2+2x+5)}dx.$ 2) $\int \frac{dx}{2\sin x-3\cos x}.$

3) $\int \frac{dx}{x(\sqrt[3]{x}+\sqrt{x})}.$ 4) $\int \frac{\sqrt[5]{(1+\sqrt{x})^4}}{x\cdot\sqrt[10]{x^9}}dx.$

5.24. 1) $\int \frac{x^2+3x+1}{(x-1)(x^2-6x+13)}dx.$ 2) $\int \frac{dx}{2\cos x-4\sin x+5}.$

3) $\int \frac{\sqrt{x+1}+1}{\sqrt{x+1}-1}dx.$ 4) $\int \frac{\sqrt[3]{(1+\sqrt[4]{x^3})^2}}{x^2\cdot\sqrt[4]{x}}dx.$

5.25. 1) $\int \frac{5x^2+6}{x^3+27}dx.$ 2). $\int \frac{dx}{5+2\sin x+3\cos x}.$

3) $\int \frac{\sqrt{x+2}}{x-\sqrt[3]{x+2}+2}dx.$ 4) $\int \frac{\sqrt{1+\sqrt[5]{x^4}}}{x^2\cdot\sqrt[5]{x}}dx.$

6-masala. Aniq integrallarni hisoblang:

6.1. 1) $\int_{-2}^{0}(x+2)^2\cos 3xdx.$ 2) $\int_{0}^{\pi}2^4\cos^8 xdx.$

6.2. 1) $\int_{1}^{e^2}\sqrt{x}\ln^2 xdx.$ 2) $\int_{0}^{\pi}2^4\sin^6 x\cos^2 xdx.$

6.3. 1) $\int_{0}^{3}(x^2-3x)\sin xdx.$ 2) $\int_{0}^{2\pi}2^4\sin^4 x\cos^4 xdx.$

6.4. 1) $\int_{1}^{2}x\ln(3x+2)dx.$ 2) $\int_{0}^{\pi}2^4\sin^8 xdx.$

6.5. 1) $\int_{1}^{2}x^2\ln xdx.$ 2) $\int_{0}^{2\pi}\sin^4\frac{x}{4}\cos^4\frac{x}{4}dx.$

6.6. 1) $\int_{0}^{\frac{\pi}{2}}(x^2+1)\cos xdx.$ 2) $\int_{0}^{2\pi}\sin^2\frac{x}{4}\cos^6\frac{x}{4}dx.$

6.7. 1) $\int_{-1}^{1}x^2e^{-\frac{x}{2}}dx.$ 2) $\int_{\frac{\pi}{2}}^{\pi}2^8\sin^2 x\cos^6 xdx.$

6.8. 1) $\int_{0}^{1}xarctgxdx;$ 2) $\int_{-\frac{\pi}{2}}^{0}2^8\sin^8 xdx.$

6.9. 1) $\int_{-2}^{0}(x-1)e^{-\frac{x}{2}}dx;$ 2) $\int_{0}^{2\pi}\sin^4 3x\cos^4 3xdx.$

6.10. 1) $\int_{1}^{e}x\ln^2 xdx.$ 2) $\int_{0}^{\pi}2^4\sin^2 x\cos^6 xdx.$

6.11. 1) $\int_{0}^{1}x^2e^{3x}dx.$ 2) $\int_{\frac{\pi}{2}}^{\pi}2^8\cos^8 xdx.$

6.12. 1) $\int_{0}^{e-1}\ln^2(x+1)dx.$ 2) $\int_{-\frac{\pi}{2}}^{0}2^8\sin^2 x\cos^6 xdx.$

6.13. 1) $\int_0^{\frac{\pi}{2}} x^2 \sin\frac{x}{2}dx.$ 2) $\int_0^{2\pi} \sin^2 x\cos^6 xdx.$

6.14. 1) $\int_0^{\frac{\pi}{3}} \frac{xdx}{\cos^2 x}.$ 2) $\int_0^{\pi} 2^4\sin^4 x\cos^4 xdx.$

6.15. 1) $\int_0^{\sqrt{e}} x^2 \ln xdx.$ 2) $\int_0^{2\pi} \cos^8\frac{x}{4}dx.$

6.16. 1) $\int_1^{e} \ln^3 xdx.$ 2) $\int_0^{\pi} 2^4\sin^2\frac{x}{2}\cos^6\frac{x}{2}dx.$

6.17. 1) $\int_0^{\pi} x^3 \sin xdx.$ 2) $\int_0^{2\pi} \sin^6 x\cos^2 xdx.$

6.18. 1) $\int_{-2}^{0} (x^2-4)\cos 3xdx.$ 2) $\int_{-\pi}^{0} 2^8\sin^6 x\cos^2 xdx.$

6.19. 1) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{xdx}{\sin^2 x}.$ 2) $\int_{\frac{\pi}{2}}^{\pi} 2^8\sin^8 xdx.$

6.20. 1) $\int_{\frac{\pi}{4}}^{3} (3x-x^2)\sin 2xdx.$ 2) $\int_{\frac{\pi}{2}}^{\pi} 2^8\sin^4 x\cos^4 xdx.$

6.21. 1) $\int_{-1}^{0} x^2 \ln(1-x)dx.$ 2) $\int_0^{\pi} 2^4\sin^4\frac{x}{2}\cos^4\frac{x}{2}dx.$

6.22. 1) $\int_0^{\pi} (x+1)^2\cos\frac{x}{2}dx.$ 2) $\int_0^{2\pi} \sin^8\frac{x}{4}dx.$

6.23. 1) $\int_1^{e} \frac{3\ln x}{x^2}dx.$ 2) $\int_{-\frac{\pi}{2}}^{0} 2^8\cos^8 xdx.$

6.24. 1) $\int_{-1}^{0} (x+1)e^{-2x}dx.$ 2) $\int_{-\frac{\pi}{2}}^{0} 2^8\sin^4 x\cos^4 xdx.$

6.25. 1) $\int_0^{1} xarctg\sqrt{x}dx.$ 2) $\int_0^{\pi} 2^4\cos^8\frac{x}{2}dx.$

7-masala. Berilgan l egri chiziqning ko‘rsatilgan o‘q atrofida aylanishidan hosil bo‘lgan sirt yuzasini hisoblang:

7.1. $l:\ x=e^t\sin t,\ y=e^t\cos t$ egri chiziqning $t=0$ dan $t=\frac{\pi}{2}$ gacha qismi, Ox.

7.2. $l:\ x=2\cos^3 t,\ y=2\sin^3 t$ astroida, Oy.

7.3. $l:\ x=3(t-\sin t),\ y=3(1-\cos t)$ sikloidaning bir arkasi, Ox.

7.4. $l:\ r=4\sin\varphi$ aylananing $\varphi=0$ dan $\varphi=\frac{\pi}{2}$ gacha qismi, Ox.

7.5. $l:\ x=\frac{t^3}{24},\ y=4-\frac{t^2}{16}$ egri chiziqning $t=0$ dan $t=2\sqrt{2}$ gacha qismi, Ox.

7.6. $l:\ y=\frac{x^2}{4}-\frac{\ln x}{2}$ egri chiziq yoyining $x=1$ dan $x=e$ gacha qismi, Ox.

7.7. $l:\ y=\sin x$ sinusoidaning $x=0$ dan $x=\pi$ gacha qismi, Ox.

7.8. $l:\ \frac{x^2}{25}+\frac{y^2}{16}=1$ ellipsning $x=0$ dan $x=5$ gacha qismi, Ox.

7.9. $l:\ y=2ch\frac{x}{2}$ zanjir chiziq yoyining $x=0$ dan $x=2$ gacha qismi, Ox.

7.10. $l:\ x^2=2y$ parabolaning $y=0$ dan $y=\frac{3}{2}$ gacha qismi, Oy.

7.11. $l:\ r=\frac{1}{\cos^2\frac{\varphi}{2}}$ egri chiziq yoyining $\varphi=0$ dan $\varphi=\frac{\pi}{2}$ gacha qismi, Ox.

7.12. $l:\ y^2=2x+1$ parabolaning $x=0$ dan $x=7$ gacha qismi, Ox.

7.13. $l:\ r^2=9\cos 2\varphi$ limniskataniing $\varphi=0$ dan $\varphi=\frac{\pi}{4}$ gacha qismi, Ox.

7.14. $l:\ r=4\cos\varphi$ egri chiziq yoyi, Ox. **9.** $y=ach\frac{x}{a},\ -a\le x\le a,\ Ox$.

7.15. $l:\ r=2(1-\cos\varphi)$ kardioidaning $\varphi=-\pi$ dan $\varphi=-\frac{\pi}{2}$ gacha qismi, Ox.

7.16. $l:\ x=e^t\sin t,\ y=e^t\cos t$ egri chiziqning $t=0$ dan $t=\frac{\pi}{2}$ gacha qismi, Oy.

7.17. $l:\ x=\dfrac{y^2}{4}-\dfrac{\ln y}{2}$ egri chiziq yoyining $y=1$ dan $y=e$ gacha qismi, Oy.

7.18. $l:\ x=\cos t,\ \ y=1+\sin t$ egri chiziq yoyi, Ox.

7.19. $l:\ x=4-\dfrac{t^2}{2},\ \ y=\dfrac{t^3}{3}$ egri chiziqning $t=0$ dan $t=2\sqrt{2}$ gacha qismi, Oy.

7.20. $l:\ \dfrac{x^2}{9}+\dfrac{y^2}{25}=1$ ellipsning $y=0$ dan $y=5$ gacha qismi, Oy.

7.21. $l:\ r=\dfrac{1}{\sin^2\dfrac{\varphi}{2}}$ egri chiziq yoyining $\varphi=0$ dan $\varphi=\dfrac{\pi}{2}$ gacha qismi, Ox.

7.22. $l:\ x=2(t-\sin t),\ \ y=2(1-\cos t)$ sikloidaning bir arkasi, Oy.

7.23. $l:\ r=5(1+\cos\varphi)$ kardioidaning $\varphi=0$ dan $\varphi=\dfrac{\pi}{2}$ gacha qismi, Oy.

7.24. $l:\ x=4\cos^3 t,\ \ y=4\sin^3 t$ astroida, Ox.

7.25. $l:\ y=e^{-x}$ egri chiziq yoyianing $x\geq 0$ ga mos qismi, Ox.

8-masala. (**8.1-8.15**). Bir jinsli l egri chiziq og‘irlik markazining koordinatalarini toping:

8.1. $l:\ x=2\cos^3\dfrac{t}{4},\ \ y=2\sin^3\dfrac{t}{4}$ astroidaning birinchi kvadrantdagi qismi.

8.2. $l:\ r=2\sin\varphi$ egri chiziqning $\varphi=0$ dan $\varphi=\pi$ gacha qismi.

8.3. $l:\ y=3ch(x-3)$ zanjir chiziq yoyining $x=-3$ dan $x=3$ gacha qismi.

8.4. $l:\ x=5\cos^3 t,\ \ y=5\sin^3 t$ astroidaning Oy o‘qdan chapda yotgan qismi.

8.5. $l:\ x^2+y^2=9$ aylananing $\varphi=60^o$ li markaziy burchagi orasidagi qismi.

8.6. $l:\ r=2(1-\cos\varphi)$ kardioidaning $\varphi=-\pi$ dan $\varphi=-\dfrac{\pi}{2}$ gacha qismi.

8.7. $l:\ x=\sqrt{3}t^2,\ \ y=t-t^3$ egri chiziq yoyining $t=0$ dan $t=1$ gacha qismi.

8.8. $l:\ x=3(\cos t+t\sin t),\ \ y=3(\sin t-t\cos t)$ $(0\leq t\leq\pi)$ egri chiziq yoyi.

8.9. $l:\ r=a\sin^3\dfrac{\varphi}{3}$ egri chiziq yoyi.

8.10. $l:\ x^2+y^2=25$ aylananing Ox o‘qdan yuqori yarim qismi.

8.11. l: $r=4(1+\cos\varphi)$ kardioidaning $\varphi=0$ dan $\varphi=\pi$ gacha qismi.

8.12. l: $y=ach\dfrac{x}{a}$ zanjir chiziq yoyining $x=-a$ dan $x=a$ gacha qismi.

8.13. l: $x^2+y^2=16$ aylananing Oy o‘qdan o‘nq tomonda yotgan yarim qismi.

8.14. l: $x=3\cos^3\dfrac{t}{2}$, $y=3\sin^3\dfrac{t}{2}$ astroidaning uchinchi kvadrantdagi qismi.

8.15. l: $r=2\cos\varphi$ egri chiziq yoyining $\varphi=-\dfrac{\pi}{4}$ dan $\varphi=\dfrac{\pi}{4}$ gacha qismi.

8-masala. (8.16-8.25). Berilgan chiziqlar bilan chegaralangan bir jinsli D yassi figura og‘irlik markazining koordinatalarini toping:

8.16. D: $r^2=9\cos2\varphi$ limniskataning birinchi halqasi bilan chegaralangan.

8.17. D: $y=\sin x$ sinusoida va Ox o‘qining $[0;\pi]$ kesmasi bilan chegaralangan.

8.18. D: $y^2=3x$ va $x^2=3y$ egri chiziqlar bilan chegaralangan.

8.19. D: $x=4\cos^3 t$, $y=4\sin^3 t$ $\left(0\le t\le\dfrac{\pi}{2}\right)$ astroida yoyi bilan chegaralangan.

8.20. D: $r=2(1-\cos\varphi)$ kardioida bilan chegaralangan.

8.21. D: $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$ ellips va koordinata o‘qlari $(y\ge0,\ x\ge0)$ bilan chegaralangan.

8.22. D: $y=(x-2)^2$, $x=0$, $y=0$ chiziqlar bilan chegaralangan.

8.23. D: $x^2+y^2=16$ aylananing $\varphi=60^o$ li markaziy burchagi bilan chegaralangan.

8.24. D: $x+y=6$, $y=0$, $x=0$ chiziqlar bilan chegaralangan.

8.25. D: $y=\cos x$ kosinusoida va koordinata o‘qlari bilan chegaralangan.

9-masala. Differensial tenglamaning umumiy yechimini toping:

9.1. 1) $(1+e^{-x})yy'=1$. 2) $y^2+x^2y'=xyy'$.

9.2. 1) $y^{'}\ln y=e^{3x}$. 2) $xy^2y'=x^3+y^3$.

9.3.1) $\cos^3 yy'-\cos(2x-y)=(\cos2x+y)$. 2) $(4y+5x)dx+(5y+7x)dy=0$.

9.4. 1) $(e^x+8)2y-ye^xdx=0$. 2) $xy'=y\left(\ln\dfrac{y}{x}-1\right)$.

9.5. 1) $3^{x^2+y}dy + xdx = 0.$ 2) $(2\sqrt{xy} - x)y' + y = 0.$

9.6. 1) $e^{-x^2}dy - x(1+y^2)dx = 0.$ 2) $y' = \frac{y}{x} + \sin\frac{y}{x}.$

9.7. 1) $e^{3y+x}dx = ydy.$ 2) $x^3y' = y(y^2 + x^2).$

9.8. 1) $x + xy + y'(y + xy) = 0.$ 2) $y' - \frac{y}{x} = tg\frac{y}{x}.$

9.9.1) $2yx^2dy = (1+x^2)dx.$ 2) $xy' - y = (x+y) + \ln\left(\frac{x+y}{x}\right).$

9.10.1) $(xy^2 + x) + y'(y - x^2y) = 0.$ 2) $xy' = y - xe^{\frac{y}{x}}.$

9.11.1) $xydy = (1-x^2)dx.$ 2) $xy' = y\cos\left(\ln\frac{y}{x}\right).$

9.12. 1) $y' + \sqrt{\frac{1-y^2}{1-x^2}} = 0.$ 2) $(x^2 - 2xy)y' = xy - y^2.$

9.13. 1) $\sin y\cos xdy = \cos y\sin xdx.$ 2) $y' = \frac{y}{x} + \frac{x}{y}.$

9.14. 1) $y' = 10^{x+y}.$ 2) $(y + \sqrt{xy}) = xy'.$

9.15. 1) $\sqrt{1-x^2}dy - x\sqrt{1-y^2}dx = 0.$ 2) $y\ln\frac{y}{x}dx - xdy = 0.$

9.16. 1) $(1+y)dx = (x-1)dy.$ 2) $xyy' = y^2 + 2x^2.$

9.17. 1) $\sqrt{4-x^2}y' + xy^2 + x = 0.$ 2) $xy + y^2 = (2x^2 + xy)y'.$

9.18. 1) $x^2dy - (2xy + 3y)dx = 0.$ 2) $(y + 2x)dy - ydx = 0.$

9.19.1) $(1+y^2)dx - \sqrt{x}dy = 0.$ 2) $(2y^2 + 3x^2)xdy = (3y^3 + 6yx^2)dx.$

9.20. 1) $1 + (1+y')e^y = 0.$ 2) $y^2 = x(x+y)y'.$

9.21. 1) $(4x + 2xy^2)dx - (3y - 3x^2y)dy = 0.$ 2) $(x^2 - 3y^2)dx + 2xydy = 0.$

9.22.1) $\sin yy' - y\cos x = 2\cos x.$ 2) $(y^2 - 2xy)dx - x^2dy = 0.$

9.23.1) $y' = (2y+1)tgx.$ 2) $ydx - xdy = \sqrt{x^2 + y^2}dy.$

9.24.1) $\sqrt{3+y^2}dx - ydy = x^2ydy.$ 2) $xy' = 4\sqrt{2x^2 + y^2} + y.$

9.25. 1) $x(4+e^y)dx - e^y dy = 0.$ 2) $\left(xye^{\frac{x}{y}} + y^2\right) = x^2 e^{\frac{x}{y}} y'.$

10-masala. Koshi masalasini yeching:

10.1. $y'x + y = \dfrac{xy^2}{3}, \quad y(1) = 3.$

10.2. $y' + y = e^{\frac{x}{2}}\sqrt{y}, \quad y(0) = \dfrac{9}{4}.$

10.3. $y' - y = xy^2, \quad y(0) = 1.$

10.4. $xy' + y = 2y^2 \ln x, \quad y(1) = \dfrac{1}{2}.$

10.5. $3xy' + 5y = (4x-5)y^4, \quad y(1) = 1.$

10.6. $y' + 2xy = 2x^3y^2, \quad y(0) = \sqrt{2}.$

10.7. $y' + y = xy^2, \quad y(0) = 1.$

10.8. $2(y' + y) = xy^2, \quad y(0) = 2.$

10.9. $y' - ytgx = y^4 \cos x, \quad y(0) = 1.$

10.10. $xyy' = y^2 + x, \quad y(1) = \sqrt{2}.$

10.11. $xy' - 2x^2\sqrt{y} = 4y, \quad y(1) = 1.$

10.12. $y' + x\sqrt[3]{y} = 3y, \quad y(0) = 1.$

10.13. $y' - ytgx = -\dfrac{2}{3}y^4 \sin x, \quad y(0) = 1.$

10.14. $xy' + y = y^2 \ln x, \quad y(1) = 1.$

10.15. $2(xy' + y) = xy^2, \quad y(1) = 1.$

10.16. $3(xy' + y) = y^2 \ln x, \quad y(1) = 3.$

10.17. $yx' + x = -yx^2, \quad x(1) = 2.$

10.18. $y' - y + y^2 \cos x = 0, \quad y(0) = 2.$

10.19. $xy^2y' = x^2 + y^3, \quad y(1) = \sqrt[3]{3}.$

10.20. $xy' - 2\sqrt{x^3}\,y = y, \quad y(2) = 8.$

10.21. $xy' + y = xy^2, \quad y(1) = 1.$

10.22. $y' - y = \dfrac{x}{y}e^x; \quad y(0) = 4.$

10.23. $xdx = \left(\dfrac{x^2}{y} - y^3\right)dy, \quad x(1) = \sqrt{2}.$

10.24. $y' - xy = -y^3e^{-x^2}; \quad y(0) = \dfrac{\sqrt{2}}{2}.$

10.25. $y'x + y = -xy^2; \quad y(1) = 2.$

11-masala. Differensial tenglamani ixtiyoriy o‘zgarmasni variatsiyalash usuli bilan yeching:

11.1. $y'' + y = ctgx$.

11.2. $y'' + 4y = tg2x$.

11.3. $y'' + y = x\cos^2 x$.

11.4. $y'' + y = tgx$.

11.5. $y'' + 4y = ctg2x$.

11.6. $y'' + 2y' + y = xe^x$.

11.7. $y'' - 4y' = e^{2x} - e^{-2x}$.

11.8. $y'' + 4y = \dfrac{1}{\sin 2x}$.

11.9. $y'' + 5y' + 6y = \dfrac{1}{1+e^{2x}}$.

11.10. $y'' - y = \dfrac{e^x}{e^x + 1}$.

11.11. $y'' + 2y' + 2y = \dfrac{e^{-x}}{\cos x}$.

11.12. $y'' + 4y' + 4y = \dfrac{e^{-x3}}{x^3}$.

11.13. $y'' + y = \dfrac{1}{\sin x}$.

11.14. $y'' - 2y' + y = \dfrac{e^x}{x}$.

11.15. $y'' - 4y' + 5y = \dfrac{e^{2x}}{\cos x}$.

11.16. $y'' + 4y = \dfrac{1}{\cos 2x}$.

11.17. $y'' + 3y' + 2y = \dfrac{1}{e^x + 1}$.

11.18. $y'' + 4y' + 4y = e^{-2x} \ln x$.

11.19. $y'' - 2y = xe^{-x}$.

11.20. $y'' - y = e^{2x} \sin(e^x)$.

11.21. $y'' - y = e^{2x} \cos(e^x)$.

11.22. $y'' - 4y' + 4y = \dfrac{e^{2x}}{x^3}$.

11.23. $y'' + 2y' = \dfrac{1}{\cos 3x}$.

11.24. $y'' + y = \dfrac{2}{\sin^2 x}$.

11.25. $y'' + \pi^2 y = \dfrac{\pi^2}{\sin \pi x}$.

12-masala. Differensial tenglamalar sistemasining umumiy yechimini toping:

12.1. $\begin{cases} y_1' = 3y_1 - y_2 + e^x, \\ y_2' = y_1 + y_2 + x. \end{cases}$

12.2. $\begin{cases} y_1' = 2y_1 - y_2 + \cos x, \\ y_2' = 3y_1 - 2y_2 + \sin x. \end{cases}$

12.3. $\begin{cases} y_1' = y_1 + y_2 + x, \\ y_2' = y_1 - 2y_2 + 2x. \end{cases}$

12.4. $\begin{cases} y_1' = -y_1 + y_2 + x, \\ y_2' = 3y_1 + y_2 + x^2. \end{cases}$

12.5. $\begin{cases} y_1' = y_1 - 3y_2 + e^{2x}, \\ y_2' = y_1 - y_2 + 2x. \end{cases}$

12.6. $\begin{cases} y_1' = 2y_1 + y_2 + 1, \\ y_2' = -5y_1 - 2y_2 + x. \end{cases}$

12.7. $\begin{cases} y_1' = y_1 + 4y_2, \\ y_2' = -y_1 + y_2 + e^{3x}. \end{cases}$

12.8. $\begin{cases} y_1' = 3y_1 + y_2 + e^x, \\ y_2' = y_1 + 3y_2 - e^x. \end{cases}$

12.9. $\begin{cases} y_1' = y_2 - \cos x, \\ y_2' = 2y_1 + y_2. \end{cases}$

12.10. $\begin{cases} y_1' = 4y_1 - 5y_2 + 4x + 1, \\ y_2' = y_1 - 2y_2 + x. \end{cases}$

12.11. $\begin{cases} y_1' = -2y_1 - y_2 + \sin x, \\ y_2' = 4y_1 + 2y_2 + \cos x. \end{cases}$

12.12. $\begin{cases} y_1' = y_1 - y_2 - e^{-x}, \\ y_2' = -4y_1 + y_2 + xe^{-x}. \end{cases}$

12.13. $\begin{cases} y_1' = 5y_1 + 4y_2 + e^{x}, \\ y_2' = 4y_1 + 5y_2 + 1. \end{cases}$

12.14. $\begin{cases} y_1' = -2y_1 - y_2, \\ y_2' = 5y_1 + 2y_2 + x^2 + 1. \end{cases}$

12.15. $\begin{cases} y_1' = 5y_1 - 3y_2 + xe^{2x}, \\ y_2' = 3y_1 - y_2 + e^{2x}. \end{cases}$

12.16. $\begin{cases} y_1' = 4y_1 - y_2, \\ y_2' = y_1 + 2y_2 + xe^{x}. \end{cases}$

12.17. $\begin{cases} y_1' = y_1 - 3y_2, \\ y_2' = y_1 + y_2 + e^{x}. \end{cases}$

12.18. $\begin{cases} y_1' = y_1 - 3y_2 + 1, \\ y_2' = -y_1 + y_2 + 2x. \end{cases}$

12.19. $\begin{cases} y_1' = 4y_1 + y_2 - e^{3x}, \\ y_2' = -y_1 + 2y_2. \end{cases}$

12.20. $\begin{cases} y_1' = 2y_1 + y_2 + x, \\ y_2' = -5y_1 - 2y_2 + x^2. \end{cases}$

12.21. $\begin{cases} y_1' = 2y_1 + y_2 - \cos 3x, \\ y_2' = -y_1 + 4y_2 + \sin 3x. \end{cases}$

12.22. $\begin{cases} y_1' = 2y_1 - 5y_2, \\ y_2' = y_1 - 2y_2 + e^{2x}. \end{cases}$

12.23. $\begin{cases} y_1' = 2y_1 + 4y_2 + \cos x, \\ y_2' = 3y_1 - 2y_2 + \sin x. \end{cases}$

12.24. $\begin{cases} y_1' = 2y_1 + 3y_2 + e^{x}, \\ y_2' = y_1 - 2y_2 + 2xe^{x}. \end{cases}$

12.25. $\begin{cases} y_1' = -2y_1 - y_2 + e^{-x}, \\ y_2' = 3y_1 + 2y_2 - e^{-x}. \end{cases}$