O’ZBEKISTON RESPUBLIKASI
OLIY VA O’RTA MAXSUS TA’LIM VAZIRLIGI

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AVTOMATIK BOSHQARISH NAZARIYASI
fanidan
amaliy mashg‘ulotlar uchun o‘quv qo’llanma

«5311000 **–Texnologik jarayonlarni ishlab chiqarishda avtomatlashtirish va boshqarish».**

Toshkent 2019

Аннотация

O’quv qo’llama “Avtomatik boshqarish nazariyasi” faninihng 5311000 - “Texnologik jarayonlar va ishlab chiqarishni avtomatlashtirish va boshqarish” yo’nalishi uchun qabul qilingan o’quv dasturiga binoan tayyorlangan. To’plamda chiziqli uzliksiz va impuls, hamda nochiziq tizimlarga doir mashqlar keltirilgan. Har bir mavzu bo’yicha mashqlar oldida, tegishli nazariy kirishlar keltirilgan.

O’quv qo’llama “Avtomatik boshqarish nazariyasi” fanini o’rganayotgan barcha talabalar, magistrlar va o`qituvchlar, soxa mutaxassislari uchun mo’ljallangan.

Аннотация

Учебное пособие подготовлено в соответствии с учебной программой курса «Теория автоматического управления» по направлению 5311000 - «Автоматизация и управление технологическими процессами и производством». В состав практических задач входят упражнения для линейных и импульсных систем и нелинейных систем. Для каждой темы перед упражнениями даны соответствующие рекомендации для выполнения задач.

Учебное пособие предназначено для всех студентов, магистров, также могут пользоваться преподаватели и инженеры технических направлений, изучающих теорию автоматического управления.

Annotation

The manual was prepared in accordance with the curriculum of the course "Theory of automatic control" in the direction 5311000 - "Automation and control of technological processes and production." The practical tasks include exercises for linear and impulse systems and non-linear systems. For each topic before the exercises, appropriate recommendations are given for completing tasks.

The manual is intended for all students, masters, teachers and engineers of technical directions which studying the theory of automatic control.

Taqrizchilar:

TATU xuziridagi “Axborot kommunikatsiya texnologiyalari ilmiy-innovatsion markazining “Amaliy axborot tizimlari va axborot xavfsizligi” laboratoriyasi mudiri, t.f.n., dotsent A.Arifjanov

TIQXMMI “Elekt ta’minoti va qayta tiklanuvchan energiya manbalari” kafedrasi dotsenti, t.f.n. Sh.Muzaffarov

SO`Z BOSHI

Hozirgi zomonda “Avtomatik boshqarish nazariyasi” qator muhandislik yo`nalishlari uchun muhim va zaruriy fanlardan biri bo`lib kelmoqda. Uning ahamiyati faqat oshib bormoqda, qo`llanish sohalari esa yildan-yilga kengaymoqda.

Bizda tarqalgan darslik, masalan [1,3-7, 9-11], va mashq to`plamlari [2,8], asosan rus tilida bosilgan. Keltirilgan darsliklar ichida ancha ilgari bosmadan chiqqanlari [1,3,4,7,9,10] va nisbatan yangiroqlari mavjud [5,6,11]. Bulardan tashqari bizdagi sotuvda bo`lmagan, alohida shaxslar tufayli kelib qolgan ingliz tilida bosilgan darsliklar uchrab qoladi [12-14]. Odatda ular tarkibida fanga doir mashqlar mavjud. Bu hol fanni o`rganishda talabalarimiz uchun ma’lum noqulayliklar hosil qilmoqda. Undan tashqari, bu kitoblar son va sifat jihatdan talabalarimizni qoniqtirmayapdi.

Bu fanning usullari mashinasozlik, kimyo` sanoati, aviasozlik va aviatsiya, asbobsozlik, qishloq xo`jaligi, suv xo`jaligi, qurilish, iqtisodiyot, moliya, meditsina va boshqa qator sohalarda unimli qo`llanishi mumkin.

Har qanday fan singari avtomatik boshqarish nazariyasini o`rganishda shu sohaga doir bo`lgan mashqlarni bajarish muhim ijobiy ahamiyatga ega.

Ushbu mashqlar to`plami “Avtomatik boshqarish nazariyasi” fanining 72 soatlik maruzalar kursiga mo`ljallangan. To`plamda avtomatik boshqarishning umumiy masalalari, chiziqli uzliksiz tizimlar, chiziqli impuls tizimlar va uzliksiz nochiziq tizimlarga tegishli bo`lgan mashqlar keltirilgan. Bu mashqlar boshqarish tizimni kirish-chiqish shaklida ifodalanishi nazarda tutib tuzilgan bo`lib, oddiy differensial tenglamalar, rekkurent tenglamalar, vazn va o`tish funksiyalar, uzatish funksiyalar, chastota tavsiflar apparatiga asoslangan. Nochiziq tizimlarni o`rganish asosan Gammersteyn tuzilmasiga keltirilgan tizimlar asosida bajarilgan va o`quv dasturiga binoan, nochiziq tizimlar nazariyasining kichik qismini qamrab olgan.

Mashqlar asosan o`qituvchi rahbarligida bajarilishi nazarda tutilgan. Shuning uchun qator mashqlar shunday umumiy korinishda tuzilganki, o`qituvchi yoki o`quvchi parametrlarga son qiymatlar berib har xil variantlar hosil qilish imkoni mavjud. Bunday imkoniyat o`qituvchilar uchun joriy va oraliq nazoratlarda qo`llaniladigan vazifalar tayyorlashda ma'lum qulayliklar hosil qiladi. Shunga qaramasdan, fanga yetarli darajada kirib borganidan so`ng, talabalar to`plamdan to`la-tokis mustaqil foydalanishi ham mumkin.

Talabalar mashqlarni qanchalik unimli bajarganini baholashda o`qituvchi shuni hisobga olishi lozim bo`ladiki, muhandislik masalalari ko`p hollarda yagona yechimga ega bo`lmaydi. Muhandislik ishini bunday xususiyati to`plamdagi "Umumiy qism" hamda sintezga doir mashqlarda yaqqol namoyon bo`ladi.

Mashqlar uchun uch o`rinli tartib son qo`llanilgan: bo`lim, paragraf, paragraf ichidagi tartib son. Masalan, 1.2.7 quyidagini bildiradi: 1 – bo`lim, 2 – paragraf, 7 – mashq. Rasmlarga ikki o`rinli tartib son berilgan: bo`lim, bo`lim ichidagi tartib son.

Toplamdagi rasmlarni magistrant A. Xusanov va talaba D.D. Bratishevlar kompyuterda bajarganlar. Muallif ularga o`zining chuqur minnatdorchiligini bildiradi.

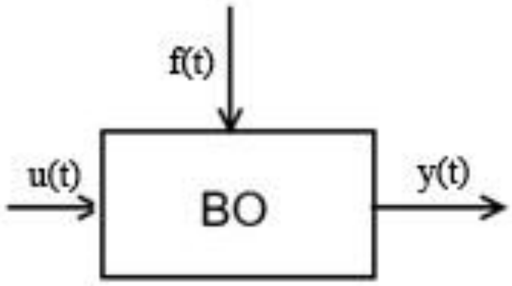
To`plamda mavjud bo`lgan barcha kamchiliklarni muallif o`z zimmasiga oladi, bu borada har qanday fikr va mulohazalarni minnatdorchilik bilan qabul qiladi.

I Umumiy qism

1.1 Ob’ektlarni boshqarish ob’ekti sifatida ifodalash

Nazariy kirish

Boshqarish ob’ekti chiqish, kirishlar va ular orasidagi bog`lanish (munosabat) birligidir. Boshqarish ob’ektini 1.0.A-rasmdagidek tasvirlash mumkin.



1.0.A-rasm. Boshqarish ob’ektining tasviri

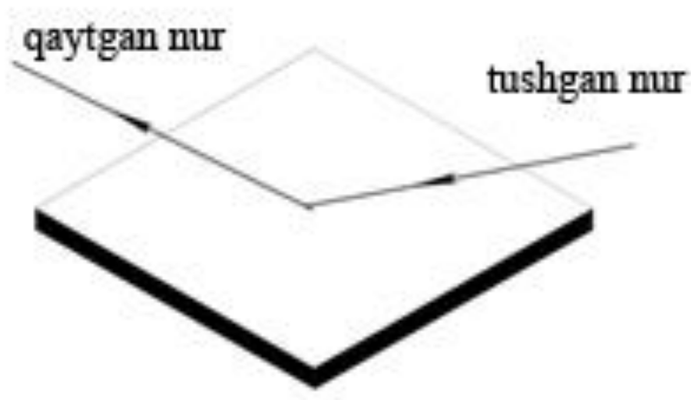
Bu yerda to`rtburchakdan tashqariga yo`naltirilgan kamon o`qi {*y(t)*} **chiqish** jarayoni, tashqaridan to`rtburchakka yo`naltirilgan kamon o`qlari {*u(t),f(t)*} esa **kirish** jarayonlarining belgilaridir. To`rtburchakning o`zi jarayonlar orasidagi **mavjud bog`lanishni** bildiradi. Ob’ektning chiqishi odatda **boshqariluvchi**, ya’ni qandaydir tarzda o`zgartirilishi lozim bo`lgan, jarayondir. Kirishlar esa chiqish jarayonini o`zgarishiga sabab bo`ladigan jarayonlardir. Kirishlar ikki xil bo`ladi. Birinchisi, masalan *u(t)*, shunday jarayonlarki, ularni istalgan tarzda o`zgartirish mumkin. Bunday kirishlarga **boshqaruvchi** kirish, yoki boshqaruvchi jarayon deyiladi. Ikkinchi xil kirishlar chiqishga ta’sir etadi, biroq uni o`zini istalgancha o`zgartirib bo`lmaydi. Bunday kirishlarga **galayonlantiruvchi** jarayonlar deyiladi.

Boshqarish ob’ektining chiqishi **boshqarish maqsadi** bilan chambarchas bog`liqdir. Aslini olganda, boshqarish maqsadi bu chiqishlar, yoki ulardan birining o`zgarishi bo`yicha bizning istagimizning ifodasidir.

Yuqorida keltirilganlardan hulosa chiqadiki, ob’ektni boshqarish ob’ekti sifatida ifodalash deganda uning chiqish va kirishlarini tayinlash va belgilash, hamda boshqarish maqsadini ifodalash tushuniladi.

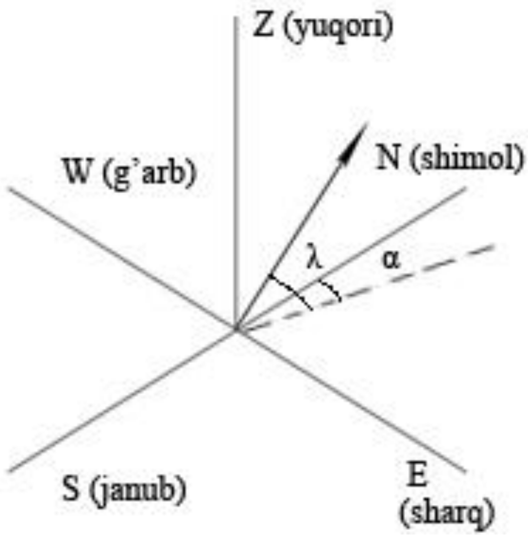
Masalan avtomobilni ko`raylik. Avtomobildan foydalanishning asosiy maqsadi bir joydan boshqa joyga yetib borishdir. Bu holda ob’ektning chiqishlari, ya’ni boshqariluvchi jarayon, sifatida avtomobilning joriy koordinatalari, tezligi, yo`nalishi ko`rilishi lozim. Unda boshqaruvchi kirish sifatida rul va pedallarning holatlari tanlanishi mumkin. Nihoyat, g`ildiraklar yo`lga ishqalanish kuchlari, yo`lda uchraydigan notekisliklar, piyodalar va h.k. g`alayonlantiruvchi kirishlar bo`ladi.

1.1.0 Nur tushib qaytayotgan ko`zgu (1.0.A-rasm) boshqrish ob’ekti sifatida tariflansin.



1.1.A-rasm. Nur tushib qaytayotgan ko`zgu.

Yechish. Ko`zgudan ***qaytayotgan nurning yo`nalishini*** chiqish, ya’ni boshqariluvchi jarayon, sifatida qabul qilamiz. Bu yo`nalish nur qaytayotgan nuqtaning ***koordinatalari*** hamda ***azimuti*** ($\alpha_n$) va ***o`rin burchagi*** (λ_n) orqali ifodalanadi (1.1.B-rasm).



1.1.B-rasm. Nur yo`nalishini ifodalovchi burchaklar.

Boshqarish maqsadi turlicha bo`lishi mumkin. Masalan, ***qaytgan nurning dog`i fazoning biror nuqtasi atrofida ushlab turilishi*** talab etilgan bo`lsin.

Qaytish nurning yonalishini o`zgarishi ***ko`zguning fazoda joylashishiga*** va ***tushish nurning yo`nalishiga*** bog`liq. Demak, bular kirish ta'sirlarini tashkil etadi. Tushish nurning yo`nalishi o`zicha o`zgaradi, shuning uchun u g`alayonlantiruvchi kirish bo`ladi.

Ma'lumki, jism fazoda joylashishi oltita erkinlik darajasi bilan belgilanadi: uch o`q bo`lab siljish va uch o`q atrofida aylanish. Ko`rilayotgan kozguning yuzasi mutlaqo silliq bo`lsa, uning sirtiga tik bo`lgan o`q atrofida aylanishi qaytish nurning yo`nalishiga ta'sir ko`rsatmaydi. Demak, beshta erkinlik darajasini nazarda tutish kifoyadir. Bulardan uch o`q bo`ylab harakatlantirish uchun uch yo`nalishda **kuch** bilan ta'sir etish kerak. Ikki o`q atrofida aylantirish uchun esa ikki **moment** bilan ta'sir etish lozim (1.1.C-rasm). Shunday qilib,bu ob'ekt uchin chiqish:

qaytgan nurning azimuti (α_n),

qaytgan nurning o`rin burchagi (λ_n),

nur qaytadigan nuqtaning (ya'ni ko`zguning) koordinatalari;

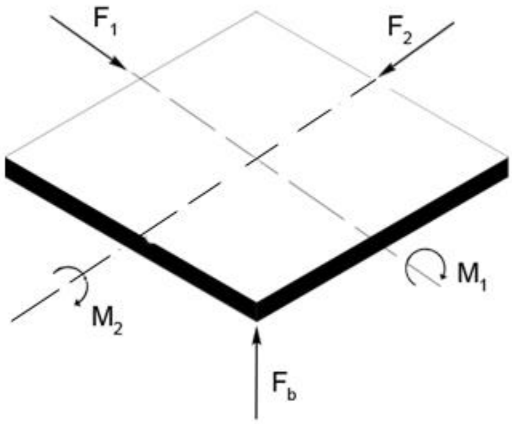
boshqaruvchi kirish:

ikki o`qqa ta’sir etadigan momentlar,

uch yo`nalish bo`yicha ta’sir etadigan kuchlar;

galayonlantiruvchi kirish:

tushish nurning yo`nalishi.

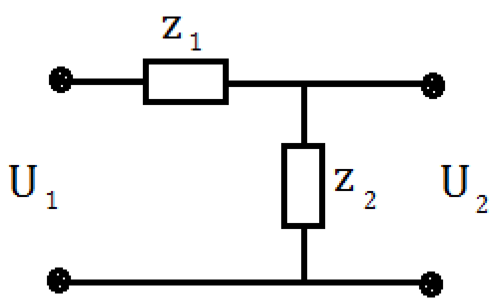


1.1.C-rasm. Ko`zguni harakatlantiruvchi ta’sirlar.

Mashqlar

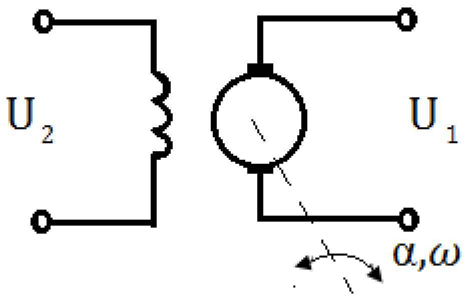
1.1.1 Ishlab chiqarish, hayot va jamiyatda uchraydigan ob’ekt tanlansin va u boshqarish ob’ekti sifatida ifodalansin. Buning uchun boshqarish maqsadi, ob’ektning chiqishi, boshqaruvchi va g`alayonlantiruvchi kirishlari ta’riflanishi lozim.

1.1.2 Boshqarish ob’ekti sifatida 1.1-rasmda keltirilgan elektr zanjir ifodalansin. Bu yerda: z_1, z_2 – kompleks qarshiliklar; u_1,u_2 – kuchlanishlar.

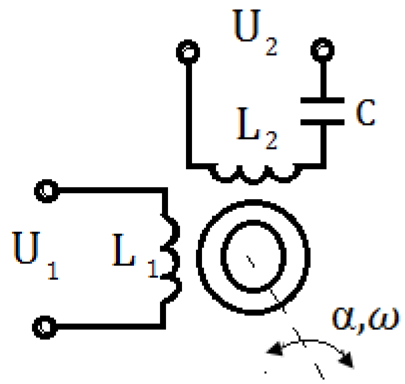


1.1-rasm. Sodda elektr zanjir

1.1.3 Erkin g`alayonlantiruvchi chulg`amli o`zgarmas tok motori (1.2-rasm) boshqarish ob'ekti sifatida ifodalansin. Bu yerda: u_1,u_2 – kuchlanishlar; α – rotorning aylanish burchagi; ω – aylanma tezlik.



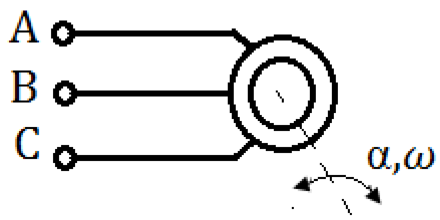
1.2-rasm. Erkin g`alayonlantiruvchi chulg`amli o`zgarmas tok motori



1.3-rasm. Ikki fazali asinxron motor

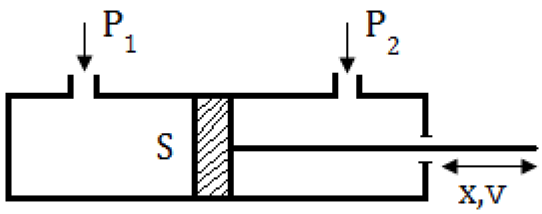
1.1.4 Ikki fazali asinxron motor (1.3-rasm) boshqarish ob'ekti sifatida ifodalansin. Bu yerda: u_1,u_2 – kuclanishlar; L_1,L_2 – chulg`amlar; C – elektr sig`im; α – rotorning aylanish burchagi; ω – rotorning aylanma tezligi.

1.1.5 Uch fazali qisqa ulangan asinxron motor (1.4-rasm) boshqarish ob'ekti sifatida ifodalansin. Bu yerda: A,B,C – kuchlanish fazalari; f – kuchlanish chastotasi; α – rotorning aylanish burchagi; ω – rotorning aylanma tezligi.

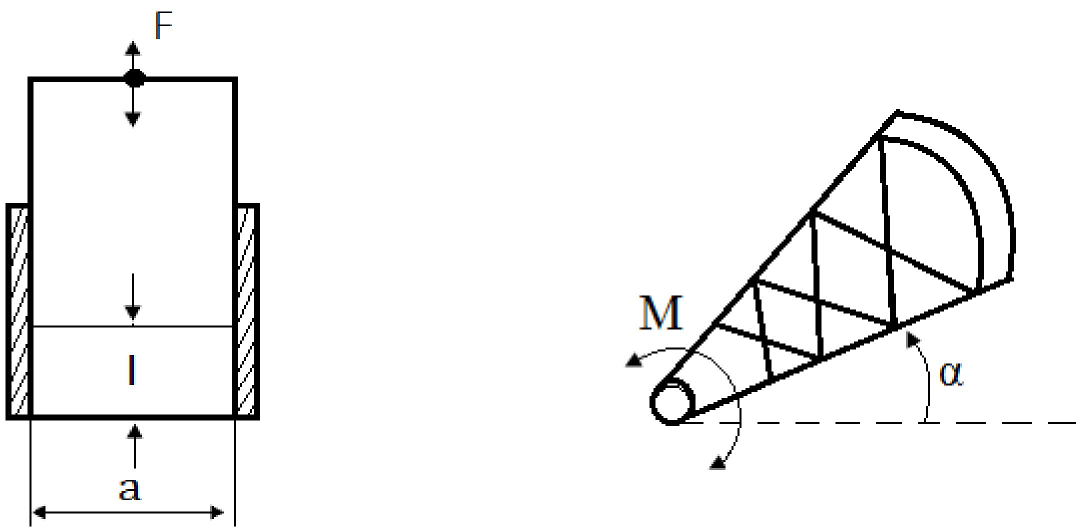


1.4-rasm. Qisqa ulangan rotorli uch fazali asinxron motor

1.1.6 Gidrosilindr (1.5-rasm) boshqarish ob’ekti sifatida ifodalansin. Bu yerda: p_1, p_2 – bosimlar; S – porshenning yuzasi; x – porshenning holati; v – porshenning chiziqli tezligi.



1.5-rasm. Gidrotsilindr

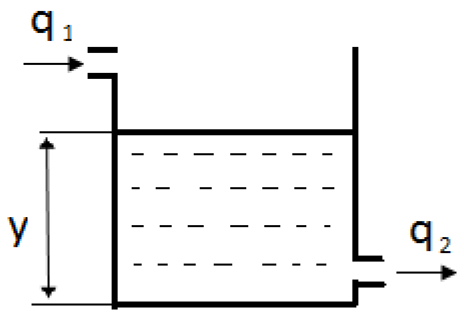


a) b)

1.6-rasm. To`sqichlar

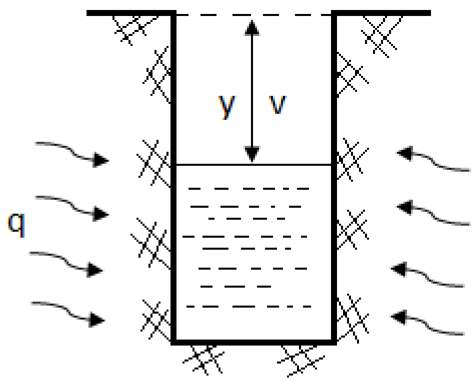
1.1.7 To`sqich (zatvor) boshqarish ob’ekti sifatida ifodalansin. 1.6-rasmda yassi (a) va sektorli (b) tosqichlarning shartli chizmalari keltirilgan. Bu yerda: F – kuch; M – moment; *a* – to`sqichning kengligi; *l,α* – to`sqichning holati.

1.1.8 Rezervuardagi suv boshqarish ob'ekti sifatida ifodalansin (1.7-rasm). Bu yerda: q_1, q_2 – rezervuarga kirib kelayotgan va undan chiqib ketayotgan suv sarflari; y – rezervuardagi suvning sadhi.



1.7-rasm. Suvli rezervuar

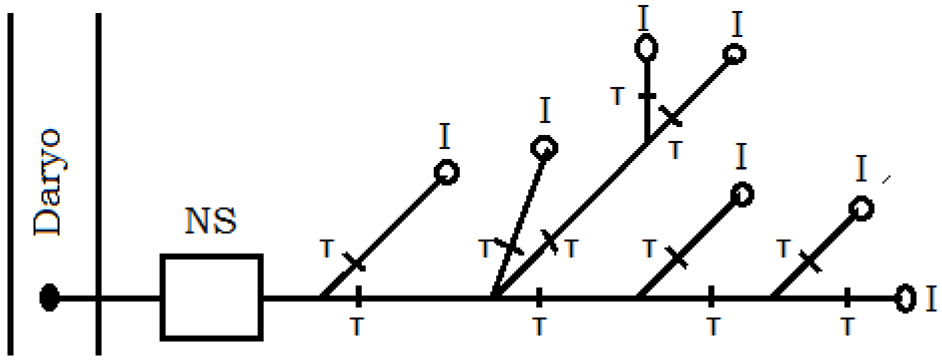
1.1.9 Quduqdagi suv boshqarish ob'ekti sifatida ifodalansin (1.8-rasm). Bu yerda: q – quduqqa silqib chiqayotgan yerosti suvlar sarfi; y – quduqdagi suvning sadhi; v – sadhni o`zgarish tezligi.



1.8-rasm. Quduqdagi suv

1.1.10 Yuqoridagi 1.1.2 – 1.1.9-mashqlar uchun rasmlarda ko`rsatilmagan ta'sirlar e'lon qilinsin.

1.1.11 Anhorlar (kanallar) tarmog`i (1.9-rasm) boshqarish ob'ekti sifatida ifodalansin. Bu yerda: NS – nasos stansiyasi; T – to`sqichlar; I – iste'molchilar. To`g`ri chiziqlar – anhorlarning shartli belgisi.



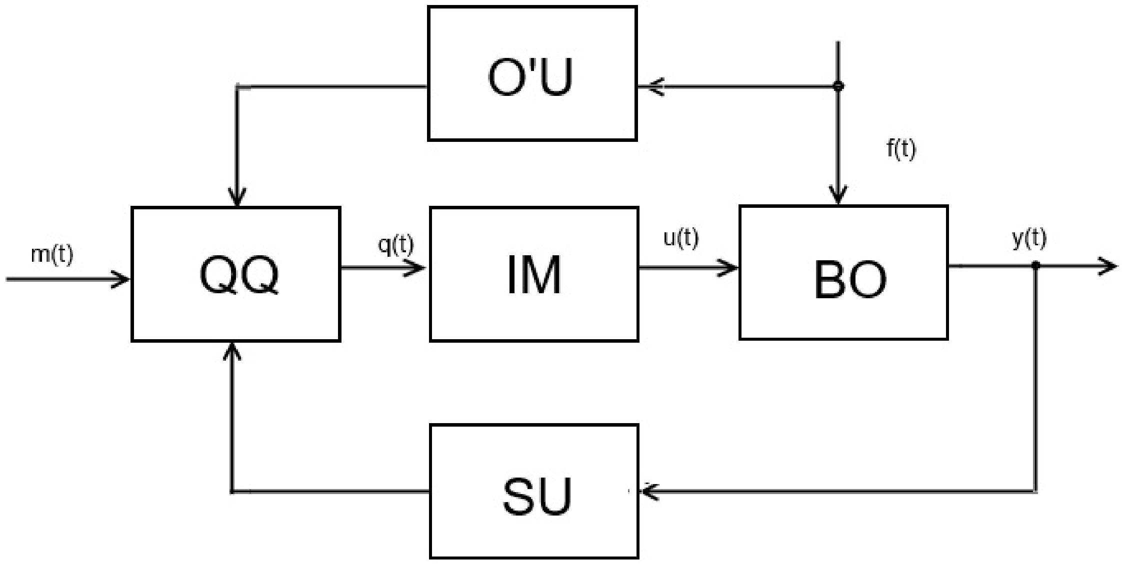
1.9-rasm. Anhorlar tarmog`i

1.2 Tizimlarning vazifaviy tuzilmalarini qurish

Nazariy kirish

Boshqarish tizimni turli tuzilmalar ko`rinishida ifodalash mumkin. Bular ichida vazifaviy tuzilma o`ziga hos o`rin tutadi. Umumiy holda vazifaviy tuzilmaning ko`rinishi 1.2.A-rasmda keltirilgan. Bu tuzilmaning asosiy o`ziga hosligi uning universalligidir. Boshqacha qilib aytganda, ixtiyoriy boshqarish tizimini shu ko`rinishda tasvirlash mumkin.

Bu tuzilmadagi har bir unsir faqat o`ziga hos vazifani bajaradi, shuning uchun ham u vazifaviy tuzilma deb nomlangan. Unda quyidagi qismlar ajratilgan:



1.2.A-rasm. Boshqarish tizimining vazifaviy tuzilmasi.

BO – boshqarish ob'ekti;

IM – ijro mexanizmi;

QQ – qaror qabul qilish qurilmasi;

SU – sezuvchi qurilma;

O`U – o`lchash qurilmasi;

y(t) – boshqariluvchi chiqish;

u(t) – boshqaruvchi kirish;

f(t) – g`alayonlantiruvchi kirish;

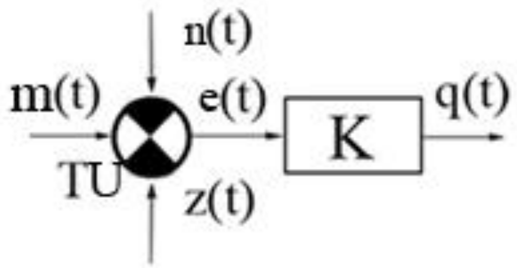
q(t) – boshqaruvchi qaror haqidagi ma'lumot;

m(t) – boshqarish maqsadi haqida ma'lumot;

z(t) – chiqish haqidagi ma'lumot;

n(t) – g`alayonlantiruvchi haqidagi ma'lumot.

Amaldagi ko`p texnik tizimlarda boshqarish haqida qaror qabul qilish faqat boshqarish maqsadi bilan natija, ya'ni boshqariluvchi chiqish, o`rtasidagi farq, tafovut asosida amalga oshiriladi. Bunday xususiy holda QQ soddalashib taqqoslash uskinasi va, u bilan ketma-ket ulangan, funksional almashtirgich ko`rinishida tasvirlanadi (1.2.B-rasm). Bunlay tizimlarga tafovut bo`yicha boshqarish tizimlar deyiladi.



1.2.B-rasm. Tafovut

boyicha boshqarish tizimining qaror qabul qilish qurilmasi.

Keltirilgan chizmada:

TU – taqqoslash uskinasi;

K – funksional almashtirgich;

e(t) – tafovut. Qolgan unsirlar 1.2.A-rasmdagidek.

Vazifaviy tuzilma nazariyada qo`llanadigan tuzilmalar orasida amaliy, haqiqiy tizimning tabiatiga eng yaqin turadiganidir. Shuning uchun u haqiqiy tizimni yaratish jarayonida shu haqiqiy tizimni dastlabki texnik tuzilmasining variantlarini ishlab chiqish uchun asos sifatida qo`llanilishi mumkin.

1.2.0 Yuqoridagi 1.1.3-mashqda keltirilgan o`zgarmas tok motori – boshqarish ob'ekti. Boshqaruvchi ta'sir – yakor zanjiriga uzatilayotgan kuchlanish $u_1$. Boshqariluvchi chiqish – motor rotorining aylanish tezligi $\omega$. Boshqarish maqsadi – aylanish tezligini ($\omega$) berilgan o`zgarmas ω_1 qiymatga teng qilish. Tizimning kamida ikki xil vazifaviy tuzilmasi taklif etilsin.

Mashqlar

1.2.1 Avvalgi 1.2.1-mashqdagi boshqarish ob'ekti uchun boshqariluvchi chiqish motor rotorining aylanish burchagi α bo`lganida vazifaviy tuzilma variantlari taklif qilinsin. Boshqarish maqsadi – motor rotorini ma'lum α_T holatga keltirish.

1.2.2 Yuqoridagi 1.2.1-mashqda keltirilgan masala boshqaruvchi ta’sir sifatida q`alayonlantiruvchi chulg`amga uzatilayotgan kuchlanish u_2 qabul qilingan hol uchun yechilsin.

1.2.3 Ikki fazali motor (1.3-rasm) boshqarish ob’ektidir. Boshqaruvchi ta’sir sifatida motorga uzatilayotgan kuchlanishning chastotasi f qabul qilingan. Motorning aylanish tezligini (ω) barqarorlashtiruvchi tizimning vazifaviy tuzilmasi taklif etilsin.

1.2.4 Qisqa ulangan rotorli uch fazali asinxron motorning (1.4-rasm) aylanish tezligini barqarorlashtiruvchi tizimning vazifaviy tuzilmasi taklif qilinsin.

1.2.5 Qisqa ulangan rotorli uch fazali asinxron motorning aylanish burchagini barqarorlashtiruvchi tizimning vazifaviy tuzilmasi taklif qilinsin.

1.2.6 Gidrisilindrning (1.5-rasm) holatini (x) barqarorlashtiruvchi tizimning vazifaviy tuzilmasi taklif qilinsin.

1.2.7 Yassi to`sqichning (1.6a-rasm) holatini (l) barqarorlashtiruvchi tizimning vazifaviy tuzilmasi taklif qilinsin.

1.2.8 Sektor to`sqichning (1.6b-rasm) holatini (α) barqarorlashtiruvchi tizimning vazifaviy tuzilmasi taklif qilinsin.

1.2.9 Rezervuardagi (1.7-rasm) suv sadhini (y) barqarorlashtiruvchi tizimning vazifaviy tuzilmasi taklif qilinsin.

1.2.10 Quduqdagi suv sadhini (1.8-rasm) rostlash tizimning vazifaviy tuzilmasi taklif qilinsin.

1.2.11 Anhorlar tarmog`ida (1.9-rasm) suv sarfini boshqarish tizimning vazifaviy tuzilmasi taklif qilinsin.

II Chiziqli uzliksiz tizimlar

2.1 Chiziqli dinamik tizimlarning differensial tenglamalari, o`tish va vazn funksiyalar

Nazariy kirish

Chiziqli statsionar parametrlari tarqoq bo`lmagan dinamik tizimlardagi jarayonlar o`zgarmas koeffitsientli oddiy differensial tenglamalar orqali ifodalanadi. Bunday tenglamalarning umumiy ko`rinishi quyidagicha

$$\sum_{i=o}^{n} a_i \frac{d^i y(t)}{dt^i} = \sum_{i=o}^{m} b_i \frac{d^i u(t)}{dt^i}.$$

Bu yerda

y(t) – boshqariluvchi chiqish,

u(t) – boshqaruvchi kirish.

Koeffitsentlar $\{a_i, b_i\}$ o`zgarmas sonlardir. Butun son *n* tizimning tartibi deyiladi. Fizik ro`yobga oshirish mumkin bo`lgan tizimlar uchun butun sonlar *n* va *m* quyidagi tengsizlikni qanoatlantirishi zarur:

$$n \geq m.$$

Boshlang`ich shartlar ma'lum bo`lsa, keltirilgan tenglama yagona yechimga ega. Bunday tenglamalarning yechimini topish matematika fani tomonidan chuqur ishlab chiqilgan. Differensial tenglama mos dinamik tizimni to`la-to`kis ifodalaydi.

Biror ma'lum *u(t)* funksiya uchun hosil bo`lgan *y(t)* yechimga tizimning shu kirishga reaksiyasi deyiladi.

Dirakning ajoib funksiyasi yo`ki δ-funksiya quydagicha ta'riflanadi

a) $\delta(t) = \begin{cases} 0, t \neq 0 \\ \infty, t = 0 \end{cases}$

b) $\int_{-\varepsilon}^{\varepsilon} \delta(t) dt = 1, \qquad \varepsilon > 0.$

Boshlang`ich shartlar no`lga teng bo`lganida tizimning δ-funksiyaga reaksiyasi $g(t)$ tizimning vazn funksiyasi deyiladi. Tizimning differensial tenglamasida boshlang`ich shartlar no`lga teng deb qabul qilib,

$$u(t) = \delta(t)$$

bo`lganida tenglamaning yechimi vazn funksiyaga teng bo`ladi, ya'ni

$$y(t) = g(t).$$

Vazn funksiya ham dinamik tizimni to`la-to`kis ifodalaydi. Agar vazn funksiya ma'lum bo`lsa, ixtiyoriy kirish ta'siri uchun tizim reaksiyasini Dyuamel integrali orqali topish mumkin:

$$y(t) = y(0) + \int_0^t g(\tau)u(t-\tau)d\tau.$$

Xevisayd funksiyasi, yo`ki birli pog`onasimon funksiya quyidagicha ta'riflanadi

$$1(t) = \begin{cases} 0, t < 0 \\ 1, t \geq 0 \end{cases}.$$

Boshlang`ich shartlar no`lga teng bo`lganda tizimning birli pog`onasimon funksiyaga reaksiyasi *h(t)* tizimning o`tish funksiyasi deyiladi. Agar boshlang`ich shartlar no`lga teng bo`lib,

$$u(t) \equiv 1(t)$$

bo`lsa, differensial tenglamaning yechimi o`tish funksiyaga teng bo`ladi, ya'ni

$$y(t) \equiv h(t).$$

O`tish funksiyaning birinchi tartibli hosilasi vazn funksiyaga teng

$$h'(t) \equiv g(t).$$

Demak, o`tish funksiya vazn funksiyaning integraliga teng

$$h(t) = \int_0^t g(\tau)d\tau.$$

O`tish funksiya ham tizimni to`la-to`kis ifodalaydi.

Mashqlar

2.1.1 Berilgan teglamalardan dinamik bo`g`inlarning o`tish va vazn funksiyalari topilsin.

a) $y(t)=ku(t)$;

b) $y(t)=ku(t-\tau)$;

c) $y(t)=k\dot{u}(t)$;

d) $T\dot{y}(t)=ku(t)$;

e) $T\dot{y}(t)+y(t)=ku(t)$;

f) $T\dot{y}(t)-y(t)=ku(t)$;

g) $y(t)=k(T\dot{u}(t)+u(t)$;

h) $T^2\ddot{y}(t)+2\xi T\dot{y}(t)+y(t)=ku(t)$;

i) $y(t)=k(u(t)+2\xi T\dot{u}(t)+T^2\ddot{u}(t))$;

j) $T^2\ddot{y}(t)+y(t)=ku(t)$;

k) $T^2\ddot{y}(t)-2\xi T\dot{y}(t)+y(t)=ku(t)$.

2.1.2 ARTning differensial tenglamasi

$$\sum_{i=0}^{n} a_i y^{(i)}(t) = ku(t).$$

Uning xarakteristik ko`phadining ildizlari karrali bo`lmasdan λ_1, λ_2, λ_3,… λ_n sonlarga teng. ARTning vazn va o`tish funksiyalarining ifodalari yozilsin.

2.2 Uzatish funksiya

Nazariy kirish

Qandaydir $f(t)$ funksiya ko`rilsin. Shu funksiya uchun Laplas almashtirishi deb quyidagiga aytiladi

$$F(p) = \int_0^\infty f(t)e^{-pt}\,dt.$$

Dastlabki *f(t)* funksiyaga **asl** deyiladi. Unga nisbatan qo`llanilgan Laplas almashtirish *F(p)* natijasiga **tasvir** deyiladi. Keltirilgan integral juda keng funksiyalar to`plami uchun mavjud va ular uchun teskari Laplas almashtirishi ham ma’noga ega. Asl bilan tasvir o`rtasidagi munosabatni

$$F(p) \Leftrightarrow f(t),$$

yoki

$$f(t) \Leftrightarrow F(p)$$

Ko`rinishda belgilash mumkin.

Boshlang`ich holati no`lga teng bo`lgan tizim, kirish ta’siri *u(t)* va chiqish reaksiyasi *y(t)* berilgan bo`lsa, tizimning uzatish funksiyasi quyidagicha ta’riflanadi

$$W(p) = \frac{Y(p)}{U(p)}.$$

Bu yerda

$$Y(p) \Leftrightarrow y(t),$$

$$U(p) \Leftrightarrow u(t).$$

O`zgarmas koeffitsientli chiziqli oddiy differensial tenglamalar bilan ifodalanadigan tizimlarning uzatish funksiyalari ko`p hollar uchun kasirli ratsional

$$W(p) = \frac{\sum_{i=0}^{m} b_i p^i}{\sum_{i=0}^{n} a_i p^i},$$

yoki

$$W(p) = \frac{\sum_{i=0}^{m} b_i p^i}{\sum_{i=0}^{n} a_i p^i} e^{-p\tau} \ ,$$

Korinishga ega bo`ladi. Bu yerda $a_i$ , $b_i$ koeffitsientlar va sof kechikish vaqti deb nomlangan $\tau$ parametr o`zgarmas sonlardir. O`zgaruvchi $p$ kompleks o`zgaruvchidir. Fizik ro`yobga oshirish mumkin bo`lgan tizimlar uchun *n* va *m* butun sonlar

$$n \geq m$$

tengsizlikni qanoatlantirishi kerak.

Uzatish funksiyaning suratida turgan ko`phadning ildizlariga tizimning **no`llari** deyiladi. Mahrajdagi ko`phad tizimning xarakteristik ko`phadi deyiladi. Uning ildizlariga tizimning **qutblari** deyiladi.

Agar tizimning no`llari $\lambda_1, \lambda_2, \dots \lambda_m$ va qutblari $\mu_1, \mu_2, \dots \mu_n$ ma'lum do`lsa kasirli ratsional uzatish funksiyani

$$W(p) = \frac{b_m \prod_{i=1}^{m}(p-\lambda_i)}{a_n \prod_{i=1}^{n}(p-\mu_i)}$$

ko`rinishda yozish mumkin.

Surat ildizlaridan $2m_k$ donasi kompleks, m-$2m_k$ donasi esa haqiqiy bo`lsin. Mahraj ildizlaridan $\nu$ donasi no`lga teng, $2n_k$ donasi kompleks va n-$2n_k$-ν donasi haqiqiy bo`lsin. Unda uzatish funksiya quyidagi ko`rinishda yozilishi mumkin

$$W(p) = \frac{K \prod_{i=1}^{m-2m_k}(1+\tau_i p) \prod_{i=m-2m_k+1}^{m}(1+2\zeta_i \tau_i p + \tau_i^2 p^2)}{p^\nu \prod_{i=\nu+1}^{n-2n_k}(1+T_i p) \prod_{i=n-2n_k+1}^{n}(1+2\xi_i T_i p + T_i^2 p^2)}.$$

Bu yerda

K – tizimning statik kuchaytirish koeffitsienti,

τ_i, T_i – vaqt doimiylari,

ζ_i, ξ_i – tebranuvchanlik koeffitsientlari.

Mashqlar

2.2.1 Yuqoridagi 2.1.1-mashqda keltirilgan tenglamalardan, rasmiy usuldan foydalanib, bo`g`inlarning uzatish funksiyalari topilsin. Uzatish funksiya bilan o`tish va vazn funksiyalar bir-biriga mos qo`yilsin.

2.2.2 ARTning differensial tenglamasi

$$\sum_{i=0}^{n} a_i y^{(i)}(t) = \sum_{i=1}^{m} b_i u^{(i)}(t).$$

Tizimning uzatish funksiyasi topilsin.

2.2.3 Yuqoridagi 2.1.2-mashqda keltirilgan ARTning uzatish funksiyasi topilsin va u qutiblar orqali ifodalansin.

2.2.4 Agar tizimda sof kechikish (τ) mavjud bo`lsa, 2.2.1-mashqdagi tenglama qanday ko`rinishga ega bo`ladi? Uzatish funksiyachi?

2.2.5 Uzatish funksiyasi

$$W(p) = \frac{ke^{-p\tau}}{1 + T_1 p + T_2^2 p^2}$$

ko`rinishga ega bo`lgan tizimning differensial tenglamasi yozilsin.

2.2.6 Konservativ bo`g`inning uzatish funksiyasi

$$W(p) = \frac{k}{p^2 + a^2}$$

ko`rinishga ega. Uning qutiblari aniqlansin va kompleks sirtda tasvirlansin.

2.2.7 Tizimning uzatish funksiyasi

$$W(p) = \frac{k(T_1^2 p^2 + 2\xi_1 T_1 p + 1)}{p(T_2 p + 1)(T_3^2 p^2 + 2\xi_2 T_3 p + 1)}$$

ko`rinishga ega. Uning qutib va nollari topilsin va kompleks sirtda tasvirlansin.

2.2.8 Avvalgi 2.2.7-mashqdagi tizim nominimal fazali va maksimal fazali bo`lish shartlari aniqlansin.

2.2.9 Ikkinchi tartibli dinamik bo`g`inning uzatish funksiyasi ikki xil ko`rinishda

$$W(p) = \frac{k}{1 + T_1 p + T_2^2 p^2}$$

va

$$W(p) = \frac{k}{1 + 2\xi T p + T^2 p^2}$$

ifodalanishi mumkin. Ular ikkalasi bir bo`g`inni ifodalagan holda, parametrlar o`rtasidagi bog`liqlik topilsin.

2.2.10 Ikki uzatish funksiya

$$W_1(p) = \frac{1}{(p + 1)(p + 3)}$$

$$W_2(p) = \frac{0,5}{p + 1} - \frac{0,5}{p + 3}$$

o`zaro ekvivalentligi ko`rsatilsin.

2.2.11 Ikki uzatish funksiya

$$W_1(p) = \frac{k}{(p+\alpha)(p+\beta)(p+\gamma)}$$

$$W_2(p) = \frac{a}{(p+\alpha)} + \frac{b}{(p+\beta)} + \frac{c}{(p+\gamma)}$$

o`zaro ekvivalent bo`lishi uchun (a, b, c) koeffitsientlar $W_1(p)$ uzatish funksiya parametrlari bilan qanday bog`langan bo`lishi kerak?

2.2.12 Ob’ektning uzatish funksiyasi quyidagicha

$$W(p) = \frac{1}{1+0{,}2p} + \frac{1}{1+0{,}5p} + \frac{1}{1+p}.$$

Shu ob’ektning a) xarakteristik ko`phadi; b) qutiblari; c) nollari topilsin. Tizim minimal fazalimi?

2.2.13 Avvalgi 2.2.12-mashqdagi ob’ektda qayta aloqa kiritilib yopiq zanjirli tizim hosil qilingan. Uning uzatish funksiyasi

$$W_y(p) = \frac{W(p)}{1+W(p)}.$$

Hosil bo`lgan tizimning a) xarakteristik ko`phadi topilsin va b) qutib bilan nollari baholansin.

2.2.14 Ob’ektning uzatish funksiyasi kasirli ratsional

$$W(p) = \frac{Q(p)}{P(p)}.$$

Bu yerda

$$P(p) = \sum_{i=0}^{n} a_i\, p^i\,,$$

$$Q(p) = \sum_{i=0}^{m} b_i\, p^i$$

algebraik ko`phadlar. Yopiq tizimning uzatish funksiyasining ko`rinishi aniqlansin.

2.2.15 Ob’ekt sof kechikish τ vaqtiga ega, uning uzatish funksiyasi quyidagicha

$$W(p) = \frac{Q(p)e^{-\tau p}}{P(p)}\,,$$

P(p) va *Q(p)* 2.2.14-mashqdagidek. Yopiq tizimning uzatish funksiyasining ko`rinishi aniqlansin.

2.2.16 Yuqorida keltirilgan 2.2.11 – 2.2.15-mashqlardagi uzatish funksiyalar bilan ifodalaniladigan dinamik ob'ektlarning differensial tenglamalari tuzilsin.

2.2.17 Quyidagi jarayonlarning tasvirlari topilsin va shu jarayonlar dinamik bo`g`inning reaksiyasi ko`rinishida tasvirlansin.

$$a)\ y(t) = \delta(t);\quad b)\ y(t) = 1(t);$$

$$\text{c) } y(t) = t \cdot 1(t);\quad \text{d) } y(t) = t^n \cdot 1(t).$$

Jarayonlarning cchizmalari tasvirlansin.

2.3 Chastota tavsiflar

Nazariy kirish

Berilgan funksiya *f(t)* uchun Fur'e almashtirishi deb quyidagi integralga aytiladi

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt\,.$$

Integral ostidagi *f(t)* funksiyaga ***asl*** deyiladi, integralga esa ***Fur'e-tasvir*** deyiladi. Muhandislik amaliyotida *F(jω)* amplituda faza chastota tavsif yoky, qisqaroq qilib, chastota tavsif deb nomlanadi.

Ko`rilayotgan funksiya *f(t)* argumentning manfiy qiymatlari uchun no`lga teng bo`lsa, Fur'e almashtirish Laplas almashtirishda $p = j\omega$ deb qabul qilinganiga mos keladi. Aslni Fur'e tasvir orqali ifodalsh ham mumkin, unga teskari Fur'e almashtirish deyiladi.

Boshlang`ich holati no`lga teng bo`lgan tizim uchun kirish ta'siri *u(t)* va chiqish reaksiyasi *y(t)* berilgan bo`lsa, tizimning amplituda faza chastota tavsifi, ya'ni chastota tavsifi, quyidagicha ta'riflanadi

$$W(j\omega) = \frac{Y(j\omega)}{U(j\omega)}\,.$$

Bu yerda

$$Y(j\omega) \Leftrightarrow y(t),$$

$$U(j\omega) \Leftrightarrow u(t).$$

Chastota tavsif kompleks funksiya bo`lgani uchun u algebraik, eksponensial va trigonometrik shakllarda ifodalanishi mumkin. Algebraik shakli:

$$W(j\omega) = R(\omega) + jI(\omega),$$

R(ω) – haqiqiy chastota tavsif;

I(ω) – mavhum chastota tavsif.

Eksponensial shakl:

$$W(j\omega) = A(\omega)e^{j\varphi(\omega)} .$$

Muhandislik amaliyotida shu shaklning qismlari ko`p qollaniladi. Bular

$A(\omega)$ – amplituda chastota tavsif (AChT),

$\varphi(\omega)$ – faza chastota tavsif (FChT).

Tizimning AChTsi shu tizim turli chastotali garmonik ta’sirning amplitudasini qanchalik kuchaytirishini ko`rsatadi. FChT esa garmonik ta’sirlarni chastotasiga qarab qanday miqdorga vaqt bo`ylab siljishini bildiradi.

Eksponensial shaklda Eyler formulasidan foydalanilsa, chastota tavsifni trigonometrik shakliga kelinadi.

$$W(j\omega) = A(\omega)\sin\varphi(\omega) + jA(\omega)\cos\varphi(\omega) .$$

Bir shakldan boshqasiga o`tish uchun quyidagi munosabatlardan foydalanish mumkin.

$$A(\omega) = \sqrt{R^2(\omega) + I^2(\omega)} ;$$

$$\varphi(\omega) = arctg\frac{I(\omega)}{R(\omega)} ;$$

$$R(\omega) = A(\omega)\sin\varphi(\omega) ;$$

$$I(\omega) = A(\omega)\cos\varphi(\omega) .$$

Kirish ta’sirining chastota tavsifi

$$U(j\omega) = A_{kr}(\omega)e^{j\varphi_{kr}(\omega)},$$

chiqish uchun esa

$$Y(j\omega) = A_{ch}(\omega)e^{j\varphi_{ch}(\omega)}$$

bo`lsa, tizimning AChT va FChTlari onson topiladi:

$$A(\omega) = \frac{A_{ch}(\omega)}{A_{kr}(\omega)},$$

$$\varphi(\omega) = \varphi_{ch}(\omega) - \varphi_{kr}(\omega).$$

Mashqlar

2.3.1 Yuqoridagi 2.2.1-mashqda topilgan uzatish funksiyalarda p ko`mpleks o`zgaruvchi o`rniga $j\omega$ mavhum o`zgaruvchini qo`yib, dinamik bo`g`inlarning chastota tavsiflari topilsin. Chastota tavsiflar algebraik, eksponensial va trigonometrik shakillarda ifodalansin.

2.3.2 Avvalgi 2.3.1-mashqda topilgan chastota tavsiflarning ifodalaridan foydalanib, parametrlarning tanlangan son qiymatlari uchun haqiqiy, mavhum, amplituda va faza chastota tavsiflarning grafik tasvirlari qurilsin. Kompleks sirtda amplituda faza chastota tavsiflarning chizmalari qurilsin.

2.3.3 Uzatish funksiyasi

$$W(p) = \frac{k}{p(1 + Tp)}$$

bo`lgan tizimning chastota tavsiflari qurilsin va tahlil qilinsin.

Ko`rsatma: hisoblarni bjarish uchun parametrlarga son qiymatlar berilishi lozim.

2.3.4 Uzatish funksiyasi

$$W(p) = \frac{k}{p^2(1 + Tp)}$$

bo`lgan tizimning chastota tavsiflari qurilsin va tahlil qilinsin.

Ko`rsatma: hisoblarni bajarish uchun parametrlarga son qiymatlar berilishi lozim.

2.3.5 Uzatish funksiyasi

$$W(p) = \frac{k}{(1 + T_1^2p^2)(1 + T_2p)}$$

bo`lgan tizimning chastota tavsiflari qurilsin va tahlil qilinsin.

Ko`rsatma: hisoblarni bjarish uchun parametrlarga son qiymatlar berilishi lozim.

2.3.6 Uzatish funksiyalari quyidagi ko`rinishga ega bo`lgan tizimlarning chastota tavsilari qurilsin.

$$a)\ W(p) = \frac{k(1+T_1p)}{1+T_2p}\ ;\quad b)\ W(p) = \frac{k(1+T_1p)}{p(1+T_2p)}\ ;$$

$$c)\ W(p) = \frac{k(1+T_1p)}{p^2(1+T_2p)}\ ;\quad d)\ W(p) = \frac{k(1+T_1p)}{(1+T_2p)(1+T_3p)}.$$

Ko`rsatma: hisoblarni bjarish uchun parametrlarga son qiymatlar berilishi lozim.

2.3.7 Uzatish funksiyalari quyidagi ko`rinishga ega bo`lgan tizimlarning chastota tavsilari qurilsin.

$$a)\ W(p) = \frac{ke^{-p\tau}}{1+Tp}\ ;\quad b)\ W(p) = \frac{ke^{-p\tau}}{p(1+Tp)}\ ;$$

$$c)\ W(p) = \frac{k(1+T_1p)e^{-p\tau}}{1+T_2p}\ ;\quad d)\ W(p) = \frac{k(1+T_1p)e^{-p\tau}}{p(1+T_2p)}\ .$$

Ko`rsatma: hisoblarni bjarish uchun parametrlarga son qiymatlar berilishi lozim.

2.3.8 Uzatish funksiyalari quyidagi ko`rinishga ega bo`lgan tizimlarning chastota tavsilari qurilsin.

$$a)\ W(p) = \frac{kp}{1+Tp}\ ;\quad b)\ W(p) = \frac{kpe^{-p\tau}}{(1+Tp)}\ ;$$

$$c)\ W(p) = \frac{kp^2}{(1+T_1p)^2(1+T_2p)}\ ;\quad d)\ W(p) = \frac{kp^2e^{-p\tau}}{(1+T_1p)^2(1+T_2p)}\ .$$

Ko`rsatma: hisoblarni bjarish uchun parametrlarga son qiymatlar berilishi lozim.

2.3.9 Uzatish funksiyasi

$$W(p) = \frac{ke^{-p\tau}}{1+Tp+ke^{-p\tau}}$$

bo`lgan tizimning chastota tavsiflari qurilsin.

Ko`rsatma: hisoblarni bajarish uchun parametrlarga son qiymatlar berilishi lozim.

2.3.10 Yuqoridagi 2.3.2 – 2.3.9-mashqlarda topilgan chastota tavsiflarining haqiqiy qisimlarining chizmalari trapetsiya va uchburchaklar bilan approksimatsiyalansin.

2.3.11 Chastota tavsif

$$W(j\omega) = \frac{\sum_{\nu=0}^{m} b_\nu (j\omega)^\nu}{\sum_{\nu=0}^{n} a_\nu (j\omega)^\nu}$$

Ko`rinishda ifodalangan tizimning haqiqiy, mavhum, amplituda va faza chastota tavsiflarining ifodalari aniqlansin. Bu yerda a_ν, b_ν - haqiqiy sonlar, n, m – butun sonlar va $n \geq m$.

2.4 Logarifmik chastota tavsiflar

Nazariy kirish

Logarifmik chastota tavsiflar chastota tavsifni logarifmik masshtabli koordinata tizimlarda ifodalash bilan bog`liq. Muhandislikda ko`proq o`nli asos bo`yicha logarifmlash qo`llaniladi. Logarifmik amplituda chastota tavsif (LAChT) quyidagicha aniqlaniladi:

$$L(\omega) = 20lgA(\omega).$$

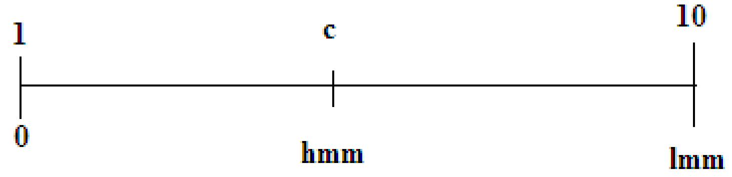
Bu holda o`lcham birligi detsibellda [db] bo`ladi.

Logarifmik faza chastota tavsif (LFChT) esa, faza chastota tavsif $\varphi(\omega)$ uchun chastota o`qi logarifmik masshtabda ifodalangan holda tasvirlanadi. O`lcham birligi o`zgarmaydi va gradus yoki radian bo`lib qolaveradi.

Logarifmik masshtabga o`tish ma’lum qulayliklardan tashqari, xususan, tasvirlashga onson bo`lgan asimptotik LAChTlardan foydalanishga yo`l ochadi. Undan tashqari, ketma-ket ulangan dinamik bo`g`inlarning LChTlari o`zaro qo`shiladi. Bunday amalni grafik usilda bajarish ancha qulay.

Logarifmik shkalani qurish tartibi quyidagicha. O`n asosli logarifmik shkala uchun bir dekadaga, masalan [1, 10] oraliqqa, *l* mm uzunlikdagi kesma mos qo`yiladi (2.4.A-rasm).

a)



2.4.A-rasm. Logarifmik masshtabni qurish.

Unda bir bilan o`n orasidagi ixtiyoriy “$c$” sonni shkalada joylashtirish o`rni kesmaning boshidan (ya’ni “0” raqamdan) h mm masofadagi nuqtada bo`ladi. Bu masofani aniqlash uchin quyidagi munosabatdan foydalanish lozim:

$$h = l \cdot lgc.$$

Boshqa dekadalar uchun shkalani shakli xuddi shunday bo`ladi, sonlari esa o`n sonining butun darajalari bilan farqlanadi.

Mashqlar

2.4.1 Quyidagi uzatsh funksiyalarga ega bo`lgan tizimlarning logarifmik chastota tavsiflari qurilsin.

$a)$ $$W(p) = \frac{k}{p(1+Tp)}\ ;$$

$b)$ $$W(p) = \frac{k(1+T_1p)}{1+T_2p}\ ;$$

$c)$ $$W(p) = \frac{k(1+T_1p)}{p(1+T_2p)}\ ;$$

$d)$ $$W(p) = \frac{k(1+T_1p)}{(1+T_2p)(1+T_3p)}\ ;$$

$e)$ $$W(p) = \frac{k(1+T_1p)}{p^2(1+T_2p)}\ ;$$

$f)$ $$W(p) = \frac{k(1+T_1p)}{p(1+T_2p)(1+T_3p)}\ ;$$

$g)$ $$W(p) = \frac{k(1+T_1p)(1+T_2p)}{(1+T_3p)}\ ;$$

$h)$ $$W(p) = \frac{k(1+T_1p)(1+T_2p)}{(1+T_3p)(1+T_4p)}\ ;$$

$i)$ $$W(p) = \frac{k}{(1+T_1p)(1+2\xi T_2p+T_2^2p^2)}\ ;$$

$j)$ $$W(p) = \frac{k(1+T_1p)}{(1+T_2p)(1+2\xi T_3p+T_3^2p^2)}\ ;$$

$k)$ $$W(p) = \frac{k(1+T_1p)}{p(1+T_2p)(1+2\xi T_3p+T_3^2p^2)}\ ;$$

$l)$ $$W(p) = \frac{k(1+2\xi_1T_1p+T_1^2p^2)}{(1+T_2p)(1+2\xi_3T_3p+T_3^2p^2)}\ ;$$

$m)$
$$W(p) = \frac{k(1+T_1p)(1+T_2p)}{p(1+T_3p)(1+2\xi_4T_4p+T_4^2p^2)}.$$

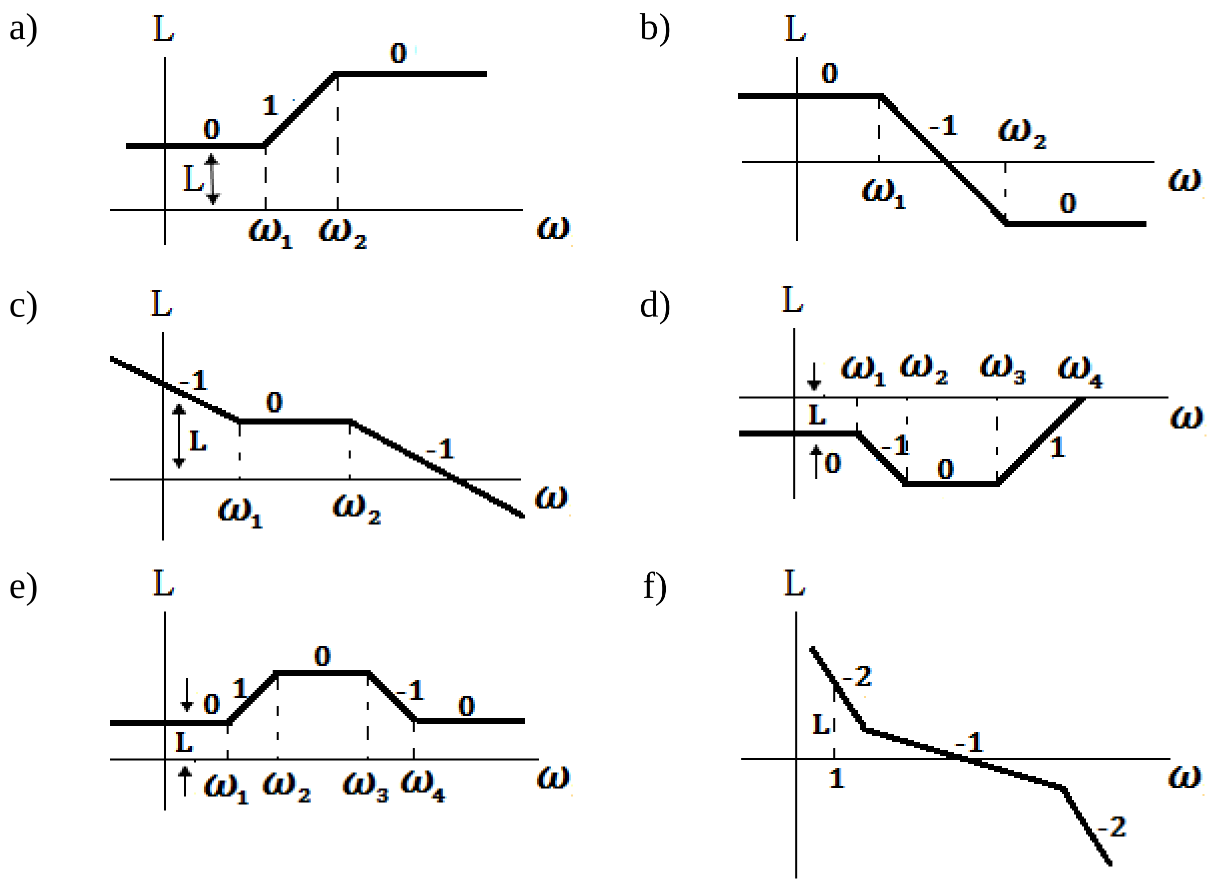
Ko`rsatma: hisoblarni bjarish uchun parametrlarga son qiymatlar berilishi lozim.

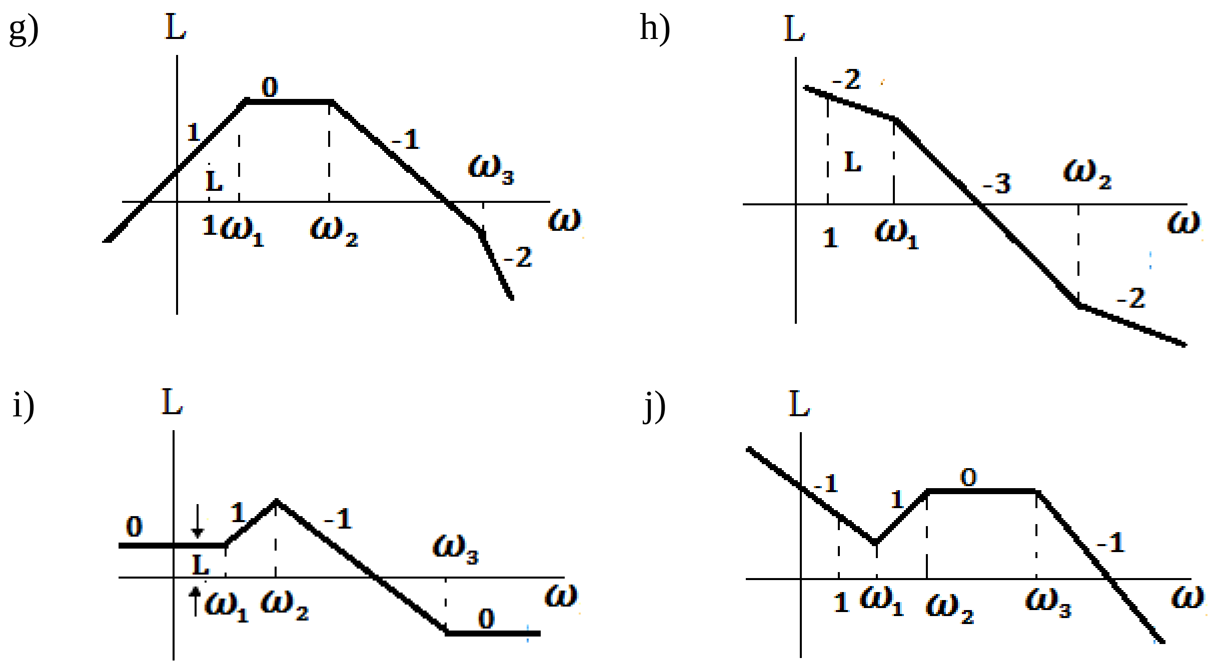
2.4.2 Avvalgi 2.4.1-mashqda keltirilgan uzatish funksiyalar tarkibiga $e^{-p\tau}$ ko`paytuvchi kiritib, natijaviy logarifmik chastota tavsiflar qurilsin.

2.4.3 Yuqoridagi 2.4.1 va 2.4.2-mashqlarda uzilgan tizimning tavsifi berilgan deb qabul qilgan holda, logarifmik chastota tavsiflardan, Nikols diagrammalari yordamida, yopiq tizimning haqiqiy va mavhum chastota tavsiflari qurilsin.

2.4.4 Berilgan (2.1-rasm) LAChTlarga mos keladigan turg`un va minimal fazali uzatish funksiyalarning ifodalari keltirilsin.

Ilova: chizmalarda 0,1,2,… sonlar, mos ravishda, 0, 20, 40,… db og`ishga to`g`ri keladi.





2.1-rasm. Logarifmik amplituda chastota tavsiflar

2.5 Chiziqli statsionar tizimlarning tuzilmalari

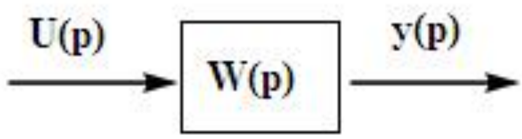
Nazariy kirish

Chiziqli statsionar uzliksiz dinamik bo`g`in yoki tizim 2.5.A-rasmdagidek tasvirlanadi. Bu yerda

$W(p)$ – tizim yoki bo`g`inning uzatish funksiyasi;

Y(p) – chiqishni tasviri;

U(p) – kirishni tasviri.



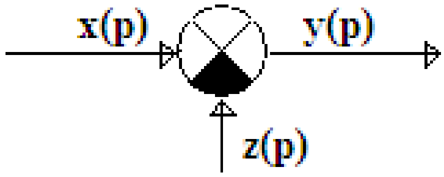
2.5.A-rasm. Dinamik tizim yoki bo`g`nning tuzilma tasviri.

Bunday tuzilma

$$Y(p) = W(p)U(p)$$

bog`lanishni bildiradi.

Alohida dinamik bo`g`inlar bir-biri bilan turlicha ulanib har xil bog`lanishlarni hosil qilishi mumkin. Chiziqli tizimlarning bog`lanishlarini hosil qilish uchun yana ikkita tuzilmaviy unsir qo`llanadi. Bular tugun (2.5.B-rasm) va summatordir (2.5.C-rasm).



2.5.B-rasm. Tugun.

2.5.C-rasm. Summator.

Tugundan oldin ham, keyin ham bir xil o`zgaruvchi. Summatorning kirishlarining soni ikkitadan kam bo`lmaydi, chiqishi esa doim bitta bo`ladi. Doiradagi bo`yalgan chorak ayrish amalini bildiradi.

Ketma-ket ulangan bo`g`inlarning umumiy uzatish funksiyasi alohida uzatish funksiyalarning ko`paytmasiga teng

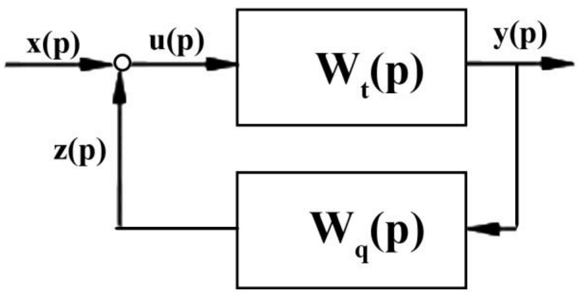
$$W_{kk}(p) = \prod_{i=1}^{n} W_i(p).$$

Parallel ulangan bo`g`inlarning umumiy uzatish funksiyasi alohida uzatish funksiylarning yig`indisiga teng

$$W_{pl}(p) = \sum_{i=1}^{n} W_i(p).$$

Teskari parallel yoki qayta aloqali (2.5.D-rasm) ulangan bo`g`inlarning natijaviy uzatish funksiyasi to`g`ri yoldagi uzatish funksiyani bir plyus (manfiy qayta aloqa) yoki minus (musbat qayta aloqa) uzilgan tizimning uzatish funksiyasiga bo`linganiga teng

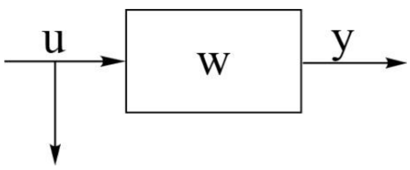
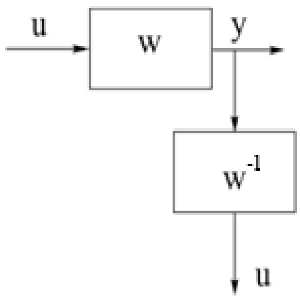
$$W_{qa}(p) = \frac{W_t(p)}{1 \pm W_t(p) W_q(p)}.$$

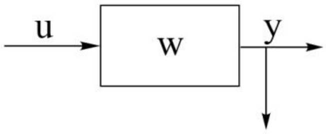
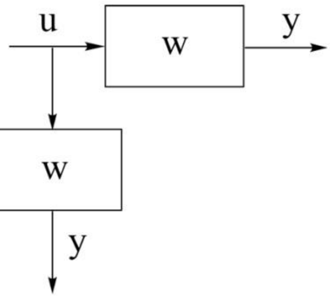
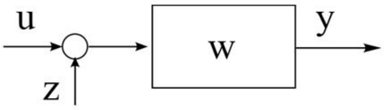
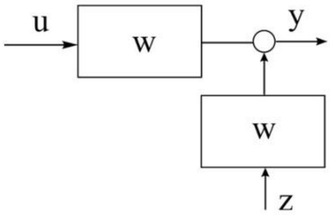
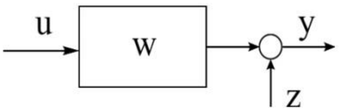
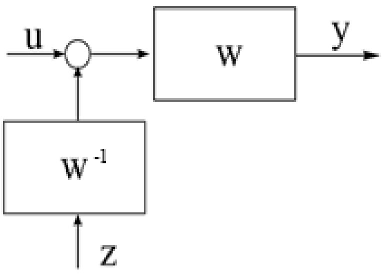
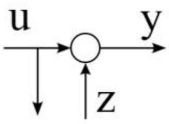
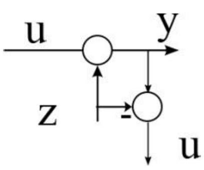


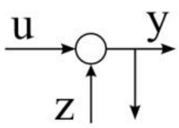
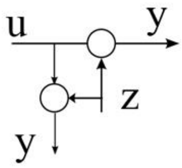
2.5.D-rasm. Qayta aloqali ulash.

Tugun va summatorlarning o`rnini ekvivalent kochirish qoidalari 2.5.A-jadvalda keltirilgan.

2.5.A-jadval. Tuzilmani ekvivalent almashtirish qoidalari.

t/r	Dastlabki tuzilma	Ekvivalent tuzilma
1		

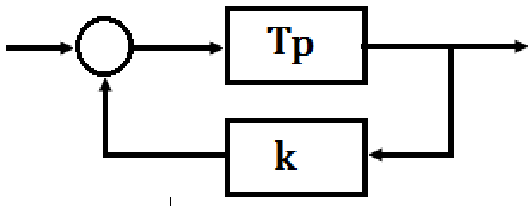
2		
3		
4		
5		

6		

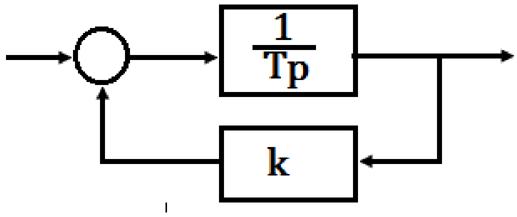
Mashqlar

2.5.1 Chizmalarda keltirilgan (2.2-rasm) tuzilmali tizimlarning uzatish funksiyalari aniqlansin.

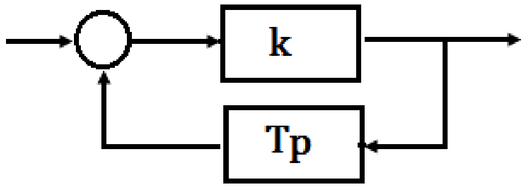
a)



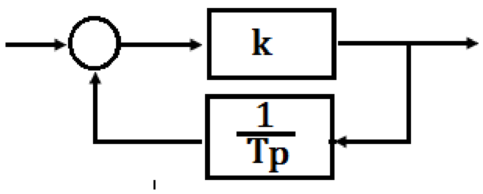
b)



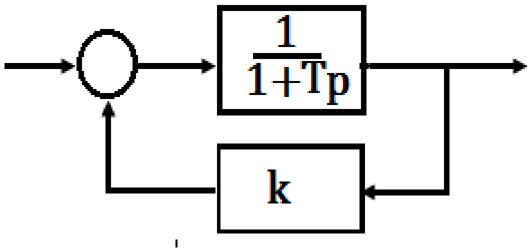
c)



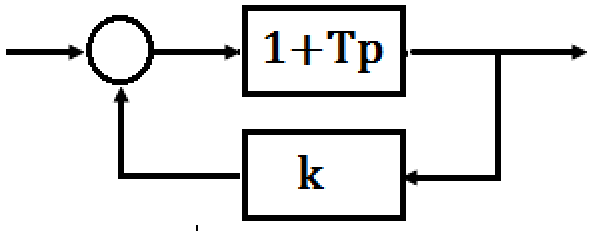
d)



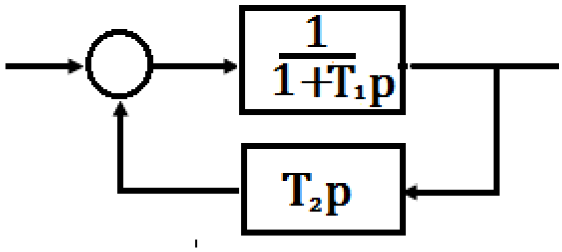
e)



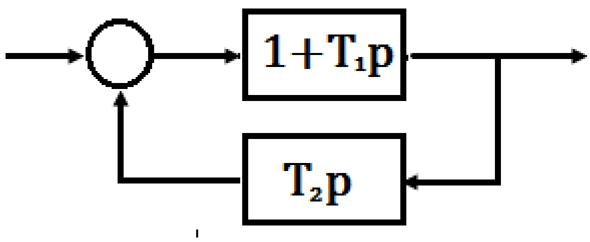
f)

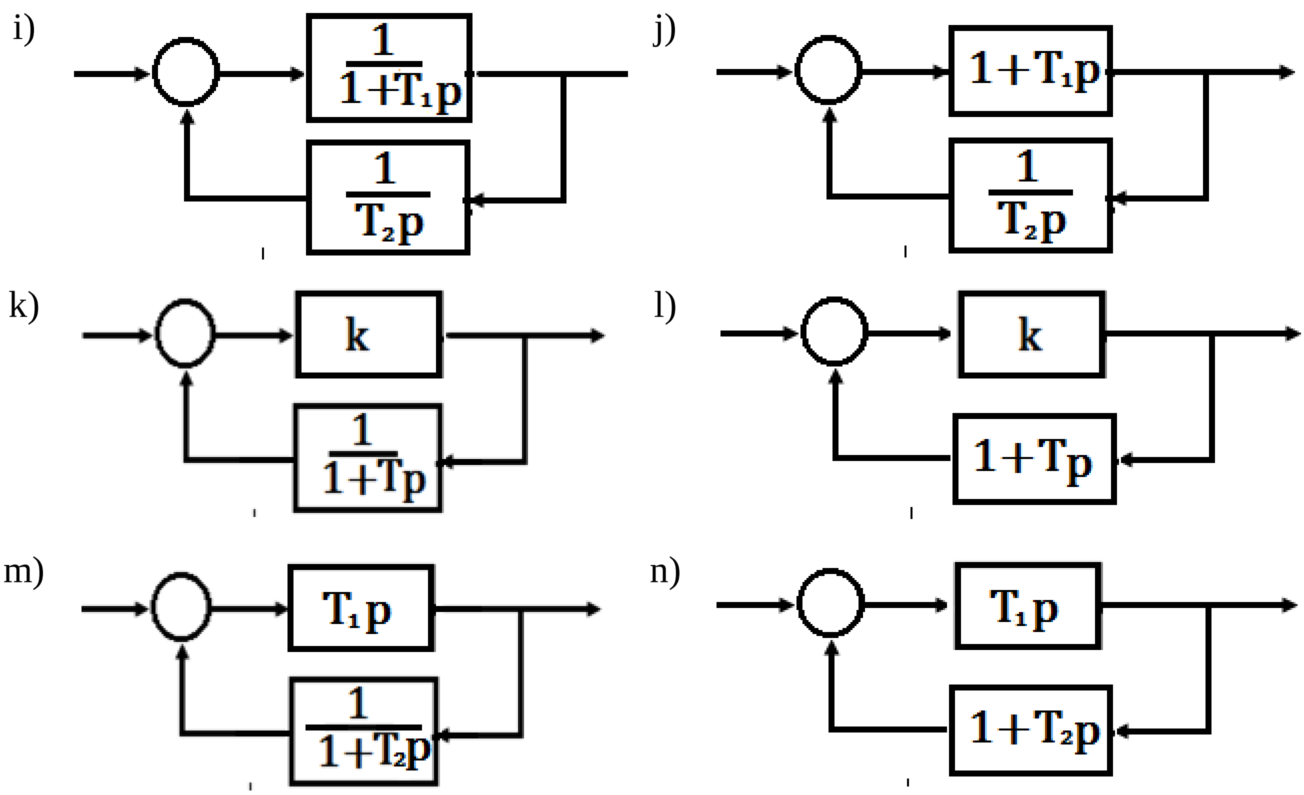


g)

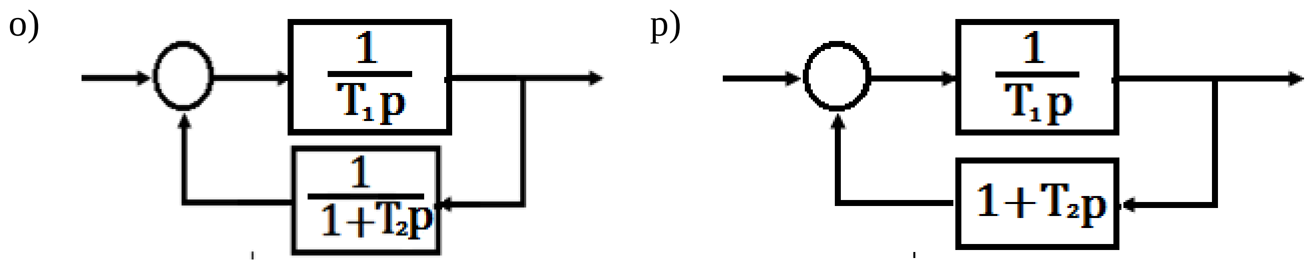


h)





2.2-rasm. Tuzilma chizmalar



2.2-rasm. Tuzilma chizmalar (davomi)

2.5.2 Integrallovchi va kuchaytiruvchi bo`g`inlardan foydalanib, nodavriy bo`g`inni hosil qiladigan tuzilmani taklif qiling.

2.5.3 Tenglamasi

$$\sum_{i=0}^{n} a_i y^{(i)} = u(t)$$

bo`lgan dinamik tizimning vazn funksiyasi *g(t)* bo`lsin. Tenglamasi

$$\sum_{i=0}^{n} a_i y^{(i)} = \sum_{i=0}^{m} b_i u^{(i)}$$

bo`lgan tizimning vazn funksiyasi topilsin.

Ko`rsatma: dinamik bo`g`inlarni ketma-ket ulash tuzilmasidan foydalanish mumkin.

2.5.4 Nodavriy bo`g`inning tenglamasi operator ko`rinishda berilgan:

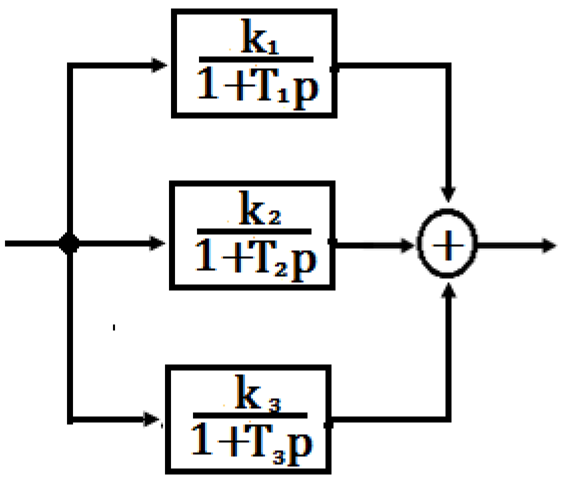
$$(1+Tp)Y(p)=kU(p).$$

Uning ichki tuzilmasi mumkin bo`lgan turli chizmalar ko`rinishida aks ettirilsin.

2.5.5 Yuqoridagi 2.5.4-mashqning vazifasi tebranuvchi bo`g`in uchin bajarilsin

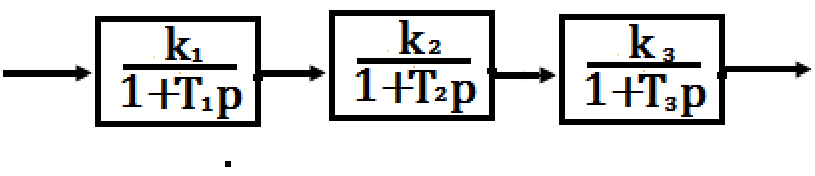
$$(1+2\xi Tp+T^2p^2)Y(p)=kU(p).$$

2.5.6 Parallel ulangan bo`g`inlar bilan tasvirlangan tuzilma (2.3-rasm) ekvivalent ketma-ket ulangan tuzilmaga almashtirilsin.



2.3-rasm. Parallel ulangan bo`g`inlar

2.5.7 Ketma-ket ulangan bo`g`inlar bilan tasvirlangan tuzilma (2.4-rasm) sodda bo`g`inlar parallel ulangan ekvivalent tuzilmaga almashtirilsin.



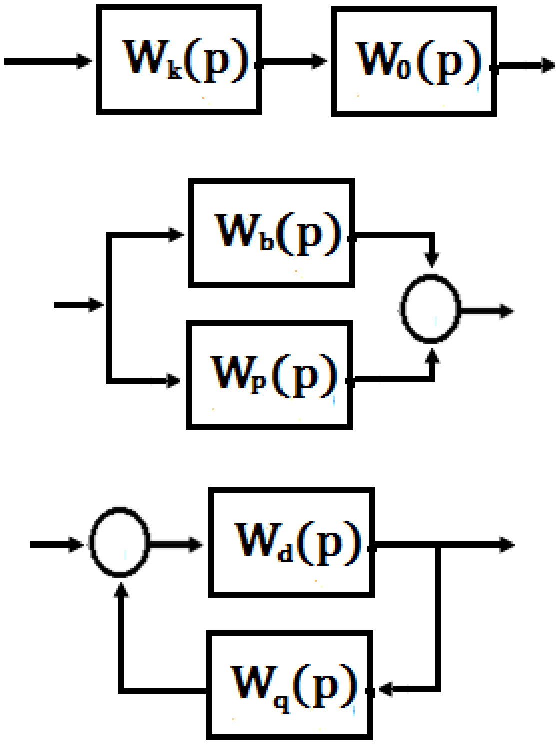
2.4-rasm. Ketma-ket ulangan bo`g`inlar

2.5.8 Ketma-ket, parallel va teskari parallel (qayta aloqali) tuzilmalar 2.5-rasmda keltirilgan. Bu yerda $W_0(p)$ – o`zgarmas qism deb qabul qilgan holda, har bir tuzilmadan boshqa ikki tuzilmaga ekvivalent o`tilsin.

a)

b)

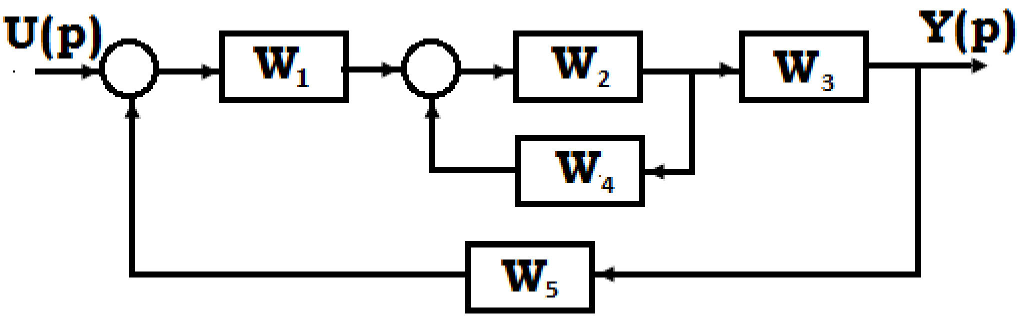
c)



2.5-rasm. Bo`g`inlarni ulash chizmalari

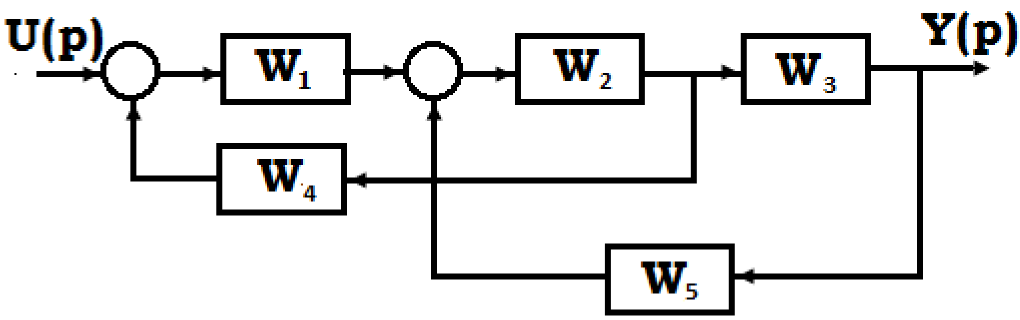
2.5.9 Keltirilgan tuzilmalar (2.6-rasm) almashtirish qoidalarini qo`llab soddalashtirilsin, chiqish bilan kirish o`rtasidagi uzatish funksiya aniqlansin.

a)

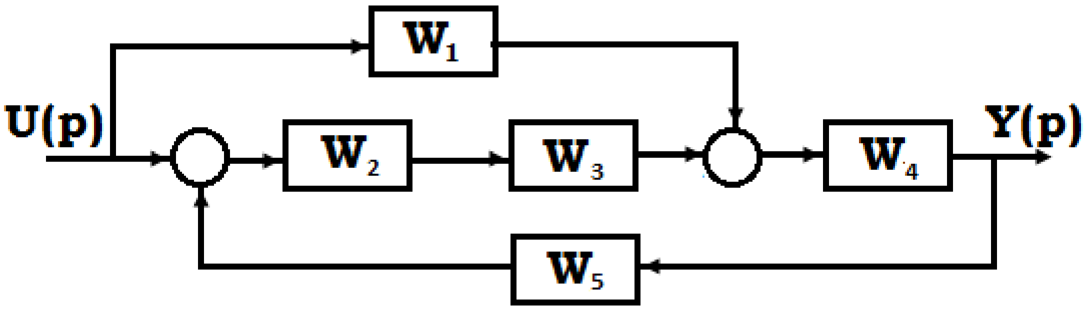


2.6-rasm. Tizimlarning turli tuzilmalari

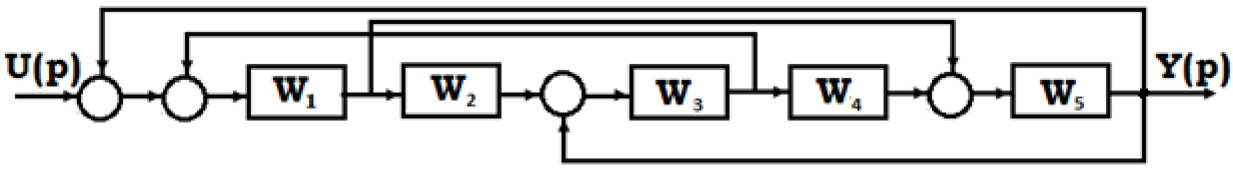
b)



c)



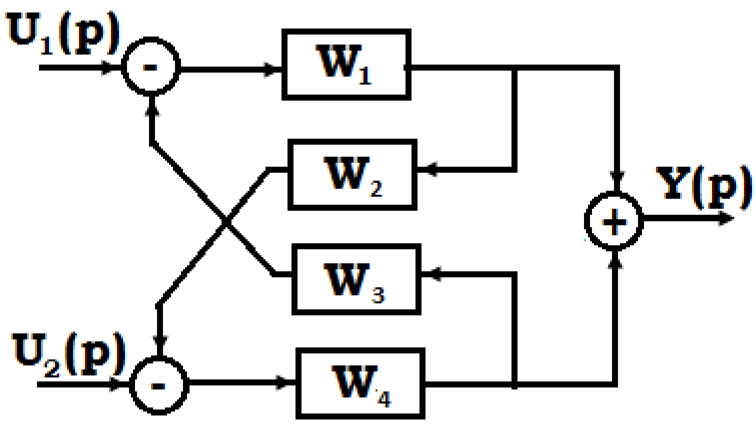
d)



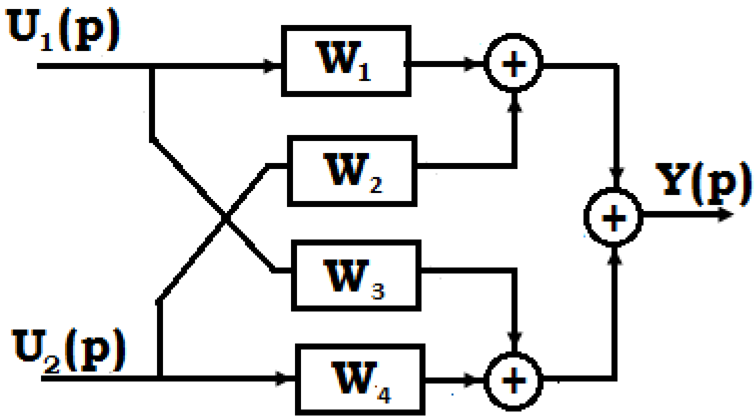
2.6-rasm. Tizimlarning turli tuzilmalari (davomi)

2.5.10 Keltirilgan tuzilmalar (2.7-rasm) almashtirish qoidalarini qo`llab soddalashtirilsin, chiqish bilan kirishlar o`rtasidagi uzatish funksiyalar aniqlansin.

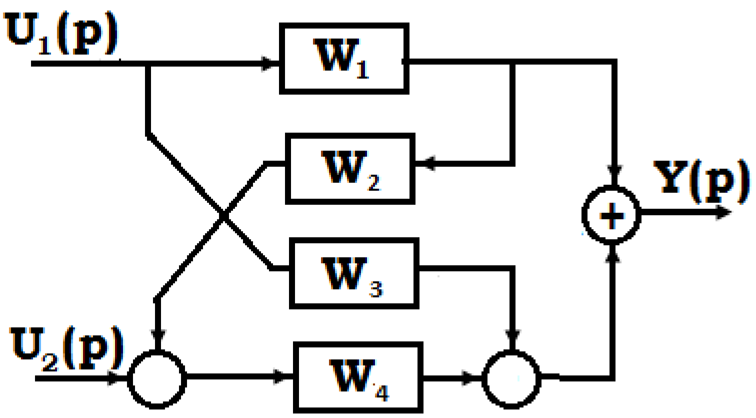
a)



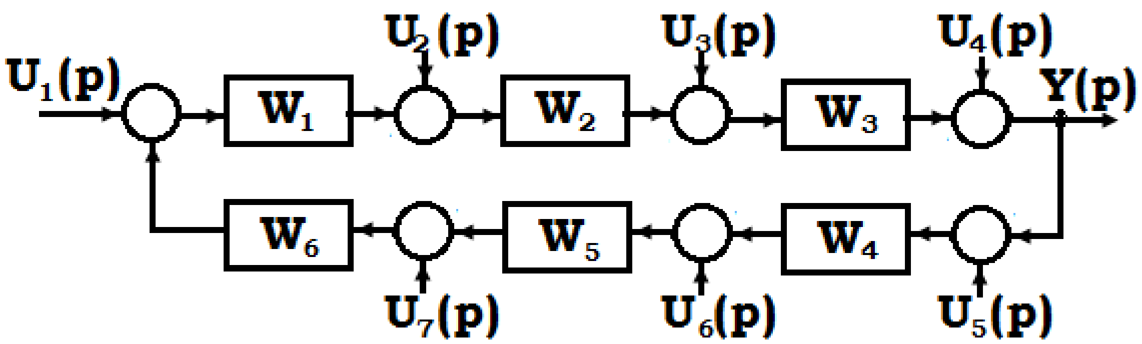
b)



c)



d)



2.7-rasm. Ko`p o`lchamli tizimlarning tuzilmalari

2.6 Turg`unlikning algebraik qoidalari

Nazariy kirish

Xarakteristik ko`phad berilgan bo`lsin

$$G(p) = a_0p^n + a_1p^{n-1} + a_2p^{n-2} + \cdots + a_{n-1}p + a_n \,.$$

Faraz qilamiz $a_0 > 0$.

Gurvits qoidasini qo`llash uchun dastlab Gurvits matritsasi qurilishi lozim. Uni quyidagi tartibda qurish mumkin.

1. Matritsaning bosh diagonal xo`jayralariga ko`phadning koeffitsientlari birinchi indekslidan boshlab joylashtiriladi.

2. Toq satrlarga toq indeksli, juft satrlarga juft indeksli koeffitsientlar joylashtiriladi. Bunda diagonal unsirdan chap tomonga indekslar kamayib boradi, o`ng tomonga indekslar ortib boradi.

3. O`lchami $n \times n$ bo`lgan matritsaning bo`sh qolgan xo`jayralari no`llar bilan to`ldiriladi.

$$\Gamma = \begin{pmatrix} a_1 & a_3 & a_5 & \cdots & 0 \\ a_0 & a_2 & a_4 & \cdots & 0 \\ 0 & a_1 & a_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & a_n \end{pmatrix}.$$

Hosil bo`lgan matritsani bosh diagonal minorlari musbat bo`lsa dinamik tizim turg`un. Shunday qilib:

$a_0 > 0;$

$\Delta_1 = a_1 > 0;$

$\Delta_2 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix} > 0;$

$\Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} > 0;$

… … … … … … … … … ….

$\Delta_n = |\Gamma| > 0.$

tengsizliklar qanoatlanishi chiziqli statsionar uzliksiz tizim turg`unligining zarur va yetarli shartidir.

Gurvits matritsasini qurish boshqacha tartibda ham bajarilishi mumkin.

Raus qoidasini qollash uchun Raus jadvali quriladi. Uning unsirlari c_{ik} quyidagi tartibda aniqlanadi.

1. Birinchi satrga ko`phadning juft indeksli koeffitsientlari joylashtiriladi:

$$c_{1k} = a_{2(k-1)}; \qquad k = 1, 2, 3, \cdots.$$

2. Ikkinchi satrga ko`phadning toq indeksli koeffitsientlari joylashtiriladi:

$$c_{2k} = a_{2k-1}; \qquad k = 1, 2, 3, \cdots.$$

3. Uchinchi va undan keyingi satrlar unsirlari quyidagi ifodadan hisoblab topiladi:

$$c_{ik} = c_{i-2,k+1} - r_i c_{i-1,k+1}; \qquad i = \overline{3, n+1}.$$

Bu ifodadagi koeffitsient har satr uchun quyidagicha hisoblanadi:

$$r_i = \frac{c_{i-2,1}}{c_{i-1,1}}; \qquad i = \overline{3, n+1}.$$

Qoida. Chiziqli statsionar uzliksiz tizim turg`un bo`lishi uchun Raus jadvalining birinchi ustun unsirlari bir xil ishorali bo`lishi zarur va yetarlidir.

Mashqlar

2.6.1 Xarakteristik ko`phadi berilgan tizimning turg`unligi Gurvits qoidasi asosida tekshirilsin.

a) $0,1p^3+0,2p^2+05p+1=0$;
b) $0,1p^3+17p^2+800p+50000=0$;
c) $5p^3+p^2+10p+1=0$;
d) $p^4+2p^3+p^2+10p+20=0$;
e) $p^4+3p^3+10p^2+2p+5=0$;
f) $p^3+2p^2+3p+1=0$.

2.6.2 Xarakteristik ko`phadi berilgan tizimning turg`unligi Raus qoidasi asosida tekshirilsin.

a) $0,1p^5+0,2p^4+0,5p^3+p^2+2p+1=0$;
b) $0,1p^5+17p^4+80p^3+300p^2+500p+5000=0$;
c) $5p^6+10p^5+20p^4+50p^3+100p^2+200p+500=0$;
d) $p^6+2p^5+8p^4+24p^3+40p^2+70p+120=0$;
e) $0,104p^7+0,33p^6+5,5p^5+15,5p^4+25p^3+25p^2+19,7p+9,5=0$;
f) $p^7+2p^6+10p^5+10p^4+26p^3+40p^2+70p+100=0$.

2.6.3 Turg`unlinking algebraik qoidalaridan foydalanib, uzilgan tizimning berilgan uzatish funksiyasidan yopiq tizimning turg`unligi tekshirilsin.

$$a)\ W(p)=\frac{10p+1}{p(3p^2+2p+1)};$$

$$b)\ W(p)=\frac{10p+1}{p^2(5p+1)};$$

$$c)\ W(p)=\frac{5p+20}{p^3+2p^2+3p+1};$$

$$d)\ W(p)=\frac{10}{(0{,}2p+1)(0{,}5p+1)(3p-1)}\ ;$$

2.6.4 To`sqich bilan yuritmaning uzatish funksiyasi

$$W_y(p)=\frac{k_y}{p(1+T_1p+T_2^2p^2)}\,.$$

Sath rostlanishi kerak bo`lsa, sof kechikish hisobga olinmagan holda, b'efning uzatish funksiyasi

$$W_b(p)=\frac{k_b}{p(1+T_3p)}$$

ko`rinishida qabul qilinishi mumkin. Bularning ketma-ket ulanishi uzilgan tizimni hosil qiladi. Shu uzilgan tizimdan hosil qilingan yopiq tizim tuzilmaviy noturginligi algebraik qoida asosida ko`rsatilsin.

Eslatma: tizim tuzilmaviy noturgin deyiladi, agar uning parametrlarining hech qaysi qiymatlari uchun tizim turg`un bolmasa.

2.7 Mixaylovning turg`unlik qoidasi

Nazariy kirish

Xarakteristik ko`phadda $p = j\omega$ deb qabul qilingan holini Mixaylov chastota tavsifi deb yuritamiz.

$$G(j\omega) = a_0(j\omega)^n + a_1(j\omega)^{n-1} + \cdots + a_{n=1}(j\omega) + a_n.$$

Bu kompleks ifodani algebraik shaklda yozish mumkin:

$$G(j\omega) = \theta(\omega) + j\eta(\omega),$$

bu yerda

$\theta(\omega)$ − Mixaylovning haqiqiy chastota tavsifi;

$\eta(\omega)$ − Mixaylovning mavhum chastota tavsifi.

Argumentning orttirmasi tamoiliga asosan

$$\Delta argG(j\omega) \uparrow_0^\infty = \frac{\pi}{2}(n - 2m).$$

Bu yerda m – xarakteristik ko`phadning o`ng ildizlar soni. Agar tizim turg`un bo`lsa $m = 0$ va yuqoridagi munosabat Mixaylovning turg`unlik qoidasining analitik ifodasini beradi:

$$\Delta argG(j\omega) \uparrow_0^\infty = \frac{\pi}{2} n\ .$$

Chastota no`ldan cheksizgacha o`zgarganida *G(jω)* funksiya kompleks sirtda qandaydir egri chiziqni hosil qiladi. Unga Mixaylov godografi deyiladi.

1-Mixaylov qoidasi. Chiziqli statsionar uzliksiz tizim turg`un bolishi uchun chastota no`ldan cheksizgacha o`zgarganida Mixaylov godografi haqiqiy o`qning musbat qiymatidan boshlanib musbat yo`nalishda, tartibni buzmasdan va koordinata boshini kesib o`tmasdan, ketma-ket *n* dona kvadrantni bosib o`tishi zarur va yetarlidir.

2-Mixaylov qoidasi. Chiziqli statsionar uzliksiz tizim turg`un bolishi uchun Mixaylovning haqiqiy chastota tavsifi musbat qiymatdan, mavhum chastota tavsifi esa no`ldan boshlanib, chastota no`ldan cheksizgacha o`zgarganida ularning no`llari galma-gal uchrashi zarur va yetarlidir.

Qurilgan Mixaylov godografidan noturg`un tizimning o`ng no`llarining soni aniqlanishi mumkin. Masalan, chizmadan topilgan argument orttirmasi $\frac{\pi}{2}k$ bo`lsin, bu yerda $k \leq n$. Unda, argument orttirmasi tamoilidan quyidagi munosabat kelib chiqadi:

$$m = \frac{n-k}{2}.$$

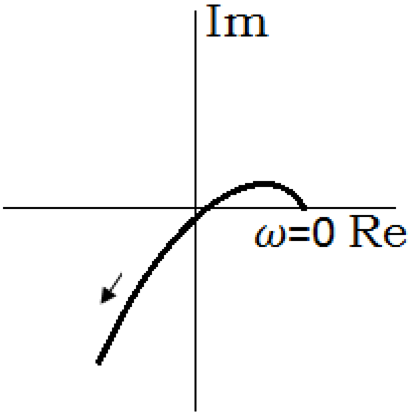
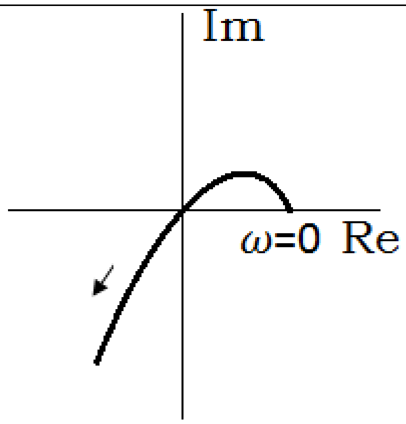
Mashqlar

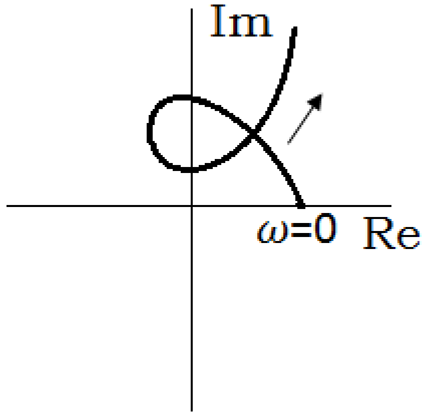
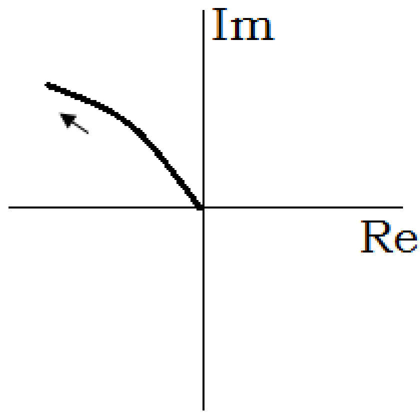
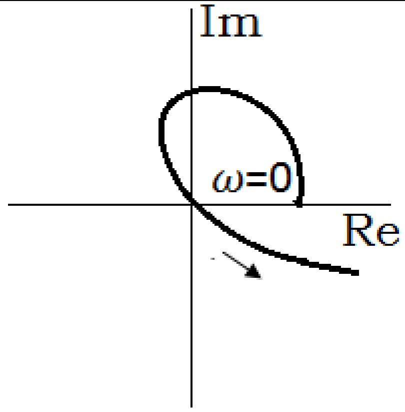
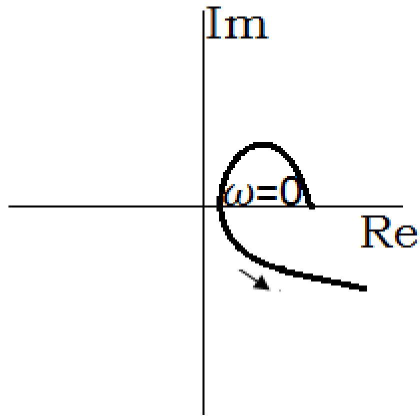
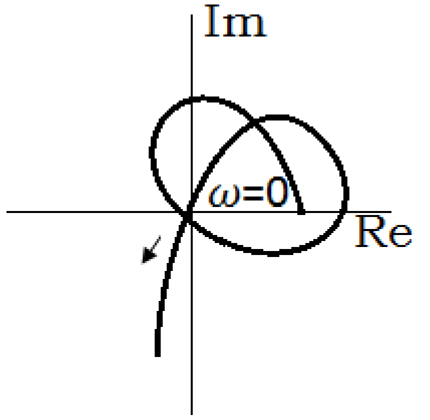
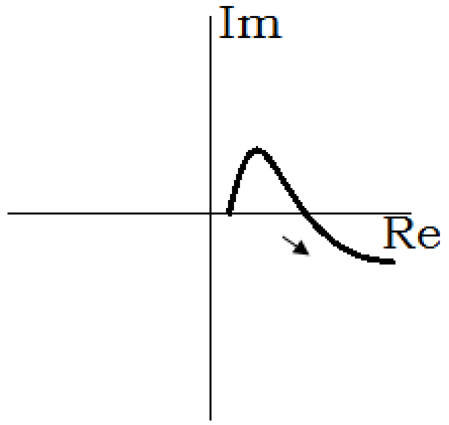
2.7.1 Yuqoridagi 2.6.1, 2.6.2-mashqlarda keltirilgan xarakteristik ko`phadlar uchun Mixaylov godograflari qurilsin, o`ng va so`l ildizlarning soni aniqlansin.

2.7.2 Yuqoridagi 2.6.1, 2.6.2-mashqlarda keltirilgan ko`phadlarda kompleks o`zgaruvchini mavhum $j\omega$ o`zgaruvchiga almashtirib, Mixaylovning haqiqiy va mavhum chastota tavsiflaridan tizimning turg`unligi tekshirilsin.

2.7.3 Mixaylovning berilgan godograflaridan tizimning tartibi, o`ng va so`l qutiblar soni aniqlansin (2.1-jadval).

2.1-jadval. Mixaylov godograflari

t/r	Godograf	t/r	Godograf
1		2	

3		4	
5		6	
7		8	

2.7.4 Berilgan kasrli ratsional uzatish funksiyaning

$$W(p) = \frac{\sum_{\nu=0}^{m} b_\nu p^\nu}{\sum_{\nu=0}^{n} a_\nu p^\nu}$$

Mixaylov godografidan foydalanib ob’ekt minimal fazaligini tekshirish yo`li taklif etilsin.

2.8 Naykvistning turg`unlik qoidasi

Nazariy kirish

Yopiq tizimning uzatish funksiyasining mahraji ko`riladi:

$$G(j\omega) = 1 + W(j\omega).$$

Bu yerda $W(j\omega) -$ uzilgan tizimning chastota tavsifi.

Naykvist qoidasining analitik ifodasi:

$$\Delta \arg[1 + W(j\omega)] \uparrow_0^\infty = \pi m.$$

Bu yerda $m -$ uzilgan tizimning o`ng qutiblar soni.

Naykvist qoidasi. Chiziqli statsionar uzliksiz yopiq tizim turg`un bo`lishi uchun, chastota no`ldan cheksizgacha o`zgarganida, Naykvist godografi *(-1, j0)* nuqtani musbat yo`nalishda (soat mili harakatiga teskari) $m/2$ marotaba qamrab olishi zarur va yetarlidir. Aks holda yopiq tizim noturg`un.

Naykvist qoidasining xususiy holi. Uzilgan tizim turg`un bo`lsa (*m=0*), chiziqli statsionar uzliksiz yopiq tizim turg`un bo`lishi uchun, chastota no`ldan cheksizgacha o`zgarganida, Naykvist godografi *(-1, j0)* nuqtani qamrab olmasligi kerak.

Uzilgan tizimning chastota tavsifi quyidagi ko`rinishda yozilishi mumkin:

$$W(j\omega) = k\overline{W}(j\omega).$$

Bu yerda

k $-$ uzilgan tizimning statik kuchaytirtish koeffitsienti;

$\overline{W}(j\omega) -$ uzilgan tizimning mo`tadillashtirilgan chastota tavsifi.

Bu hol uchun **Naykvist qoidasi** quyidagicha ifodalanadi.

Chiziqli statsionar uzliksiz yopiq tizim turg`un bo`lishi uchun, chastota no`ldan cheksizgacha o`zgarganida, mo`tadillashtirilgan Naykvist godografi $(-1/k\ , j0)$ nuqtani musbat yo`nalishda $m/2$ marotaba qamrab olishi zarur va yetarlidir. Aks holda yopiq tizim noturg`un.

Agar uzilgan tizimning statik kuchaytirish koeffitsienti $[r_0, k_0]$ oraliqqa tegishli bo`lsa, ya’ni

$$k \in [r_0, k_0]\ ,$$

oraliq turg`unlik qoidasidan foydalanish lozim. U quyidagicha ifodalanadi.

Chiziqli statsionar uzliksiz yopiq tizim turg`un bo`lishi uchun, chastota no`ldan cheksizgacha o`zgarganida, mo`tadillashtirilgan Naykvist godografi haqiqiy o`qning $[-\frac{1}{r_0}, -\frac{1}{k_0}]$ kesmasini, hech qayerda kesib o`tmasdan, musbat yo`nalishda $m/2$ marotaba qamrab olishi zarur va yetarlidir.

Naykvist godografi haqiqiy o`qning $(-\infty, -1]$ oraligini kesib o`tishiga **o`tish** deyiladi. Yuqoridan pastga o`tish musbat deyiladi, pastdan yuqoriga o`tish manfiy deyiladi. Musbat o`tishlar soni μ_+ , manfiy o`tishlar soni $\mu_-$ bo`lsin. Unda quyidagi qoida o`rinli.

Chiziqli statsionar uzliksiz yopiq tizim turg`un bo`lishi uchun Naykvist godografining musbat o`tishlari bilan manfiy o`tishlari ayirmasi $m/2$ ga teng bo`lishi kerak, ya'ni

$$\mu_+ - \mu_- = \frac{m}{2}.$$

Agar mo`tadillashtirilgan Naykvist godografidan foydalanilsa, o`tishlar $(-\infty, -1/k]$ oraliqda sanaladi.

Keying qoidalarni uzilgan tizim turg`un bo`lgan hol uchun onsonlik bilan ifodalash mumkin.

Ko`rinib turibdiki, yopiq tizimning turg`unligini Naykvist qoidasi asosida tekshirish uchun uzilgan tizimning chastota tavsifi va uning o`ng qutiblar soni ma'lum bo`lishi kerak.

Mashqlar

2.8.1 Yuqoridagi 2.6.3-mashqda keltirilgan uzilgan tizimning uzatish funksiyalaridan foydalanib, Naykvist qoidasi asosida yopiq tizimning turg`unligi tekshirilsin.

2.8.2 Uzilgan tizimning amplituda faza chastota tavsifining godografi, mavhum o`qda va kompleks sirtning o`ng yarmida joylashgan qutiblar sonlari, hamda tizimning astatizm darajasi berilgan. Naykvist qoidasi yordamida yopiq tizimning turg`unligi tekshirilsin. Variantlar 2.2-jadvalda keltirilgan. U yerda: $v$ – astatizm darajasi; $m$ – o`ng qutiblar soni; i – mavxum o`qdagi qutiblar soni [2].

2.2-jadval. Naykvist godograflari

v	m	i	Variant	Chastota tavsif	Variant	Chastota tavsif
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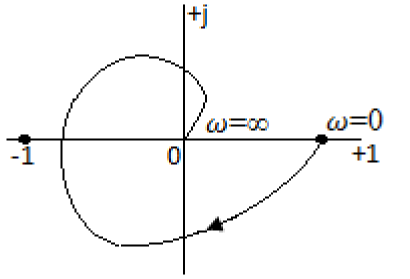
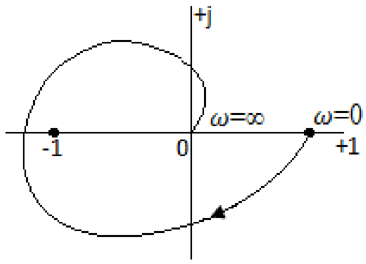
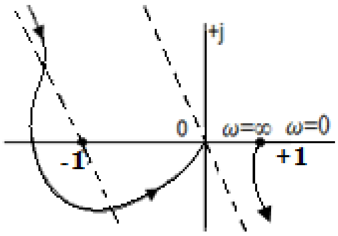
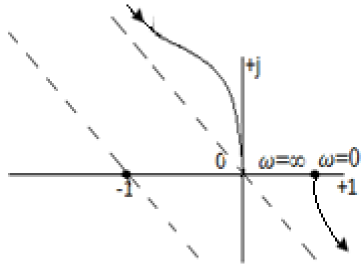
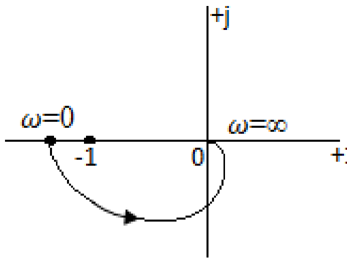
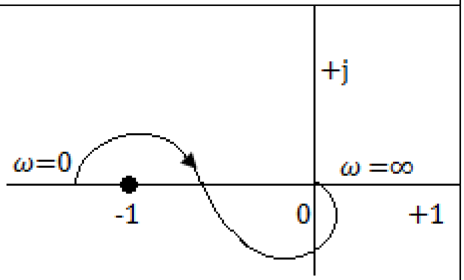
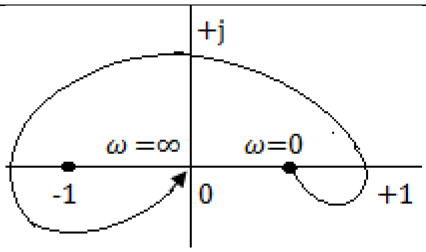
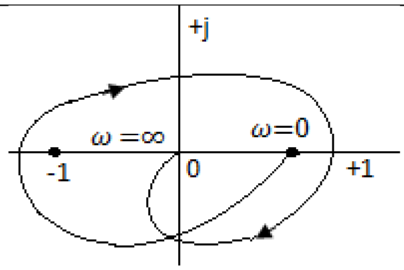
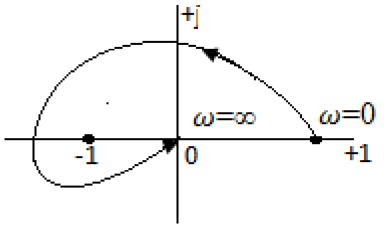
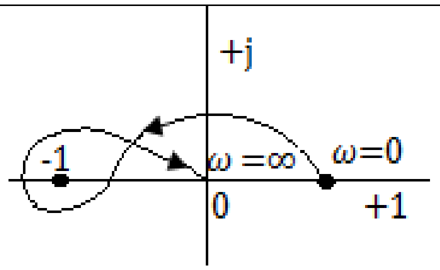
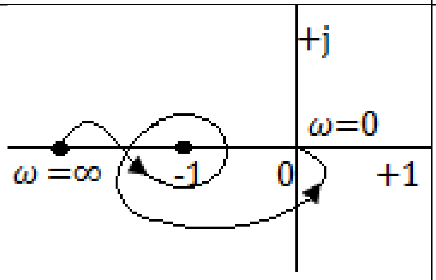
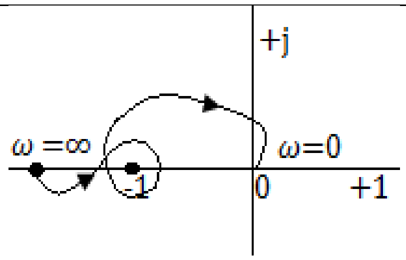
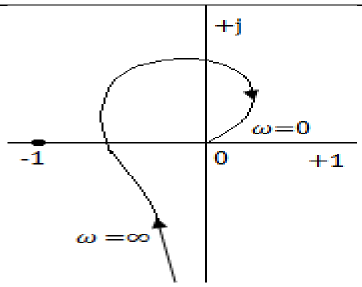
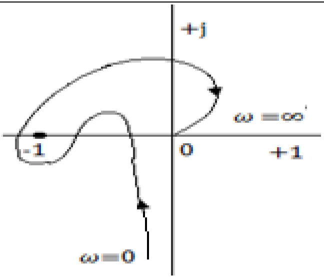
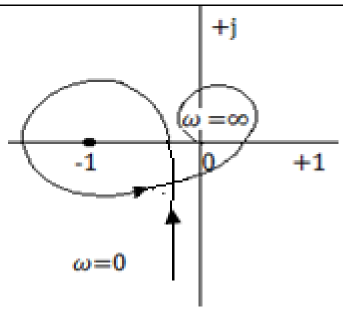
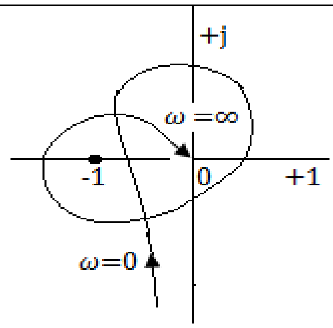
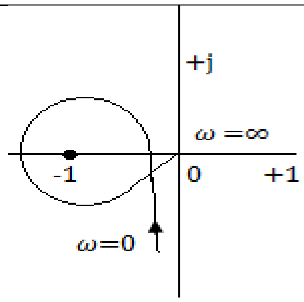
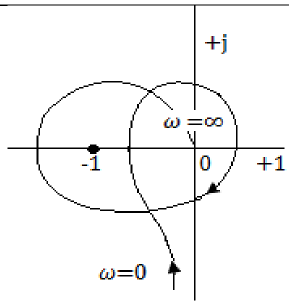
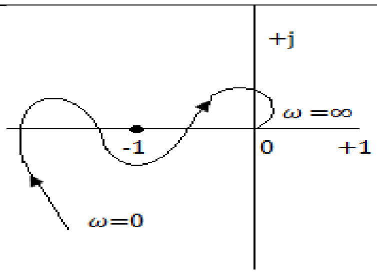
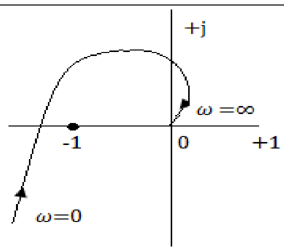
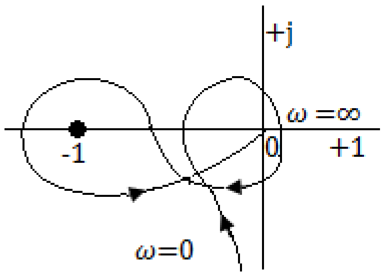
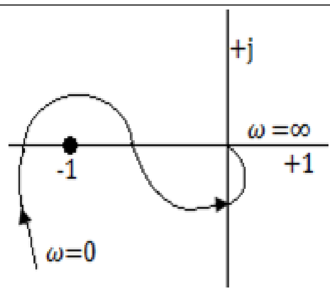
0	0	0	1		2	
0	0	2	3		4	
0	1	0	5		6	
0	2	0	7		8	

0	2	0	9		10	
0	3	0	11		12	

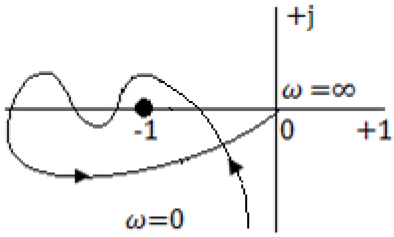
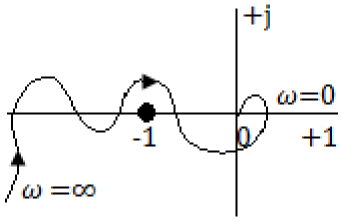
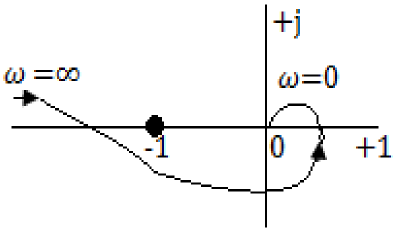
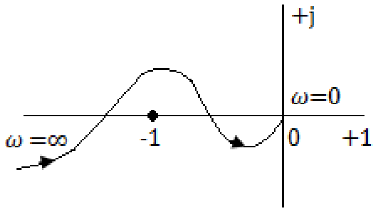
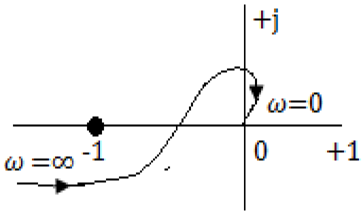
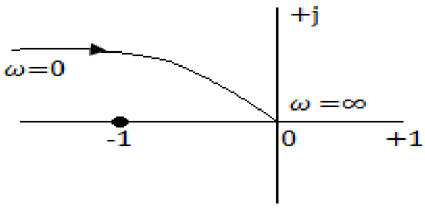
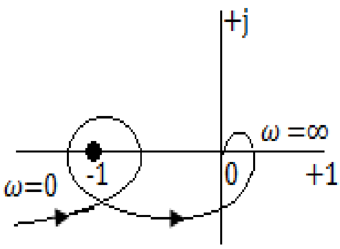
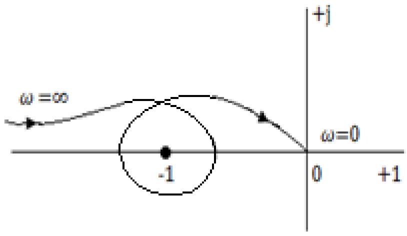
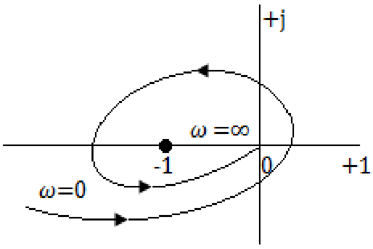
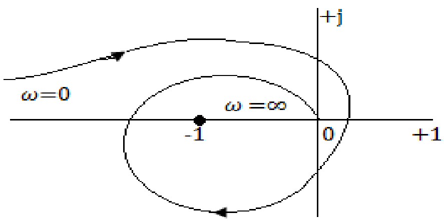
2.2-jadval (davomi)

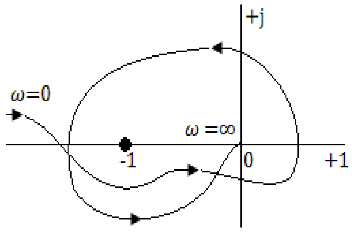
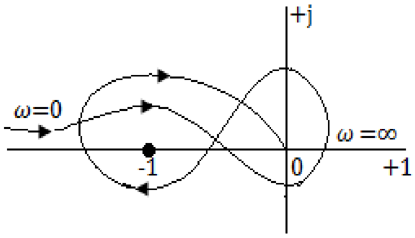
v	*m*	*i*	Variant	Chastota tavsif	Variant	Chastota tavsif
1	0	1	13		14	

1	2	1	15		16	
1	2	1	17		18	
1	0	1	19		20	
1	2	1	21		22	

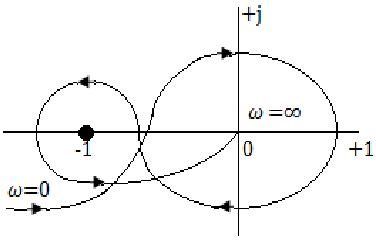
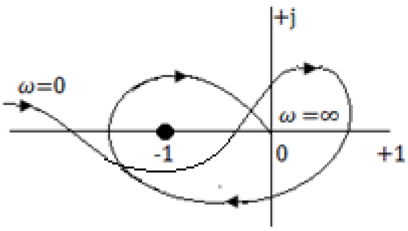
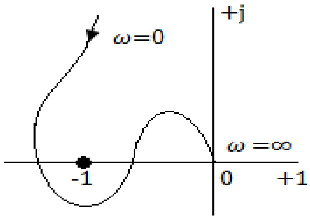
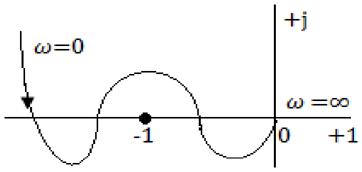
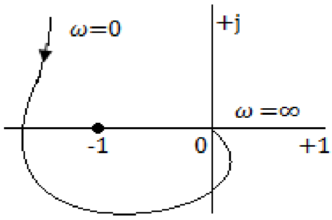
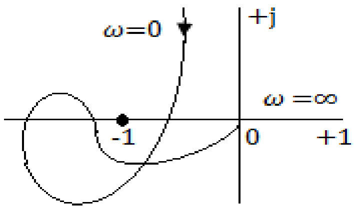
2.2-jadval (davomi)

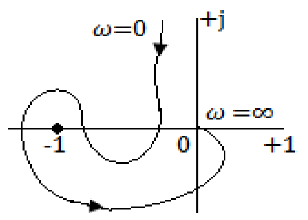
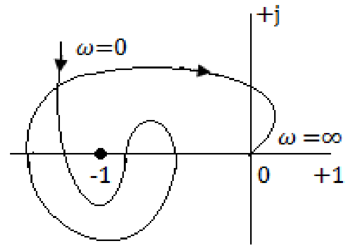
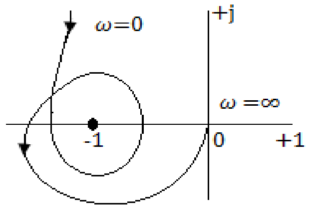
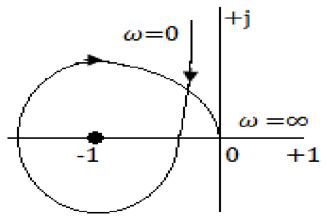
v	*m*	*i*	Variant	Chastota tavsif	Variant	Chastota tavsif

1	2	1	23		24	
2	0	2	25		26	
2	0	2	27		28	
2	2	2	29		30	
2	2	2	31		32	

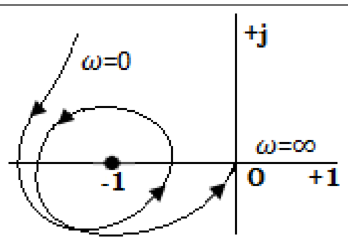
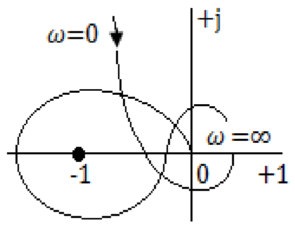
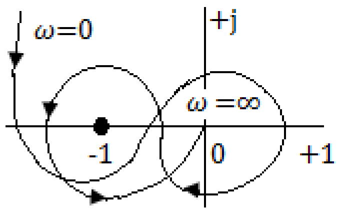
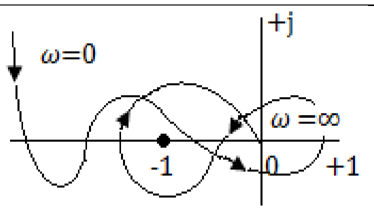
2	2	2	33		34	

2.2-jadval (davomi)

v	*m*	*i*	Variant	Chastota tavsif	Variant	Chastota tavsif
2	2	2	35		36	
3	0	3	37		38	
3	0	3	39		40	

3	0	3	41		42	
3	2	3	43		44	

2.2-jadval (davomi)

v	*m*	*i*	Variant	Chastota tavsif	Variant	Chastota tavsif
3	2	3	45		46	
3	2	3	47		48	

2.8.3 Uzilgan tizimning uzatish funksiyalari berilgan

a) $W(p) = \frac{10p+15}{p(3p^2+2p+1)}$;

b) $W(p) = \frac{10p+15}{p(3p^2+2p+0,1)}$;

c) $W(p) = \frac{5p+20}{p^3+2p^2+3p+1}$;

d) $W(p) = \frac{5p+20}{p^3+2p^2+3p+0,2}$;

e) $W(p) = \frac{10}{(0,2p+1)(0,5p+1)(3p-1)}$;

g) $W(p) = \frac{100}{(0,2p+0,1)(0,5p+0,2)(3p-2)}$.

Yopiq tizimning turg`unligi mo`tadillashtirilgan Naykvist godografi yordamida tekshirilsin.

2.8.4 Uzilgan tizimning uzatish funksiyasi quyidagi ko`rinishga ega

a) $W(p) = \frac{k(10p+1)}{p(3p^2+2p+1)}$; b) $W(p) = \frac{k(5p+1)}{p^3+2p^2+3p+1}$;

c) $W(p) = \frac{k}{(0,2p+1)(0,5p+1)(3p-1)}$; d) $W(p) = \frac{k(5p-1)}{p^3+2p^2+3p+1}$.

Statik kuchaytirish koeffitsienti $k \in [a, b]$ oraliqqa tegishli deb, yopiq tizimning turg`unligini ta'minlaydigan oraliqning chegaraviy qiymatlari a, b sonlar baholansin.

2.9 Turg`unlinking logarifmik qoidasi

Nazariy kirish

LAChT abssissa o`qini $\omega_k$ nuqtada kesib o`tadi, shu chastota kesish chastotasi deyiladi. Kesish chastotasidan kichik chastotalar sohasida LAChT musbat, undan katta chastotalar sohgasida LAChT manfiy qiymatlarni qabul qiladi. Ya’ni

$$L(j\omega) > 0, \quad \omega < \omega_k;$$
$$L(j\omega) < 0, \quad \omega > \omega_k\,.$$

LFChT $\omega < \omega_k$ sohasida $-\pi$ ($-180°$) qiymatni kesib o`tishi **o`tish** deyiladi. O`tish yuqoridan pastga bo`lsa (kattaroq qiymatlardan kichikroq qiymatlar tarafiga) u manfiy deyiladi, pastdan yuqoriga bo`lsa (kichikroq qiymatlardan kattaroq qiymatlar tarafiga) u musbat deyiladi. Musbat o`tishlar soni μ_+ , manfiy o`tishlar soni $\mu_-$ bo`lsin. Unda quyidagi turg`unlik qoidasi o`rinli.

Chiziqli statsionar uzliksiz yopiq tizim turg`un bo`lishi uchun uzilgan tizimning LFChTsi hosil qilayotgan musbat va manfiy o`tishlar sonlarining ayirmasi $m/2$ ga teng bo`lishi zarur va yetarlidir, ya’ni

$$\mu_+ - \mu_- = \frac{m}{2},$$

Bu yerda m – uzilgan tizimning o`ng qutblar soni.

Agar uzilgan tizim turg`un bo`lsa ($m=0$), ohirgi tenglik quyidagi shaklga keladi:

$$\mu_+ = \mu_-.$$

Bu holda musbat o`tishlar soni manfiy o`tishlar soniga teng bo`lishi kerak. Xususiy holda o`tishlar bo`lmasligi mumkin, ya’ni

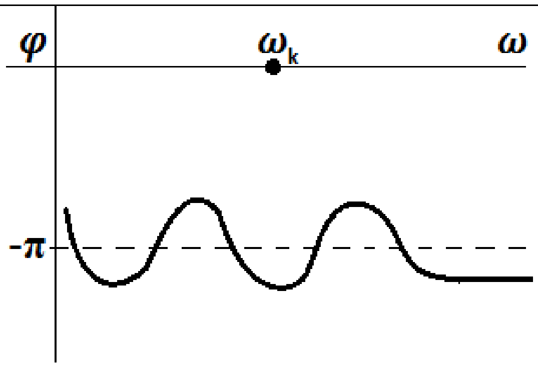
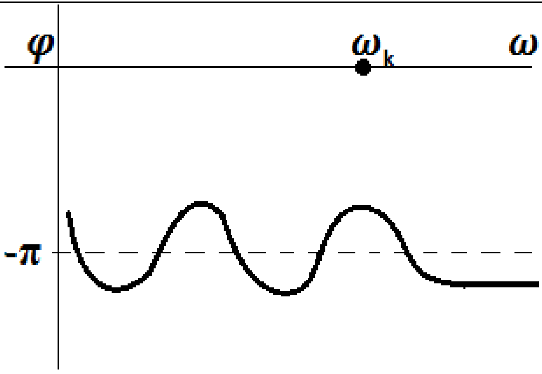
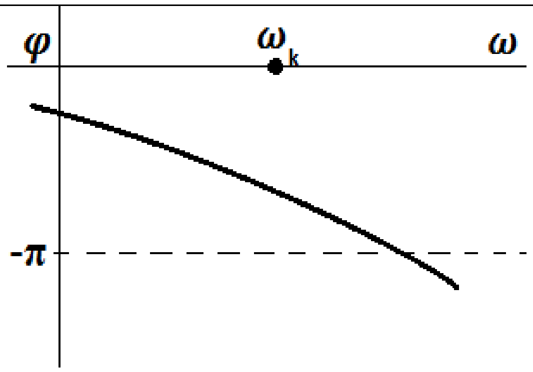
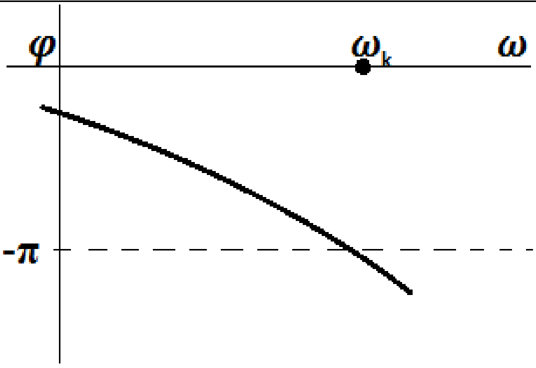
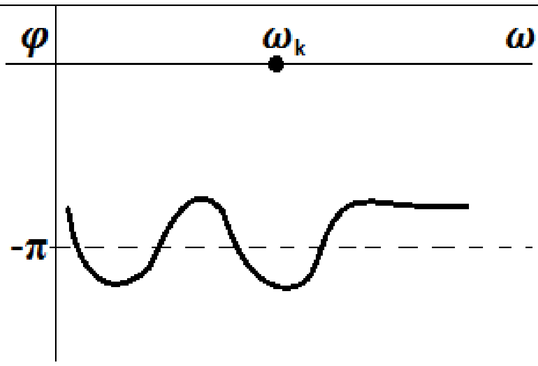
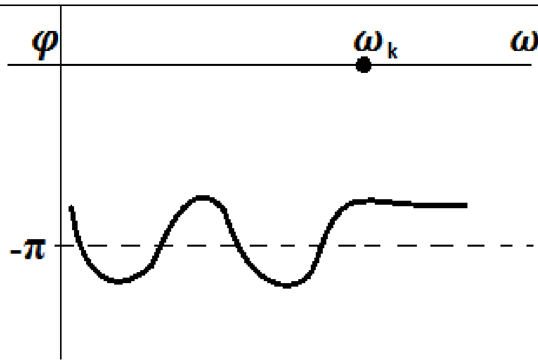
$$\mu_+ = \mu_- = 0.$$

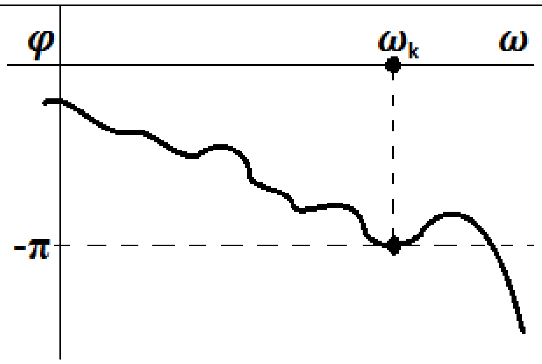
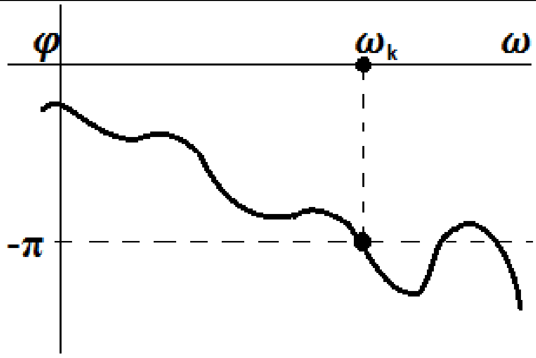
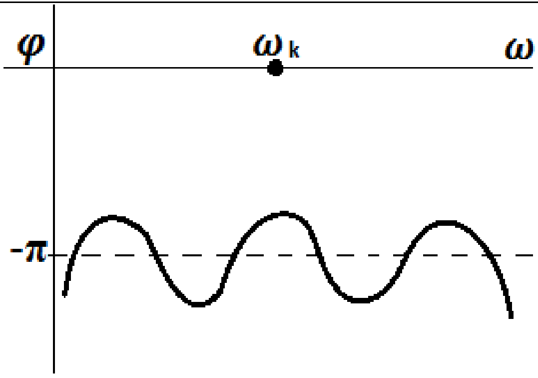
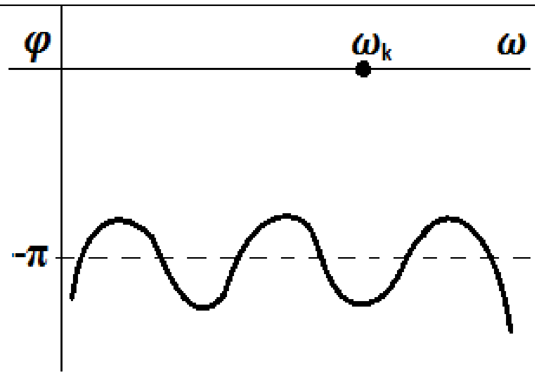
Mashqlar

2.9.1 Yuqoridagi 2.6.3-mashqda keltirilgan uzilgan tizimning uzatish funksiyasidan foydalanib, logarifmik qoida asosida yopiq tizimning turg`unligi tekshirilsin.

2.9.2 Uzilgan tizimning o`ng qutiblar soni $m$=0,1,2,3 bo`lgan hollar uchun yopiq tizimning turg`unligi 2.3-jadvalda keltirilgan LFChT va kesish chastotasi ω_k yordamida aniqlansin.

2.3-jadval. Logarifmik faza chastota tavsiflar

t/r	LFChT	t/r	LFChT
1		2	
3		4	
5		6	

7		8	
2.3-jadval (davomi)			
9		10	

2.10 D-ajratish

Nazariy kirish

Xarakteristik kophad bir parametrdan chiziqli bog`liq bo`lsa xarakteristik tenglama quyidagi ko`rinishda yozilishi mumkin

$$R(p) + \theta Q(p) = 0.$$

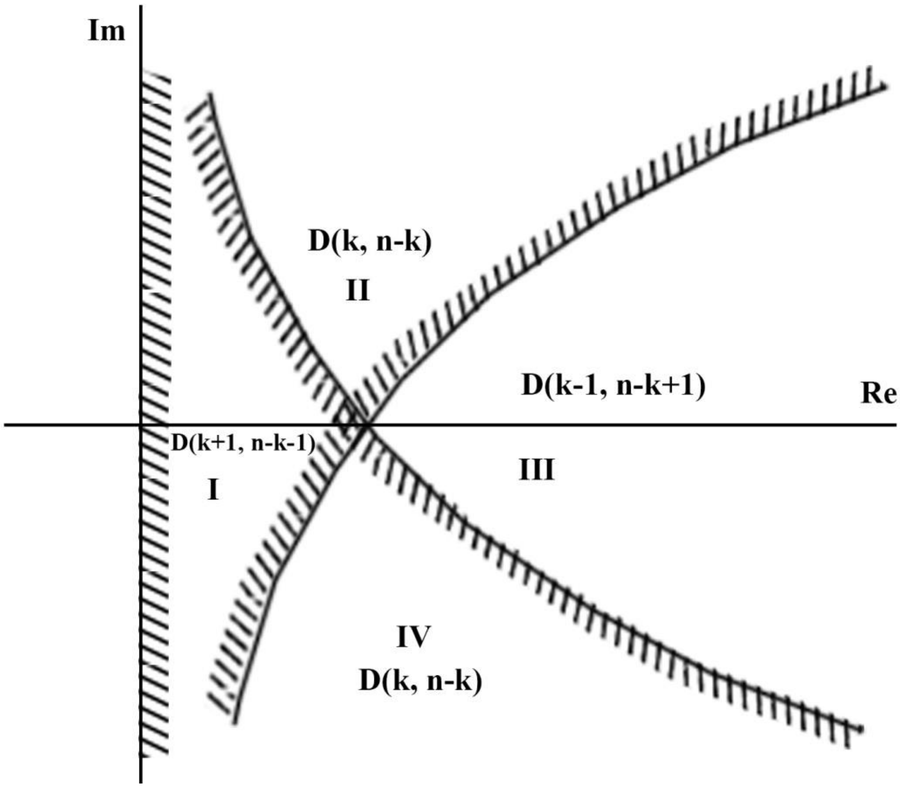
Unda θ parametr ikki ko`phad orqali topiladi

$$\theta = -\frac{R(p)}{Q(p)}.$$

Parametrni kompleks miqdor deb qabul qilib, $p = j\omega$ uchun yozish mumkim

$$\theta(j\omega) = -\frac{R(j\omega)}{Q(j\omega)} = U(\omega) + jV(\omega).$$

Chastota minus cheksizdan plyus cheksizgacha o`zgarganida, keltirilgan ifoda parametrning kompleks sirtida D-ajratish chizig`ini hosil qiladi, masalan 2.10.A-rasmdagidek. Bu, haqiqiy o`qqa nisbatan simmetrik, egri chiziq ildizlar sirtining mavhum o`qini parametr sirtiga aks ettirilganidir. Chastota oshish yo`nalishida D-ajratish chizig`i bo`ylab harakatlanilganda shu chiziqning chap tomoni ildizlar sirtidagi chap yarimsirtga mos keladi. Shu tomon shtrixlab chiqiladi. Natijada parametr sohasi turli darajada shtrixlangan nimsohalarga ajraladi. Har bir nimsohaga xarakteristik ko`phadning so`l va ong ildizlarining soni turlicha taqsimlanishi to`g`ri keladi. Yeng ko`p tomondan shtrixlangan sohadaginada tizim turg`un bo`lishi mumkin, chunki aynan shu sohaga eng ko`p so`l ildizlar to`g`ri keladi. Shu nimsohada parametrning haqiqiy qiymatlari uchungina tizimning turg`unligini tekshirishni davom etish ma’noga ega.



2.10.A-rasm. Bir parametr sirtida D-ajratish chizig`i.

Xarakteristik ko`phad ikki parametrdan chiziqli bog`liq bo`lsa xarakteristik tenglama quyidagi ko`rinishga ega bo`ladi

$$\nu P(p) + \mu Q(p) + R(p) = 0.$$

Bu yerda$\{\nu, \mu\}$ – tekshirilanayotgan parametrlar, $\{P(p), Q(p), R(p)\}$ – polinomlar. Tenglamada $p = j\omega$ deb qabul qilinsa

$$P(j\omega) = P_1(\omega) + jP_2(\omega),$$

$$Q(j\omega) = Q_1(\omega) + jQ_2(\omega),$$

$$R(j\omega) = R_1(\omega) + jR_2(\omega)$$

boladi. Natijada ikki parameterga nisbatan chiziqli tenglamalar tizimiga kelinadi

$$\begin{cases} \nu P_1(\omega) + \mu Q_1(\omega) = -R_1(\omega) \\ \nu P_2(\omega) + \mu Q_2(\omega) = -R_2(\omega) \end{cases}.$$

Bu tizimning aniqlovchisi

$$\Delta = \begin{vmatrix} P_1(\omega) & Q_1(\omega) \\ P_2(\omega) & Q_2(\omega) \end{vmatrix} \neq 0$$

shartni qanoatlantirsa, keltirilgan tenglamalar tizimi yagona yechimga ega. Chastota $(-\infty, +\infty)$ oraliqda o`zgarsa, yechimlar ikki parameter sirtida D-ajratish chizig`ini hosil qiladi. Bu chiziq ikki sidra shtrixlanadi. Agar $\Delta > 0$ bo`lsa, chap taraf ikki sidra shtrixlanadi, $\Delta < 0$ bo`lsa, o`ng taraf ikki sidra shtrixlanadi.

Chastotaning qaysidir qiymatida $\Delta = 0$ bo`lib qolishi mumkin. Shu $\omega_0$ qiymat uchun maxsus to`g`ri chiziq quriladi. Agar $\omega_0$ maxsus chastotaning qiymati 0, $-\infty$ yoki $+\infty$ bo`lsa shtrixlash bir sidra bajariladi, bordiyu boshqa chekli songa teng bo`lsa ikki sidra shtrixlash bajariladi. Umumiy nuqta atrofida maxsus to`g`ri chiziqning shtrixlanishi D-ajratish chizig`ining shtrixiga qaratilgan bo`ladi va umumiy nuqtadan o`tganda shtrixlashning yo`nalishi almashinadi. Ikki sidra shtrixlashni bosib o`tilganda xarakteristik ko`phadning o`ng va so`l ildizlar taqsimoti ikkitaga o`zgaradi.

Mashqlar

2.10.1 Keltirilgan xarakteristik ko`phad uchun tanlangan bir parameter sirtida, D-ajratish usuli yordamida, ildizlarning taqsimot sohalari qurilsin.

Ko`rsatma: ko`phadning tanlangandan boshqa parametrlariga son qiymatlar beriladi.

a) $D(p)=1+a_1p+a_2p^2+a_3p^3+a_4p^4$;

b) $D(p)=1+(T_1+T_2+T_3+T_4)p+(T_1T_2+T_1T_3+T_1T_4+T_2T_3+T_2T_4+T_3T_4)p^2+$
$+(T_1T_2T_3+T_1T_2T_4+T_1T_3T_4+T_2T_3T_4)p^3+T_1T_2T_3T_4p^4$.

2.10.2 Keltirilgan yopiq tizimning xarakteristik ko`phadi uchun uzilgan tizimning kuchaytirish koeffitsienti sirtida, D-ajratish usuli yordamida, ildizlarning taqsimot sohalari qurilsin.

Ko`rsatma: ko`phadning boshqa parametrlariga son qiymatlar beriladi.

$$D(p) = 1 + \sum_{i=1}^{4} a_i p^i + k(1 + \sum_{i=1}^{3} b_i p^i).$$

2.10.3 Keltirilgan xarakteristik ko`phad uchun tanlangan ikki parameter sirtida, D-ajratish usuli yordamida, ildizlarning taqsimot sohalari qurilsin.

Ko`rsatma: ko`phadning tanlangandan boshqa parametrlariga son qiymatlar beriladi.

$$D(p) = 1 + a_1 p + a_2 p^2 + a_3 p^3 + a_4 p^4.$$

2.10.4 Statik kuchaytirish koeffitsienti va tanlangan yana bir parameter sirtida keltirilgan ko`phad uchun D-ajratish bajarilsin.

Ko`rsatma: ko`phadning boshqa parametrlariga son qiymatlar beriladi.

$$D(p) = 1 + \sum_{i=1}^{4} a_i p^i + k(1 + \sum_{i=1}^{3} b_i p^i).$$

2.10.5 Statik kuchaytirish koeffitsienti va tanlangan bir vaqt doimiysi sirtida keltirilgan ko`phad uchun D-ajratish bajarilsin.

$D(p)=1+(T_1+T_2+T_3+T_4)p+(T_1T_2+T_1T_3+T_1T_4+T_2T_3+T_2T_4+T_3T_4)p^2+$

$+(T_1T_2T_3+T_1T_2T_4+T_1T_3T_4+T_2T_3T_4)p^3+T_1T_2T_3T_4p^4+k(1+(T_5+T_6+T_7)p+$

$+(T_5T_6+T_5T_7+T_6T_7)p^2+T_5T_6T_7p^3).$

2.11 Statsionar chiziqli boshqarish tizimning aniqligi

Nazariy kirish

Tafovut bo`yicha uzatish funksiya

$$\Phi_\Delta(p) = \frac{1}{1 + W(p)}.$$

Bu yerda $W(p)$ −uzilgan tizimning uzatish funksiyasi.

Tafovut bo`yicha uzatish funksiyani Makloren qatori ko`rinishida yozish mumkin:

$$\Phi_\Delta(p) = \Phi_\Delta(0) + \frac{d\Phi_\Delta(0)}{dp}p + \frac{1}{2!}\frac{d^2\Phi_\Delta(0)}{dp^2}p^2 + \cdots + \frac{1}{k!}\frac{d^k\Phi_\Delta}{dp^k}p^k + \cdots$$

Bu yerda

$$\frac{d^i\Phi_\Delta(0)}{dp^i} \equiv \lim_{p\to 0}\frac{d^i\Phi_\Delta(p)}{dp^i}, \quad i = 0,1,2,\ldots$$

Tafovutning tasviri quyidagicha ifodalanishi mumkin

$$E(p) = \Phi_\Delta(0)X(p) + \frac{d\Phi_\Delta(0)}{dp}pX(p) + \cdots + \frac{1}{k!}\frac{d^k\Phi_\Delta(0)}{dp^k}p^kX(p) + \cdots,$$

$X(p) -$ tizim kirishining tasviri.

Belgilash kiritiladi:

$$c_i = \frac{1}{i!}\frac{d^i\Phi_i}{dp^i},$$

$$i = 0,1,2, \ldots .$$

Bu sonlar hatolik koeffitsientlari deb nomlanadi. Unda tafovutning asli uchun quyidagi o`rinlidir

$$e(t) = c_0x(t) + c_1x^{'}(t) + c_2x^{''}(t) + \cdots + c_kx^{(k)}(t) + \cdots .$$

Hatolik koeffitsientlarni rekkurent munosabatlar yordamida hisoblab topish mumkin:

$$c_0 = \Phi_\Delta(p)|_{p=0};$$

$$c_1 = (\Phi_\Delta(p) - c_0)\frac{1}{p}\bigg|_{p=0};$$

$$c_2 = (\Phi_\Delta(p) - c_0 - c_1p)\frac{1}{p^2}\bigg|_{p=0};$$

… … … … … … … … … … … … … … … ….

$$c_k = (\Phi_\Delta(p) - \sum_{i=0}^{k-1} c_ip^i)\frac{1}{p^k}\bigg|_{p=0};$$

… … … … … … … … … … … … … … … … … …

Agar ko`rilayotgan tizimning astatizm darajasi $\nu$ bo`lsa, ya’ni uzilgan tizim tarkibida ν dona integrallovchi bo`g`in bo`lsa, $c_0 = 0$; $c_1 = 0$; … $c_{\nu-1} = 0$ bo`ladi. Nazariy jixatdan bunday tizimda vaqtning $\nu - 1$ tartibli polinom korinishidagi kirish signali uchun barqarorlashgan hatolik no`lga teng bo`ladi. Amalda astatizm darajasi oshgan sari tizimning turg`unligini ta’minlash og`irlashib boradi.

Mashqlar

2.11.1 Berilgan kirish ta'sirlar uchun uzatish funksiyasi

$$W(p) = \frac{kp}{1 + Tp}$$

bo`lgan haqiqiy differensiallovchi bo`g`in signalni ishlovlash uchun qo`llanilsin. Istalgan almashtirishning uzatish funksiyasi

$$W_{is}(p) = kp$$

bo`lsa, almashtirishning xatosi aniqlansin. Kirish ta'sitlar:

a) *u(t)=1(t);*

b) *u(t)=at·1(t);*

c) *u(t)=*$\frac{bt^2}{2}$.

2.11.2 Kirish signalini ishlovlash

$$W_{is}(p) = k(1 + T_1 p)$$

uzatish funksiyasi ko`rinishida bo`lgan izodrom bo`g`in orqali ifodalangan. Ishlovlashni ro`yobga oshirish esa, uzatish funksiyasi

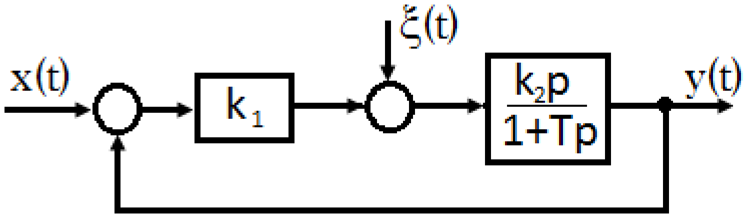
$$W(p) = \frac{k(1 + T_1 p)}{1 + T_2 p}$$

bo`lgan haqiqiy izodrom bo`g`in yordamida amalga oshirilgan. Parametrlarning berilgan son qiymatlari uchun almashtirish xatoligi topilsin.

2.11.3 Signalni ishlovlash tizimining tuzilma chizmasi 2.8-rasmda keltirilgan. Istalgan ishlovlash

$$W_{is}(p) = k_1 k_2 p$$

uzatish funksiyasi bilan belgilangan. Uzatish funksiyalarning farqi va g`alayonlantiruvchi *ξ(t)* tufayli hosil bo`ladigan xatolik topilsin.



2.8-rasm. Signalni ishlovlash tizimining tuzilmasi

2.11.4 Signalni istalgan ishlovlash

$$W_{is}(p) = k(1 + 2\xi Tp + T^2p^2)$$

uzatish funksiya yordamida ifodalangan. Aslida signal uzatish funksiyasi

$$W(p) = \frac{k(1 + 2\xi Tp + T^2p^2)}{(1 + T_1p)(1 + T_2p)}$$

bo`lgan haqiqiy tezlatuvchi bo`g`in yordamida ishlovlanadi. Ishlovlash xatoligi aniqlansin.

2.11.5 Birinchi tartibli astatizmga ega bo`lgan uzilgan tizimning tezlik bo`yicha kuchaytirish koeffitsienti aniqlansin, agar majburiy harakatning yo`l qo`yilishi mumkin bo`lgan xatoligi $|\varepsilon_1| = 0{,}01m,$ topshiruvchi signalning maksimal tezligi esa $|\dot{x}_{max}| = 0{,}06\, m/s$ bolsa.

2.11.6 Ikkinchi tartibli astatizmga ega bo`lgan uzilgan tizimning tezlanish bo`yicha kuchaytirish koeffitsienti aniqlansin, agar majburiy jarayonning yo`l qo`yilishi mumkin bo`lgan xatoligi $|\varepsilon_2| = 0{,}01m,$ topshiruvchi signalning maksimal tezlanishi esa $|\ddot{x}_{max}| = 0{,}5\, m/s^2$ bo`lsa.

2.11.7 Manfiy qayta aloqali kuchaytirgichning lozim bo`lgan statik kuchaytirish koeffitsienti aniqlansin, agar yo`l qo`yilgan statik xatolik $|\varepsilon_0| = 0{,}002\text{v},$ maksimal kirish kuchlanishi esa $U_{max} = 110\text{v}$ bo`lsa.

2.11.8 Uzilgan tizimning uzatish funksiyasi ma’lum.

$$a)\ W(p) = \frac{k(1 + Tp)}{p(1 + a_1p + a_2p^2 + a_3p^3)};$$

$$b)\ W(p) = \frac{k(1+Tp)}{1+a_1p+a_2p^2}.$$

Yopiq tizimning birinchi ν dona xatolik koeffitsientlari aniqlansin.

2.11.9 Kuzatuvchi tizimning tafovut bo`yicha uzatish funksiyasi berilgan

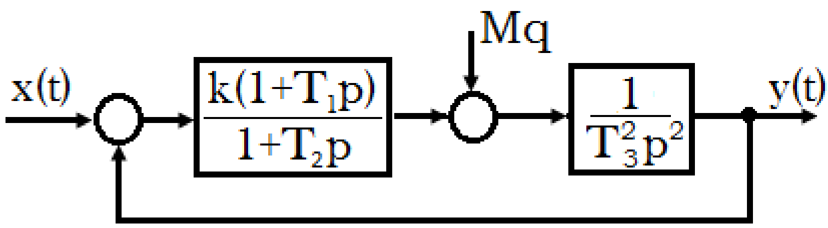
$$\Phi_\varepsilon(p) = \frac{k_\varepsilon(1+b_1p+b_2p^2)}{1+a_1p+a_2p^2}.$$

Tizimning kirishiga chastotasi ω_0 va amplitudasi A_0 bo`lgan sinusoidal signal uzatilganidagi barqarorlashgan xatolik (tafovut) topilsin.

2.11.10 Kuzatish tizimining tuzulma chizmasi (2.9-rasm) berilgan. Kirish signali

$$x(t) = \frac{at^2}{2} \cdot 1(t)$$

va motor o`qiga ta’sir etuvchi o`zgarmas g`alayonlantiruvchi M_q tizimga uzatilganida barqarorlashgan xatolik aniqlansin,

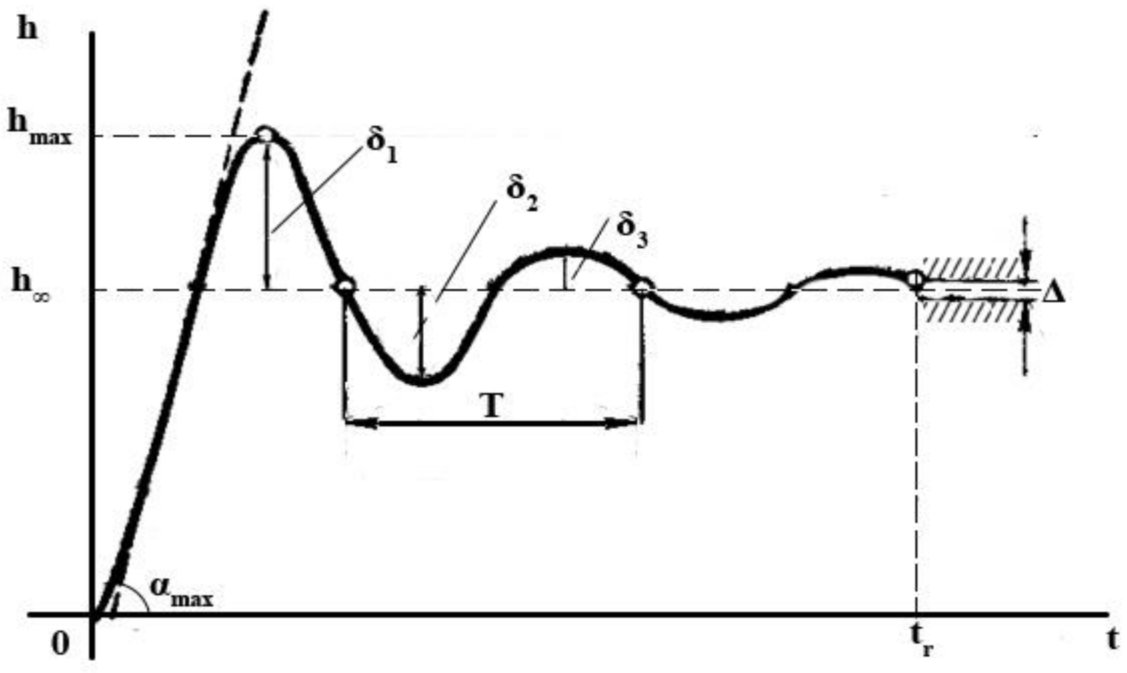


2.9-rasm. Kuzatish tizimining tuzulma chizmasi

2.12 O`tish jarayonining sifati

Nazariy kirish

O`tish jarayoni va uning asosiy sifat ko`rsatgichlari 2.12.A-rasmda ko`rsatilgan.



2.12.A-rasm. O`tish jarayonining tasviri.

1. $h_\infty -$ o`tish jarayonining barqarorlashgan qiymati.

2. $h_{max} -$ o`tish jarayonining maksimal qiymati.

3. $\Delta -$ ruxsat etilgan statik hatolik. Odatda u barqarorlashgan qiymatning (h_∞) foyizi ko`rinishida beriladi. Amalda $\Delta \leq 5\%$.

4. $t_r -$ rostlash vaqti. Bu vaqt o`tish jarayoni $h(t)$ Δ-naycha ichiga kirib uning ichidan qaytib chiqmaydigan vaqt daqiqasi bilan belgilanadi.

5. $\left(\frac{dh}{dt}\right)_{max} = tg\alpha_{max} -$ o`tish jarayonining maksimal tezligi.

6. $\sigma = \frac{h_{max}-h_\infty}{h_\infty} \cdot 100\% = \frac{\delta_1}{h_\infty} \cdot 100\% -$ oshirib rostlash.

7. $T -$ tebranish davri.

8. $N -$ rostlash vaqti davomidagi to`la tebranishlar soni.

9. $\delta_i -$ barqarorlashgan qiymat (h_∞) atrofidagi tebranishning amplitudalari.

10.So`nish dekrementi:

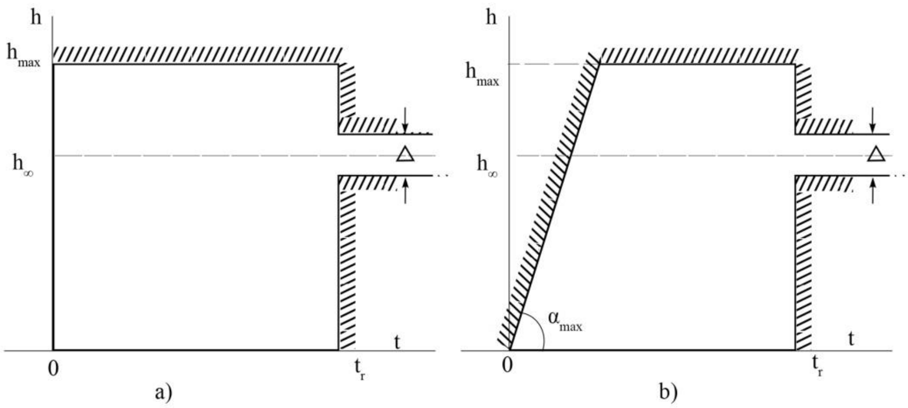
$$\kappa = \frac{\delta_i}{\delta_{i-1}}.$$

Logarifmik so`nish dekrementi:

$$\kappa_{ln} = ln\delta_i - ln\delta_{i+1} \,.$$

So`nish dekrementi qanchalik katta bo`lsa, tebranish shunchalik tezroq susayib boradi.

O`tish jarayonining sifat ko`rsatgichlariga talablar qandaydir tengsizliklar ko`rinishida qoyilishi mumkin. Hususan bu talablar Solodovnikov qutilari (2.12.B-rasm) ko`rinishida qo`yilishi mumkin.

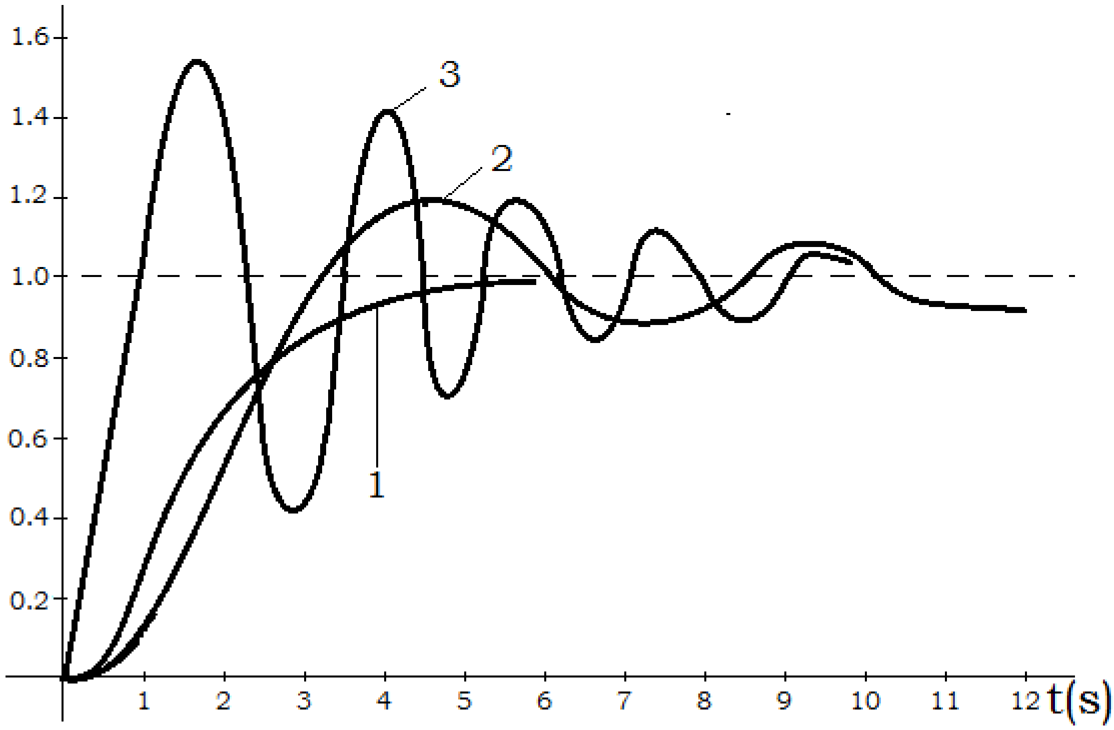


2.12.B-rasm. Solodovnikov qutilari.

Bu qutilardan a) bilan belgilanganida dastlabki to`rt sifat ko`rsatgichlarga nisbatan, b) bilan belgilanganida esa dastlabki besh sifat korsatgichlarga nisbatan qo`ilgan talablar ifodalangan. Solodovnikov qutilarini mohiyati o`tish jarayonlar shu qutidan tashqariga chiqib ketmasligidir.

Mashqlar

2.12.1 Tajribada olingan o`tish jarayonining ossilogrammalaridan (2.10-rasm) ruxsat etilgan xatolik Δ=5% bo`lganida rostlash vaqti (t_r), oshirib rostlash (σ), tebranuvchanlik (N), so`nish dekrementi (κ), maksimal tezlik baholansin.



2.10-rasm. O`tish jarayonlari

2.12.2 Ikkinchi tartibli tizimning uzatish funksiyasi

$$W(p) = \frac{1}{1 + 2\xi Tp + T^2p^2}$$

ko`rinishga ega bo`lganida uning o`tish funksiyasi quyidagicha

$$h(t) = 1 - \frac{1}{r} e^{-\xi \frac{t}{T}} \sin\left(\frac{r}{T} t + \varphi\right);$$

$$\varphi = arctg \frac{r}{\xi}.$$

Bu yerda $r = \sqrt{1 - \xi^2}$. oshirib rostlash σ parametrlar ξ va T qiymatlaridan bog`liqligi topilsin.

2.12.3 Avvalgi 2.12.2-mashqdagi tizim uchun ruxsat etilgan xatolik Δ berilgan bo`lsa, rostlash vaqti $t_r$ parametrlar ξ va T qiymatlaridan bog`liqligi topilsin.

2.12.4 Ikkinchi tartibli tizimning uzatish funksiyasi

$$W(p) = \frac{k}{T^2p^2 + 2\xi Tp + 1 + k}$$

ko`rinishga ega. Oshirib rostlash σ parametrlar ξ va T qiymatlaridan bog`liqligi topilsin.

2.12.5 Avvalgi 2.12.4-mashqdagi tizim uchun ruxsat etilgan xatolik Δ berilgan bo`lsa, rostlash vaqti $t_r$ parametrlar ξ vaT qiymatlaridan bog`liqligi topilsin.

2.12.6 Yuqoridagi 2.12.2-mashqda keltirilgan tizimning o`tish funksiyasi erishadigan maksimal tezlik parametrlar orqali ifodalansin

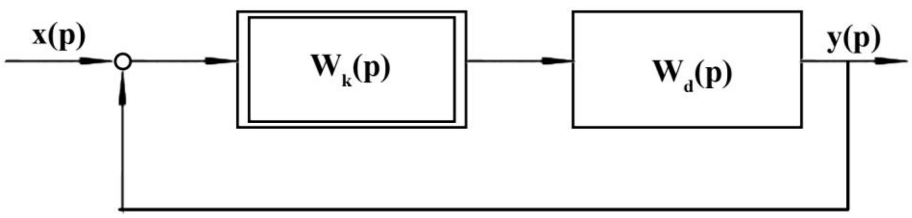
2.13 Chiziqli boshqarish tizimlarni korreksiyalash

Nazariy kirish

Tizimning dastlabki ma'lum qismi uzatish funksiya $W_d(p)$ orqali berilgan. Uzilgan tizimning istalgan uzatish funksiyasi $W_i(p)$ qandaydir yo`l bilan tanlab olingan.

Korreksiyalovchi bo`g`in ketma-ket kiritilganida (2.13.A-rasm), uning chastota tavsifi quyidagicha aniqlanadi:

$$W_k(j\omega) = \frac{W_i(j\omega)}{W_d(j\omega)}.$$



2.13.A-rasm. Ketma-ket korreksiyalash.

Logarifmik chastota tavsigflar uchun yuqoridagi ifoda quyidagi ko`rinishga ega bo`ladi:

$$L_k(\omega) = L_i(\omega) - L_d(\omega),$$

$$\varphi_k(\omega) = \varphi_i(\omega) - \varphi_d(\omega).$$

Dastlabki qismni ketma-ket ulangan ikki dinamik bo`g`in ko`rinishida tasvirlash mumkin:

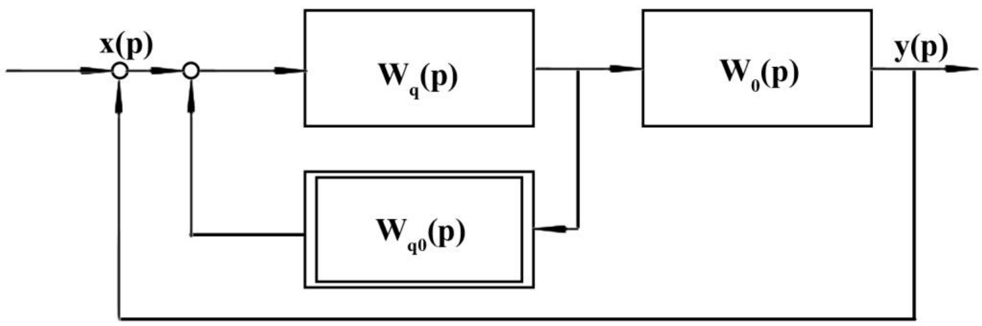
$$W_d(p) = W_q(p)W_o(p).$$

Bu yerda

$W_q(p) -$ qamrab olinadigan qism,

$W_o(p) -$ qamrab olinmaydigan qism deb nomlanadi.

Qayta aloqali (teskari parallel) korreksiyalovchi bo`g`in $W_{qa}(p)$ 2.13.B-rasmdagidek kiritiladi.



2.13.B-rasm. Qayta aloqali korreksiyalash.

Istalgan uzilgan tizimning chastota tavsifi $W_i(j\omega)$ bilan qayta aloqali korreksiyalovchi bo`g`inning chastota tavsifi $W_{qa}(j\omega)$ orasida quyidagi munosabat mavjud:

$$W_i(j\omega) = \frac{W_d(j\omega)}{1 + W_q(j\omega)W_{qa}(j\omega)}.$$

Agar ma'lum chastota sohasida

$$\left|W_q(j\omega)W_{qa}(j\omega)\right| \ll 1$$

bo`lsa, unda shu soha uchun

$$W_i(j\omega) \approx W_d(j\omega).$$

Chastotaning boshqa sohasi uchun

$$\left|W_q(j\omega)W_{qa}(j\omega)\right| \gg 1$$

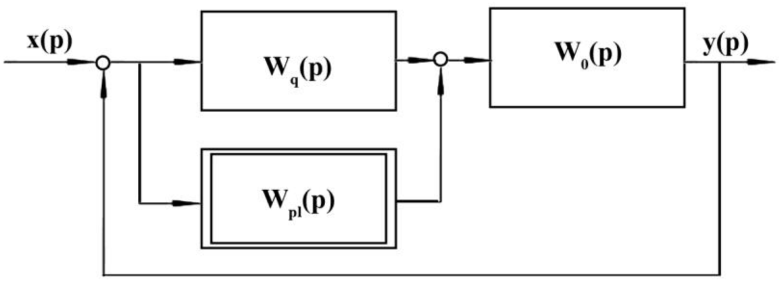
o`rinli bo`lsa, quyidagi taqribiy tenglik o`rinli

$$W_i(j\omega) \approx \frac{W_d(j\omega)}{W_q(j\omega)W_{qa}(j\omega)}.$$

Ohirgi ifodadan korreksiyalovchi qurilmaning logarifmik chastota tavsiflarini taqriban topish mumkin:

$$L_{qa}(\omega) = L_d(\omega) - L_i(\omega) - L_q(\omega),$$
$$\varphi_{qa}(\omega) = \varphi_d(\omega) - \varphi_i(\omega) - \varphi_q(\omega).$$

Korreksiyalovchi bo`g`in $W_p(p)$ qamrab olinuvchi $W_q(p)$ bo`g`inga parallel kiritilsa, tizimning tuzilmasi 2.13.C-rasmdagidek bo`ladi.



2.13.C-rasm. Parallel korreksiyalash.

Chastota tavsiflar o`rtasida quyidagi munosabat o`rinli bo`loadi:

$$W_i(j\omega) = \left[W_q(j\omega) + W_p(j\omega)\right]W_o(j\omega).$$

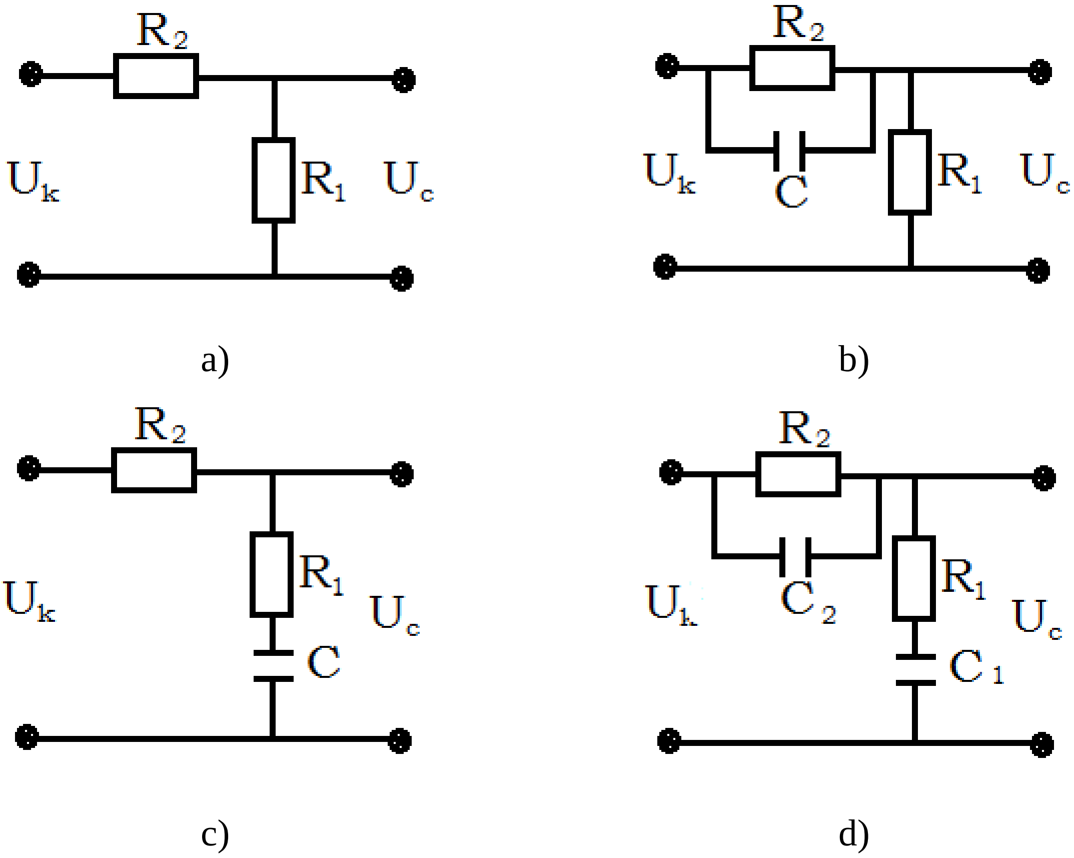
Bundan korreksiyalovchi bo`g`inning chastota tavsifini aniqlash mumkin:

$$W_p(j\omega) = \frac{W_i(j\omega) - W_d(j\omega)}{W_o(j\omega)}.$$

Parallel korreksiyalashda logarifmik chastota tavsiflar qulaylik yaratmaydi.

Mashqlar

2.13.1 Chizmasi 2.11-rasmda keltirilgan korreksiyalovchi zanjirlarning uzatish funksiyasidagi koeffitsientlar elektr unsirlarning parametrlari orqali ifodalansin.



2.11-rasm. Korreksiyalovchi zanjirlar.

a) proporsional; b) haqiqiy tezlatuvchi (izodrom);

c) haqiqiy integrallovchi; d) haqiqiy integro-differensiallovchi.

2.13.2 Uzilgan tizimning daslabki uzatish funksiyasi quyidagi ko`rinishga ega

$$W_d(p) = \frac{k_d}{p(T_1 p + 1)(T_2 p + 1)}.$$

Ketma-ket korreksiya kiritib, faza (φ_z) va amplituda (H_z) bo`yicha talab etilgan turg`unlik zaxiralari ta’minlansin. Sintez qilingan tizimning aniqligi va tezkorligi baholansin. Quyidagi passiv korreksiyalovchi zanjirlar qo`llansin:

a) proporsional;
b) haqiqiy tezlatuvchi;
c) haqiqiy integrallovchi;
d) haqiqiy integro-differensiallovchi.

Ko`rsatma: a) hisoblarni bajarishda dastlabki uzatish funksiya parametrlariga son qiymatlar berilishi lozim; b) korreksiyalash uchun logarifmik chastota tavsiflarni qo`llash tavsiya etiladi.

2.13.3 Sintez qilingan tizimga nisbatan quyidagi talablar qo`yilgan:

1) tizim birinchi tartibli astatizmga ega bo`lishi kerak;

2) tezlik va tezlanish bo`yicha xatolik koeffitsientlari mos ravishda c_1, va c_2 qiymatlardan oshmasligi kerak;

3) darqarorlashgan xatolik berilgan Δ qiymatga teng bo`lganida (Δ-naycha), rostlash vaqti berilgan t_r qiymatdan oshmasligi kerak;

4) oshirib rostlash berilgan σ% sondan oshmasligi kerak.

Yuqoridagi 2.13.2-mashqda berilgan dastlabki uzatish funksiya uchun korreksiya amali bajarilsin.

Ko`rsatma: hisoblarni bajarishda talablar parametrlariga va dastlabki uzatish funksiya parametrlariga son qiymatlar berilishi lozim.

III Impuls tizimlar

3.1 Panjarasimon funksiyalar va chekli ayirmalar

Nazariy kirish

Argumentning sanoqli qiymatlarida mavjud bo`lgan funksiyaga **panjarasimon funksiya** deyiladi. Masalan,

$$f(t_i), \quad i = 0, 1, 2, 3, \dots .$$

Bu yerda t_i funksiya mavjud bo`lgan tartib raqami “i”ga teng vaqt daqiqasi. Muhandislikda ikkita ketma-ket keladigan vaqt daqiqalar farqi o`zgarmas bo`lgan hol qo`llaniladi, ya’ni

$$t_{i+1} - t_i = T = const.$$

Parameter T diskretlash yoki kvantlash **davri** deyiladi. Bu holda panjarasimon funksiya $f[nT]$, yoki, T=1 deb qabul qilinsa, $f[n]$ ko`rinishda ifodalanadi. Bu yerda argument n butun qiymatlarni qabul qiladi. Panjarasimon funksiyaning bir daqiqadagi qiymatiga **diskreta** deyiladi.

Panjarasimon funksiyaning birinchi tartibli chekli ayirmasi:

$$\Delta f[nT] = f[nT] - f[(n-1)T];$$

k – tartibli chekli ayirmasi esa:

$$\Delta^k f[nT] = \Delta^{k-1} f[nT] - \Delta^{k=1} f[(n-1)T]$$

ko`rinishda ta’riflanadi.

3.1.0 Uzliksiz funksiyalar

$$f_1(t) = at^2 ,$$
$$f_2(t) = 1(t)at^2$$

uchun $T = 1$ bo`lganda panjarasimon funksiyalar ifodalansin va ularning birinchi va ikkinchi tartibli ayirmalari topilsin.

Bajarish.

$$f_1[n] = an^2 ; \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots$$

$$f_2[n] = \begin{cases} an^2; & n = 1, 2, 3, \ldots \\ 0 ; & n = 0, -1, -2, -3, \ldots \end{cases}$$

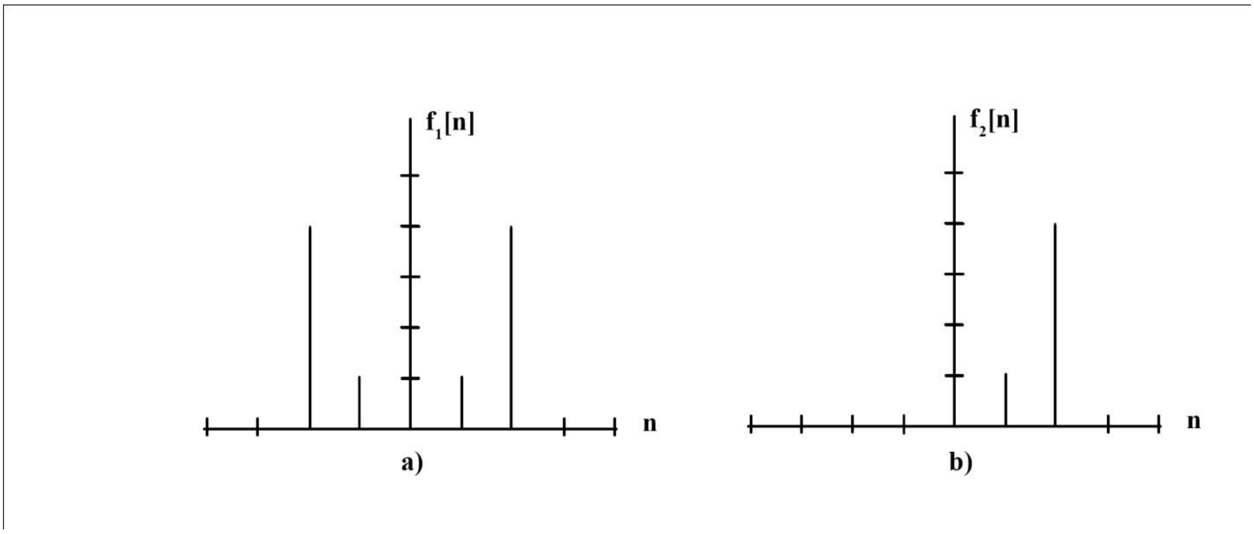
$$\Delta f_1[n] = an^2 - a(n-1)^2 = a(2n-1), \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots$$

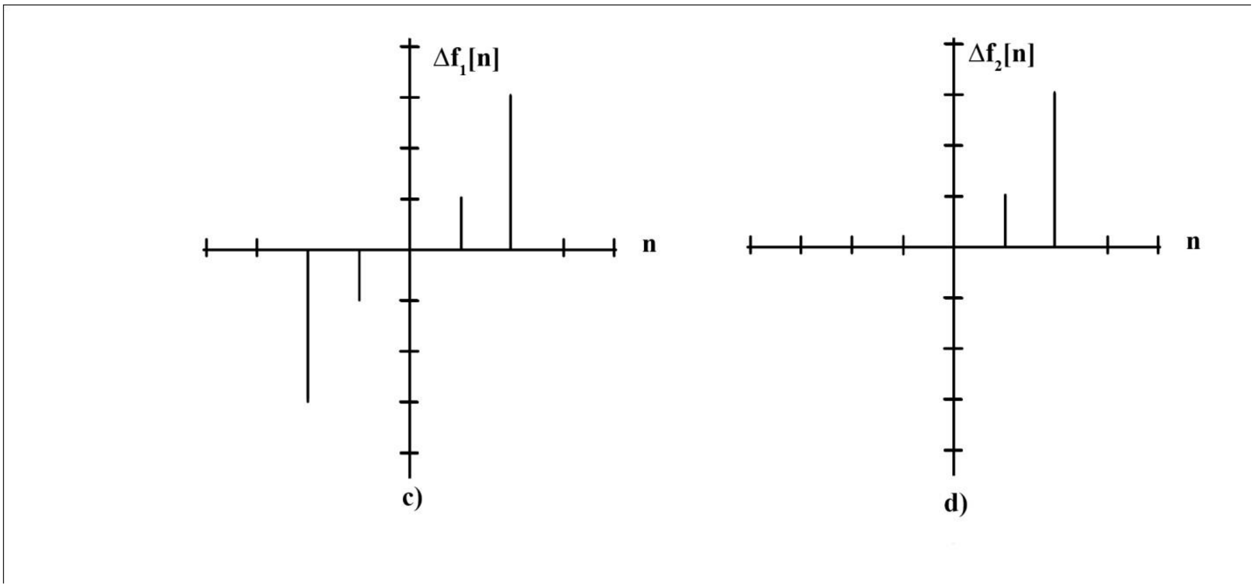
$$\Delta f_2[n] = \begin{cases} a(2n-1), & n = 1, 2, 3, \ldots \\ 0, & n = 0, -1, -2, \ldots \end{cases}$$

$$\Delta^2 f_1[n] = a(2n-1) - a(2(n-1)-1) = 0,$$

$$\Delta^2 f_2[n] = 0 ,$$

Bu funksiyalar va ularning birinchi tartibli ayirmalarining chizmalari, $a = 1$ bo`lgan hol uchun, 3.1.A-rasmda keltirilgan.





3.1.A-rasm. Panjarasimon funksiyalar va ularning ayirmalari.

Mashqlar

3.1.1 Kvantlash davrining ikki T_1, T_2 qiymatlari uchun quyidagi uzliksiz funksiyalardan hosil qilinadigan panjarasimon funksiyalar topilsin va qurilsin. Hosil bo`lgan panjarasimon funksiyalarning juftliklari o`zaro taqqoslanib tahlil qilinsin.

a) $f(t) = \delta(t)$; b) $f(t) = 1(t)$; c) $f(t) = kt$; d) $f(t) = kt \cdot 1(t)$;

e) $f(t) = ke^{-\lambda t}$; f) $f(t) = ke^{-\lambda t} \cdot 1(t)$; g) $f(t) = A \sin \omega t$;

h) $f(t) = A \sin \omega t \cdot 1(t)$; i) $f(t) = 1(t) - 1(t - \tau)$;

j) $f(t) = 1(t) - k1(t - \tau)$.

3.1.2 Avvalgi 3.1.1-mashqdagi funksiyalar (n+ε)T vaqt daqiqalarida kvantlanganida panjarasimon funksiyalar topilsin va qurilsin. Bu yerda $0 \leq \varepsilon \leq 1$; n=0,1,2,…

3.1.3 Yuqoridagi 3.1.1-mashqda topilgan panjarasimon funksiyalarning har biriga mos keladigan, mashqda berilgan funksiyalardan farqli bo`lgan, uch xil uzliksiz funksiyning grafigi qurilsin.

3.1.4 Panjarasimon funksiyaning birinchi tartibli ayirmasi

$$\Delta f[nT] = f[nT] - f[(n-1)T]$$

ko`rinishda ta'riflanadi. Yuqoridagi 3.1.1-mashqda topilgan panjarasimon funksiyalarning birinchi tartibli ayirmalari topilsin va ularning panjarasimon chizmalari qurilsin.

3.1.5 Panjarasimon funksiyaning k-tartibli ayirmasi

$$\Delta^k f[nT] = \Delta^{k-1} f[nT] - \Delta^{k-1} f[(n-1)T]$$

ko`rinishda ta'riflanadi. Ikkinchi va uchinchi tartibli ayirmalar panjarasimon funksiyaning diskretalari, y'ani $f[nT], f[(n-1)T], f[(n-2)T], \dots,$ orqali ifodalansin.

3.1.6 Birinchi tartibli ayirmalar quyidagi to`rt ko`rinishda ta'riflanishi mumkin:

a) $\Delta f[nT] = f[nT] - f[(n-1)T],$

b) $\Delta_1 f[nT] = f[nT] - f[(n+1)T],$

c) $\Delta_2 f[nT] = f[(n-1)T] - f[nT],$

d) $\Delta_3 f[nT] = f[(n+1)T] - f[nT].$

Bu ayirmalarning har jufti orasidagi munosabat aniqlansin.

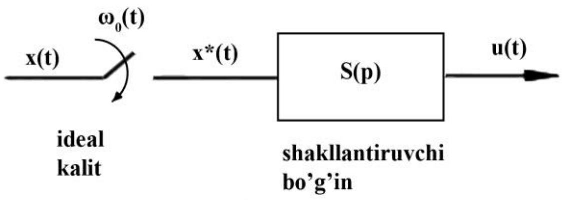
3.2 Impuls unsirlarning tavsiflari

Nazariy kirish

Impuls unsirning statik va dinamik tavsifi haqida gap yuritish mumkin. Statik tavsif modulyatsiyalangan o`zgaruvchi bilan modulyatsiyalovchi o`zgaruvchi o`rtasidagi bog`lanishdir. Bu bog`lanish har xil bo`lishi mumkin. Chiziqli tizimlarda shu o`zgaruvchilar o`rtasida **proporsional** bog`lanish o`rinli bo`ladi.

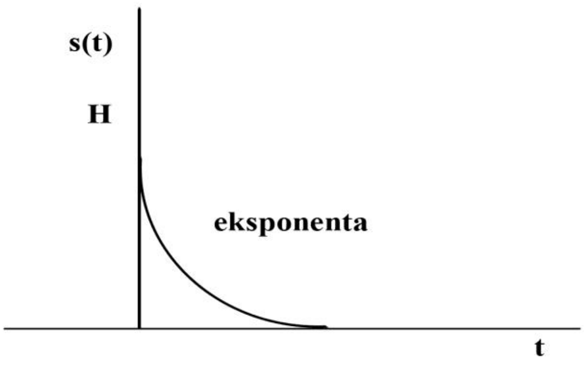
Impuls unsirning dinamik tavsifi deganda shakllantiruvchi bo`g`inning dinamik tavsifi tushuniladi. Uni uzliksiz vazn funksiya yoki uzatish funksiya ko`rinishida ifodalash mumkin.

Shunday qilib, chiziqli tizimlarda haqiqiy impuls unsir ketma-ket ulangan ideal kalit va chiziqli dinamik bo`g`in ko`rinishida modellashtiriladi (3.2.A-rasm).



3.2.A-rasm. Impuls unsirning modeli.

3.2.0 Impuls unsirning chiqishidagi impulsning shakli 3.2.B-rasmdagidek. Shakllantiruvchi bo`g`inning dinamik tavsifi aniqlansin.



3.2.B-rasm. Impuls unsirning chiqishidagi impuls.

Yechim. Nodavriy bo`g`inning vazn funksiyasi:

$$w(t) = \frac{K}{T} e^{-\frac{t}{T}}.$$

Uning uzatish funksiyasi:

$$W(p) = \frac{K}{1 + Tp}.$$

Unda impulsning amplitudasi H joriy diskretaning qiymatiga proporsional bo`ladi:

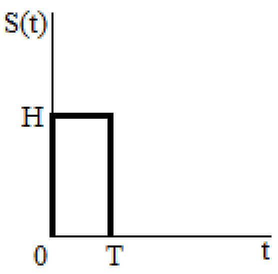
$$H = \frac{K}{T} x[nT_0].$$

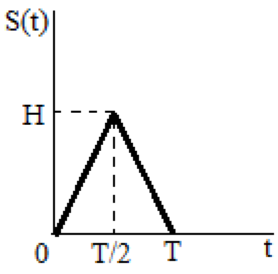
Nodavriy bo`g`inning vaqt doimiysi T kvantlash davridan (T_0) ancha kichik bo`lishi kerak

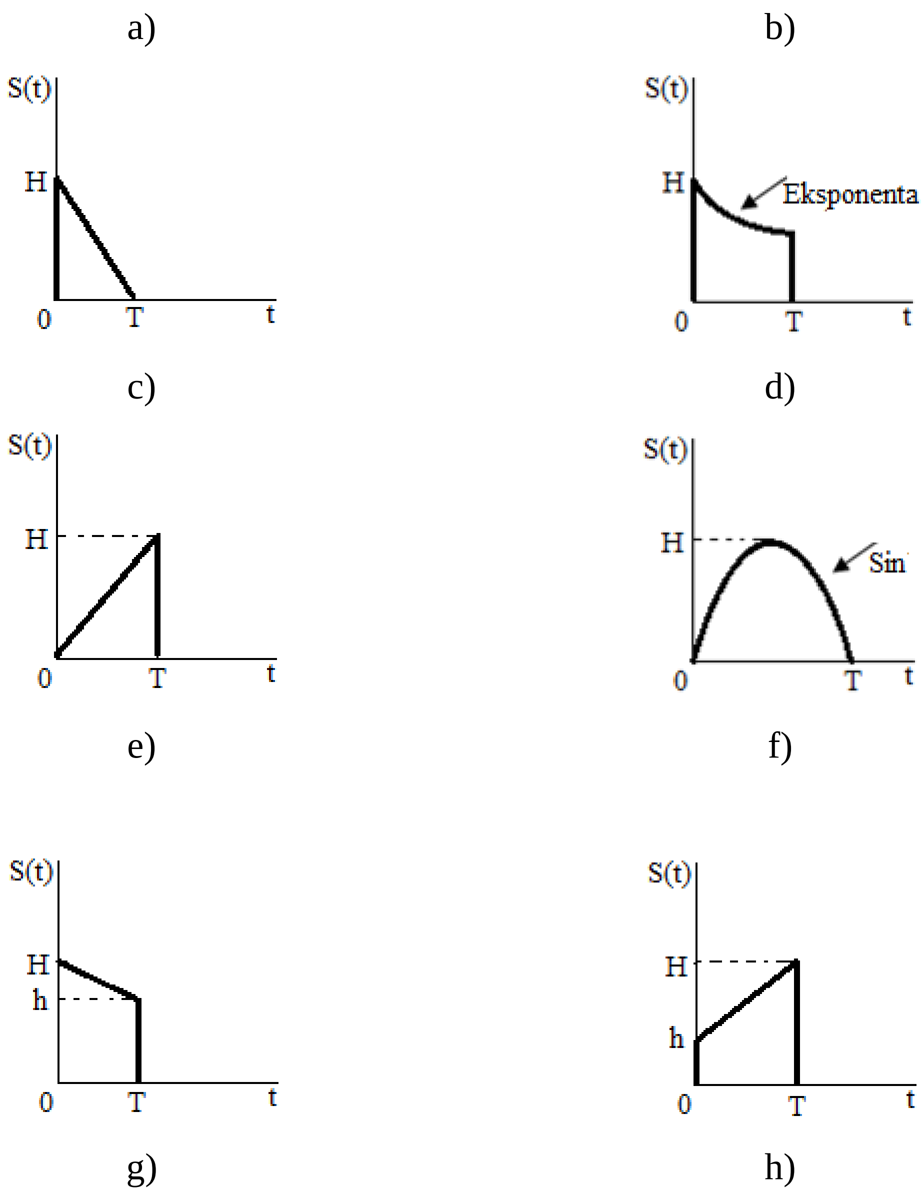
$$T \ll T_0.$$

Mashqlar

3.2.1 Impuls unsirning chiqishidagi impulslar 3.1-rasmda keltirilgan shaklga ega. Shakllantiruvchi bo`g`inning uzatish funksiyasi aniqlansin.



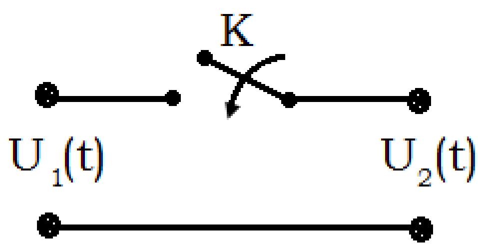




3.1-rasm. Impuls unsirning chiqishidagi impulslar.

3.2.2 Avvalgi 3.2.1-mashqdagi impuls unsirlarning chastota tavsiflari aniqlansin va qurilsin.

3.2.3 impuls modulyatsiyani bajaruvchi unsirning shartli chizmasi 3.2-rasmda keltirilgan.



3.2-rasm. Impuls unsirning shartli chizmasi

Ushbu unsir uchun

a) qaysi parameter modulyatsiyalanuvchi;

b) qanday shartlar bajarilganida modulyatsiya I-turga tegishli deb qarash mumkin;

c) qanday sharoitda modulyatsiya II-tur bo`ladi;

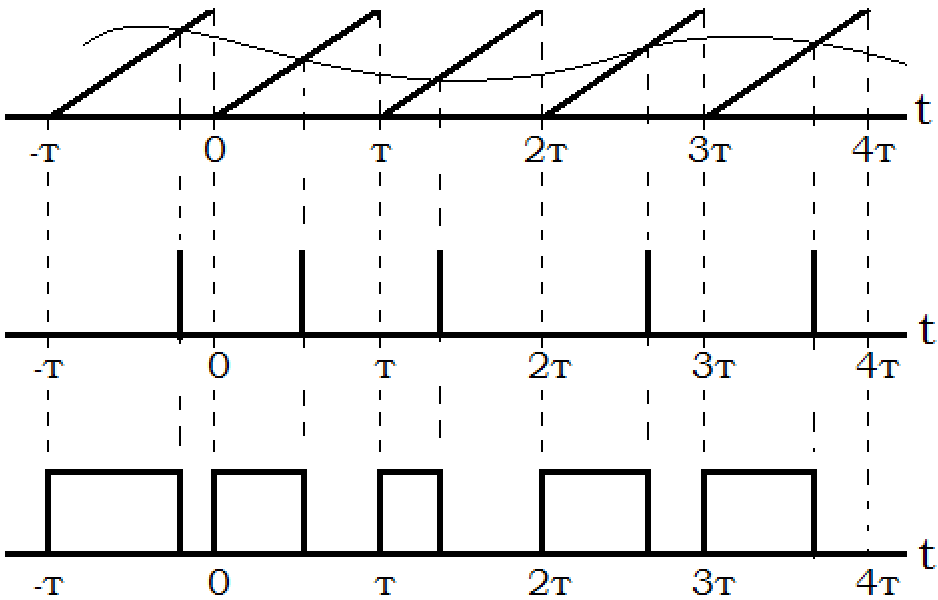
d) statik tavsifi qanday?

3.2.4 Impuls modulyatsiyalash uchun kuchlanishi chiziqli o`zgaruvchi generator (KChO`G) keng qo`llaniladi. Ushbu generatorning chiqishidagi kuchlanish arrasimon shaklga ega (3.3-rasm). Uchburchaklarning boshlag`ich nuqtasi fazaning $0, \pm 2\pi, \pm 4\pi, \ldots$ qiymatlarga to`g`ri keladi. Modulyatsiyalovhi signal $u(t)$ arrasimon signalning chiziqli qismi bilan tenglashgan daqiqada impuls shakllanadi. Impulslarning vaqt diagrammalari 3.3 a) va 3.3 b) chizmalardagi ko`rinishga ega bo`lishi mumkin. Har bir hol uchun

a) modulyatsiyalanuvchi parameter ko`rsatilsin,

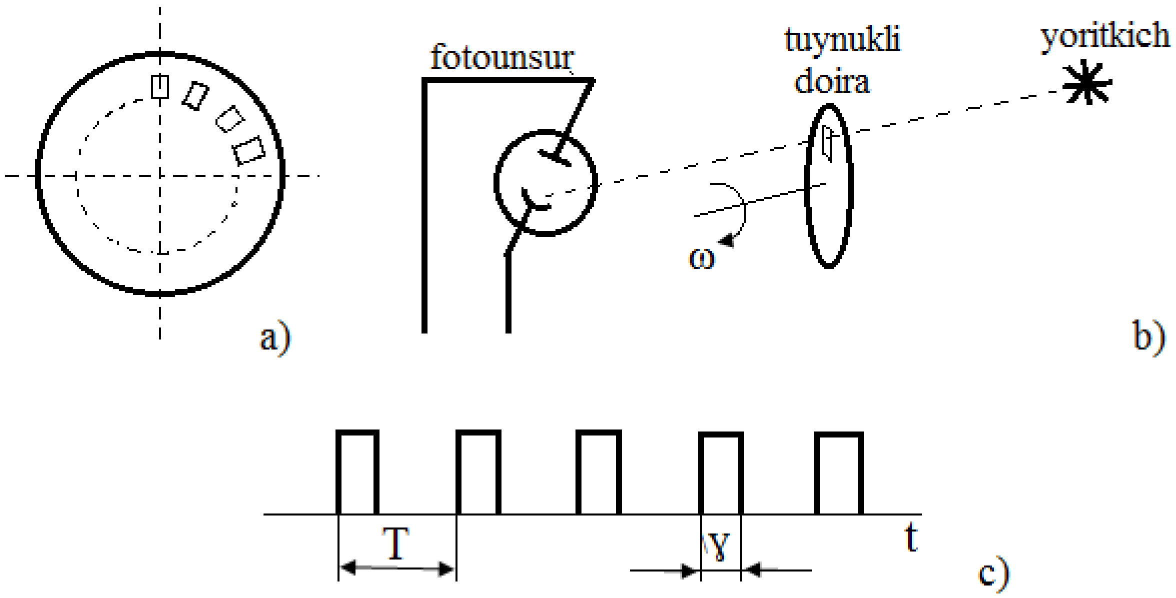
b) modulyatsiya turi aniqlansin,

c) impuls unsirning statik tavsifining ko`rinishi aniqlansin.



3.3-rasm. Impuls unsirning vaqt diagrammalari

3.2.5 Aylanma tezlikni o`lchash uchin 3.4-rasmning a) chizmasida ko`rsatilgan tuynikchali doira qo`llanishi mumkin. Aylanayotgan doira nur manbai bilan fotounsir orasiga shinday joylashtiriladiki, tuynukcha nur yo`liga to`g`ri kelganida fotounsir unga tushgan aktivlashadi (b-chizma).



3.4-rasm. Doirali impuls unsir. a) – konstruksiyasi; b) – ish tartibi;
c) – modulyatsiyalangan impulslar ketma-ketligi.

Quyidagilar aniqlansin:

a) modulyatsiyalovchi sifatida qaysi o`zgarish qatnashmoqda?
b) Qaysi parametrlar modulyatsilanuvchi sifatida qabul qilinishi mumkin?
c) Impuls unsirning statik tavsifi qanday?

3.3 Diskret almashtirishlar

Nazariy kirish

Impuls tizimlarni o`rganishda diskret almashtirishlardan foydalanish qo`l keladi.

D-almashtirish:

$$X^*(p) = \sum_{m=0}^{\infty} x[mT_0]e^{-pmT_0},$$

bu yerda $x[mT_0]$ – panjarasimon funksiyaning diskretalari, T_0 – kvantlash davri.

Z-almashtirish yuqoridagi ifodadan

$$e^{pT_0} = z$$

deb qabul qilinsa hosil bo`ladi.

$$X^*(z) = \sum_{m=0}^{\infty} x[m]z^{-m}.$$

$\mathfrak{D}$ − almashtirish:

$$X^* = \frac{\omega_0}{2\pi} \sum_{k=-\infty}^{\infty} X(p - jk\omega).$$

Bu yerda

$X(p)$ − uzliksiz funksiyaning Laplas tasviri;

$\omega_0 = \frac{2\pi}{T_0}$ − kvantlash chastotasi.

Mashqlar

3.3.1 Yuqoridagi 3.1.1-mashqda olingan panjarasimon funksiyalarning D-tasvir va Z-tasvirlari topilsin.

3.3.2 Integrallovchi va differensiallovchi bo`g`inlarning D va Z sirtlarda uzatish funksiyalari topilsin.

3.3.3 Nodavriy bo`g`inning uzatish funksiyasining ko`rinishi quyidagicha

$$W(p) = \frac{1}{p + a},$$

Diskret uzatish funksiyalariniki esa

$$W^*(p) = \frac{e^{pT}}{e^{pT} - e^{-aT}};$$

$$W^*(z) = \frac{z}{z - e^{-aT}}.$$

Bu yerda: a – bo`g`inning vaqt doimiysiga teskari bo`lgan son; T – diskretlash davri.

Ketma-ket ulangan ikkita har xil vaqt doimiyli nodavriy bog`inlarning diskret uzatish funksiyalari aniqlansin.

Ko`rsatma: ketma-ket ulangan unsirlar tuzilmasidan ekvivalent parallel ulangan unsirlar tuzilmasiga o`tilsin.

3.3.4 Yuqoridagi 3.2.1-mashqda keltirilgan impulslar uchun z-tasvirlar topilsin. Topilgan z-tasvirlardan D-tasvirlar aniqlansin.

3.3.5 Panjarasimon funksiyaning tasviri $F^*(z)$ ma’lum. Shu funksiyaning birinchi va ikkinchi tartibli ayirmalarining tasvirlari aniqlansin.

3.4 Chiziqli impuls tizimlarning tenglamalari

Nazariy kirish

Uzilgan impuls tizimning aslga nisbatan tenglamasi:

$$y[nT] = \sum_{m=0}^{\infty} g[(n-m)T]x[mT].$$

Bu yerda:

$g[nT]$ – vazn funksiya;

$y[nT]$ – chiqish jarayoni;

$x[nT]$ – kirish jarayoni.

D-almashtirish qo`llanilsa

$$Y^*(p) = G^*(p)X^*(p).$$

Uzatish funksiya uchun

$$G^*(p) = \frac{Y^*(p)}{X^*(p)}$$

o`rinli.

Z-almashtirish qollanilsa

$$Y^*(z) = G^*(z)X^*(z),$$

va bundan uzatish funksiya

$$G^*(z) = \frac{Y^*(z)}{X^*(z)}.$$

z-sirt bilan p-sirt orasidagi bog`lanish

$$z = e^{pT}$$

tenglik bilan aniqlanadi.

Yopiq zanjirli tizim uchun uzatish funksiya

$$K^*(p) = \frac{W^*(p)}{1 + W^*(p)},$$

yoki

$$K^*(z) = \frac{W^*(z)}{1 + W^*(z)}$$

ko`rinishga ega.

Asllar sohasida

$$y[nT] = \sum_{s=0}^{n} k[(n-s)T]x[sT],$$

agar

$$x[sT] = \sigma[sT] = \begin{cases} 1, & s = 0 \\ 0, & s \neq 0 \end{cases},$$

unda

$$y[nT] = k[nT].$$

Mashqlar

3.4.1 Uzilgan tizim ketma-ket ulangan sodda (ideal) impuls unsir va uzliksiz dinamik bo`g`indan iborat. Vaqt bo`yicha kvantlash davri $T_0$ songa teng. Uzliksiz qismining uzatish funksiyasi quyidagi ko`rinishga ega:

$a)$ $$W(p) = \frac{k}{p} \ ;$$

$b)$ $$W(p) = \frac{k}{1 + Tp} \ ;$$

$c)$ $$W(p) = \frac{k}{p(1 + Tp)};$$

$d)$ $$W(p) = \frac{k}{(1 + T_1 p)(1 + T_2 p)};$$

$e)$ $$W(p) = \frac{k}{p(1 + T_1 p)(1 + T_2 p)} \ ;$$

$f)$ $$W(p) = \frac{k(1 + T_1 p)}{1 + T_2 p)} \ ;$$

$g)$ $$W(p) = \frac{k(1 + T_1 p)}{p(1 + T_2 p)} \ ;$$

$h)$ $$W(p) = \frac{k}{p^2(1 + Tp)} \, .$$

Tizimning diskret uzatish funksiyasi z-sirtda aniqlansin va tenglamasi tasvirlar orqali ifodalansin.

3.4.2 Avvalgi 3.4.1-mashqdagi uzilgan tizimda qayta aloqa kiritilgandan so`ng hosil bo`ladigan tizimning chiqishi va tafovuti bo`yicha uzatish funksiylari aniqlansin va tasvirlar orqali ifodalansin.

3.4.3 Uzilgan tizim ketma-ket ulangan, galma-gal uchraydigan sodda impuls unsir va uzliksiz dinamik bo`g`inlarning ikki zanjiridan iborat. Ikkala impuls unsirning kvantlash davri T_0 , hamda ular sinxron ishlaydi. Uzliksiz qismlarning uzatish funksiyalarining ko`rinishi yuqoridagi 3.4.1-mashqdagidek. Tizimning diskret uzatish funksiyasi z-sirtda va p-sirtda aniqlansin va tenglamasi tasvirlar orqali ifodalansin.

3.4.4 Avvalgi 3.4.3-mashqdagi uzilgan tizimlarni yopish natijasida hosil bo`lgan qayta aloqali tizimning chiqishi va tafovut bo`yicha uzatish funksiyalari aniqlansin va tenglamasi tasvirlar orqali ifodalansin.

3.4.5 Impuls unsir chiqishida shakli to`g`ri to`rtburchak impulslar bo`lgan hol uchun 3.4.1-mashqdagi topshiriq bajarilsin. Impulslarning kengligi γT_0 . Bu yerda $0 < \gamma < 1$.

3.4.6 Impuls unsir chiqishida shakli teng yonli uchburchak impulslar bo`lgan hol uchun 3.4.1-mashqdagi topshiriq bajarilsin. Impulslarning asosi γT_0 qiymatga ega. Bu yerda $0 < \gamma < 1$.

3.4.7 Avvalgi 3.4.5, 3.4.6-mashqlardagi uzilgan tizimlardan hosil qilingan yopiq tizimning chiqishi va tafovutiga nisbatan tenglamalari tasvirlar orqali ifodalansin.

3.4.8 Yuqoridagi 3.4.1 – 3.4.7-mashqlardagi tizimlarning rekkurent tenglamalari tuzilsin.

3.5 Impuls unsirlar va tizimlarning chastota tavsiflari

Nazariy kirish

Agar jarayonning tasviri p-sirtda berilgan bo`lsa, chastota tavsifni olish uchun $p = j\omega$ deb qabul qilinadi:

$$X^*(p)|_{p=j\omega} = X^*(j\omega).$$

Tasvir z-sirtda berilgan bo`lsa $z = e^{j\omega T}$ almashtirish qo`llaniladi

$$X^*(j\omega) = X^*(z)|_{z=e^{j\omega T}} .$$

Agar jarayonning Laplas tasviri berilgan bo`lsa, $\mathfrak{D}$-almashtirish ifodasidan quyidagini olish mumkin:

$$X^*(j\omega) = \frac{\omega_0}{2\pi} \sum_{k=-\infty}^{\infty} X\big(j(\omega - k\omega_0)\big).$$

Mashqlar

3.5.1 Impuls unsir chiqishida γT_0 kenglikka ega bo`lgan to`g`ri to`rtburchakli impulslar kuzatiladi. Unsirning chastota tavsifi aniqlansin. Chastota tavsif modulining grafigi qurilsin. Bu yerda T_0 − diskretlash davri va $0 < \gamma < 1$.

3.5.2 Impuls unsir chiqishida γT_0 asosga ega bo`lgan teng yonli uchburchak impulslar kuzatiladi. Bu yerda T_0 − diskretlash davri va $0 < \gamma < 1$. Unsirning chastota tavsifi aniqlansin. Chastota tavsif modulining grafigi qurilsin.

3.5.3 Berilgan ikkihadning

$$G_2(z) = a_0 + a_1 z$$

(a_0, a_1) koeffitsientlari mumkin bo`lgan barcha ishoralar kombinatsiyalariga ega bo`lgan hollar uchun chastota tavsiflar qurilsin va o`zaro solishtirilsin.

Ko`rsatma: koeffitsientlar ishoralari to`rt xil kombinatsiyaga ega.

3.5.4 Avvalgi 3.5.3-mashqdagi topshiriq

$$W(z) = \frac{1}{a_0 + a_1 z}$$

Uzatish funksiya uchun bajarilsin.

3.5.5 Quyidagi uzatish funksiyalar orqali ifodalangan tizimlarning chastota tavsiflari topilsin va qurilsin.

$$a)\ \frac{e^{pT_0}}{a_0 + a_1 e^{pT_0}}\ ;$$

$$b)\ \frac{b_0 + b_1 e^{pT_0}}{a_0 + a_1 e^{pT_0} + a_2 e^{2pT_0}}\ ;$$

$$c)\ \frac{b_0}{a_0 + a_1 e^{pT_0} + a_2 e^{2pT_0} + a_3 e^{3pT_0}}\ ;$$

$$d)\ \frac{b_0 + b_1 e^{pT_0} + b_2 e^{2pT_0}}{a_0 + a_1 e^{pT_0} + a_2 e^{2pT_0} + a_3 e^{3pT_0}}\ ;$$

3.6 Chiziqli impuls tizimlar turg`unligining algebraik qoidalari

Nazariy kirish

Impuls tizimning xarakteristik ko`phadi

$$G^*(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_n\ .$$

Shu ko`phadga nisbatan algebraik turg`unlik qoidalarini bevosita qo`llab bo`lmaydi. Bununing uchun avval ko`phadga nisbatan **bichiziqli** almashtirishni qo`llash lozim bo`ladi, yani

$$z = \frac{v+1}{v-1}$$

deb qabul qilish kerak.

Natijada

$$G^*(v) = A_0 v^n + A_1 v^{n-1} + \cdots + A_n$$

ko`phad hosil boladi. Shu ko`phadga nisbatan algebraik turg`unlik qoidalarini, jumladan Gurvits yoki Raus qoidalarini, qo`llash mumkin.

Ikki ko`phad koeffitsientlari o`rtasida quyidagi bog`lanish mavjud.

$$A_\mu = \sum_{\nu=0}^{n} a_\nu \sum_{s=0}^{\nu} \binom{\nu}{s} \binom{n-\nu}{\mu+s} (-1)^s \,,$$

agar $\mu + s > n - \nu$ bo`lsa, unda

$$\binom{n-\nu}{\mu+s} \equiv 0.$$

Mashqlar

3.6.1 Xarakteristik ko`phadi berilgan tizimning turg`unligi Gurvits qoidasiga asosan tekshirilsin.

a) $z - 1$; b) $z^2+0,3z+1$;

c) $z^3 - 0,5z^2+2z - 1$; d) $z^3 + 0,5z^2 - 2z + 1$;

e) $4z^5+3z^4+2z^3+8z^2+10z+3$; f) $4z^5-3z^4+2z^3-8z^2+10z-3$.

e) $7z^5+3z^4 - 4z^3+3z^2+2z+5$; f) $7z^5-3z^4 + 4z^3-3z^2+2z-5$.

3.6.2 Xarakteristik ko`phadi berilgan tizimning turg`unligi Raus qoidasiga asosan tekshirilsin.

a) $z+0,8$; b) $4z^2+2z+1$;

c) $5z^3+z^2-3z+2$; d) $5z^3-z^2+3z+2$;

e) $7z^5+3z^4 - 4z^3+3z^2+2z+5$; f) $7z^5-3z^4 + 4z^3-3z^2+2z-5$.

3.6.3 Ikkinchi tartibli tizimning turg`unlik shartlari xarakteristik ko`phad

$$G^*(z)=a_0+a_1z+a_2z^2$$

koeffitsientlari orqali ifodalansin.

3.6.4 Uchinchi tartibli tizimning turg`unlik shartlari xarakteristik ko`phad

$$G^*(z)=a_0+a_1z+a_2z^2+a_3z^3$$

koeffitsientlari orqali ifodalansin.

3.6.5 Beshinchi tartibli

$$G^*(z)=a_0+a_1z+a_2z^2+a_3z^3+a_4z^4+a_5z^5$$

ko`phad z-sirtdan bichiziqli

$$z=\frac{v+1}{v-1}$$

almashtirishni qo`llab v-sirtga aks ettirilsin.

3.7 Chiziqli impuls tizimlar turg`unligining chastota qoidalari

Nazariy kirish

Impuls tizimning xarakteristik ko`phadi *z*-sirtda:

$$G^*(z)=a_0z^n+a_1z^{n-1}+\cdots+a_{n-1}z+a_n\ ,$$

yoki *p*-sirtda:

$$G_1^*(z)=a_0e^{pn}+a_1e^{p(n-1)}+\cdots+a_{n-1}e^p+a_n\ .$$

Chastota sohasiga o`tish uchun birinchisida $z=e^{j\omega}$, ikkinchisida esa $p=j\omega$ deb qabul qilinadi. U holda

$$G_1^*(j\omega)=a_0e^{j\omega n}+a_1e^{j\omega(n-1)}+\cdots+a_{n-1}e^{j\omega}+a_n$$

ifodaga ega bo`linadi.

Mixaylovning turg`unlik qoidasi impuls tizimlar uchun quyidagi ko`rinishga ega:

$$\Delta arg G_1^*(j\omega)|_{\omega=0}^{\frac{\omega_0}{2}} = \pi n.$$

Bu yerda $\omega_0 -$ kvantlash chastotasi.

Impuls tizimlar uchun, uzliksiz tizimlardan farqli ularoq, Mixaylov godografi *n* kvadrantni emas, balki 2*n* kvadrantni bosib o`tishi kerak.

Naykvist qoidasi impuls tizimlar uchun uzliksiz tizilarnikiga o`xshash. Faqat chastota cheksiz oraliqda emas, balki chekli $[0, \frac{\omega_0}{2}]$ oraliqda o`zgaradi. Bu qoidaning analitik ifodasi quyidagicha:

$$\Delta arg[1 + W_1^*(j\omega)]|_{\omega=0}^{\frac{\omega_0}{2}} = \pi n.$$

Bu yerda $W_1^*(j\omega) -$ uzilgan impuls tizimning chastota tavsifi. Shu chastota tavsifni mo`tadillashgan chastota tavsif orqali ifodalansa, ya'ni

$$W_1^*(j\omega) = \overline{W_1^*}(j\omega),$$

Naykvist qoidasi quyidagi ko`rinishda ifodalanadi:

$$\Delta arg\,[\frac{1}{k} + \overline{W_1^*}(j\omega)]\Big|_{\omega=0}^{\frac{\omega_0}{2}} = \pi n\,.$$

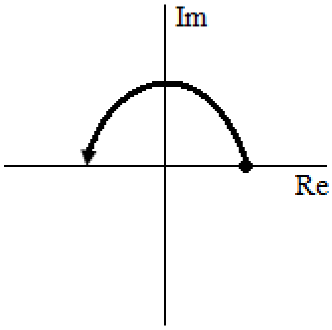
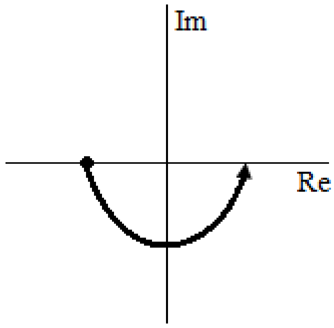
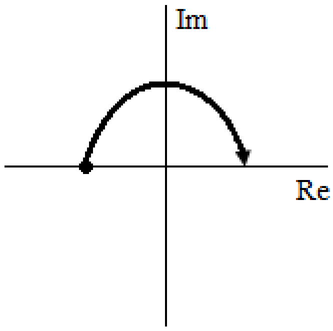
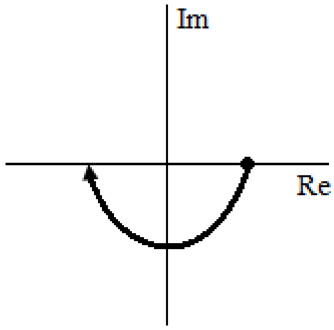
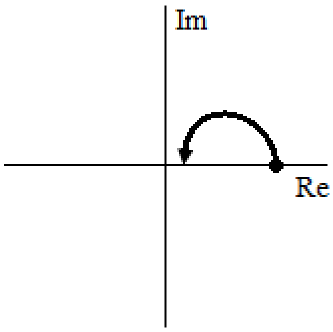
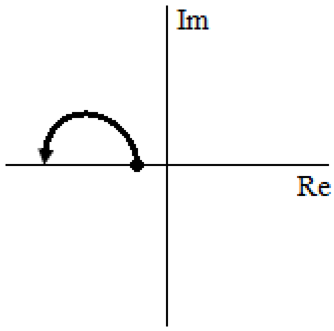
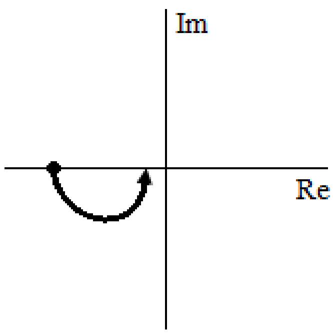
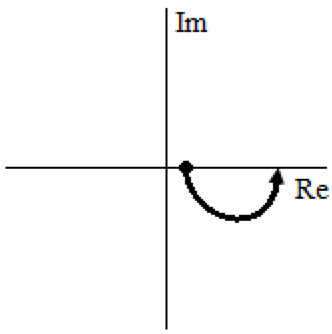
Mashqlar

3.7.1 Yuqoridagi 3.6.1, 3.6.2-mashqlarda keltirilgan impuls tizimlarning xarakteristik ko`phadlari uchun Mixaylov godograflari qurilsin, ichki va tashqi ildizlarning soni aniqlansin.

3.7.2 Avvalgi mashqdagi ko`phadlarga nisbatan bichiziqli almashtirish qo`llab, hosil bo`lgan ko`phadlar uchun Mixaylov godografi qurilsin, o`ng va so`l ildizlar soni aniqlansin. Natijalar 3.7.1-mashqdagi natijalar bilan solishtirilsin.

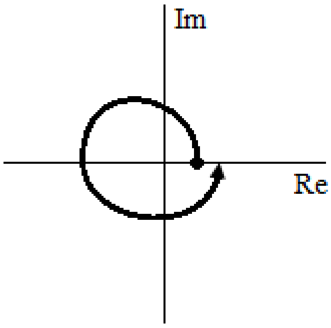
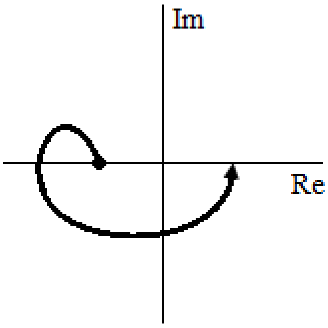
3.7.3 Birinchi tartibli xarakteristik ikkihadning turli ko`rinishlari uchun 3.1-jadvalda keltirilgan Mixaylov godograflari tahlil qilinsin. Bo`g`inning turg`unligi va ikkihadning ko`rinishi haqida xulosalar qilinsin.

3.1-jadval. Ikkihad uchun Mixaylov godograflari

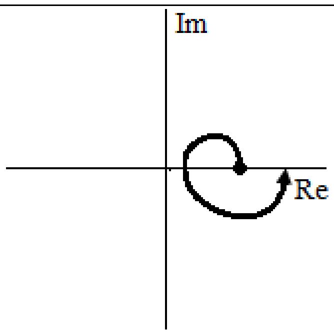
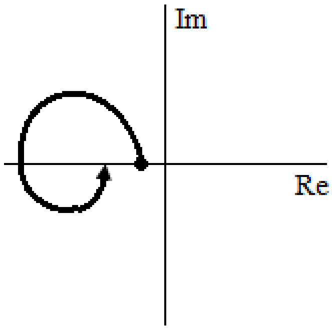
t/r	Godograf	t/r	Godograf
1		2	
3		4	
5		6	
7		8	

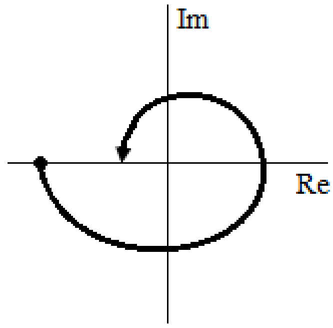
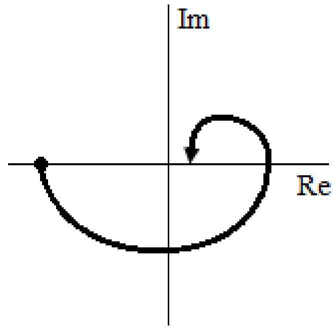
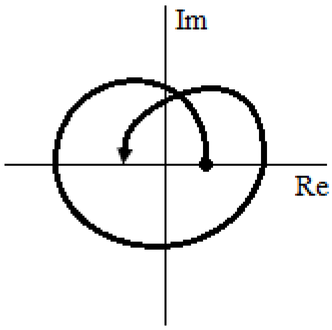
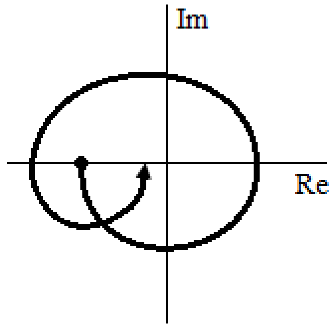
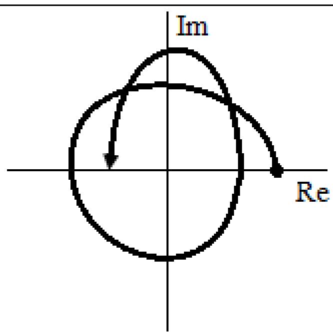
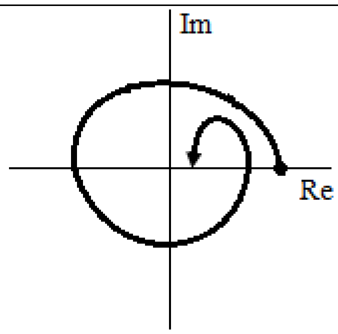
3.7.4 Impuls tizim uchun berilgan (3.2-jadval) Mixaylov godograflaridan tizimning turgunligi, tartibi, ichki va tashqi qutiblar soni haqida xulosalar qilinsin.

3.2-jadval. Impuls tizimlarning Mixaylov godograflari

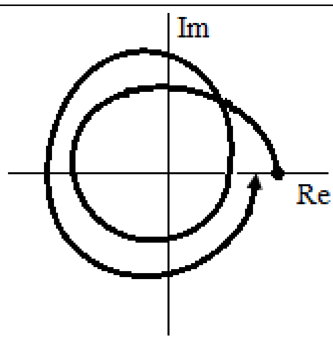
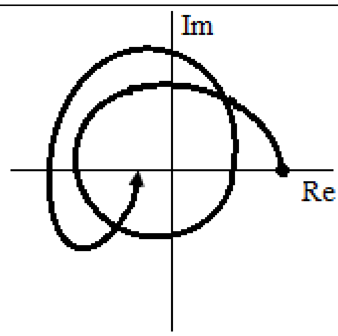
t/r	Godograf	t/r	Godograf
1		2	

3.2-jadval. Impuls tizimlarning Mixaylov godograflari (davomi)

3		4	

5		6	
7		8	
9		10	

3.2-jadval. Impuls tizimlarning Mixaylov godograflari (davomi)

11		12	

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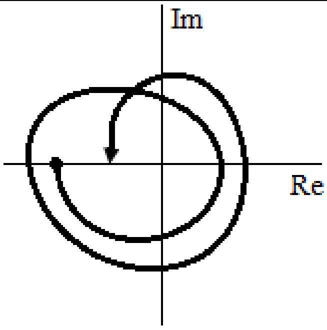

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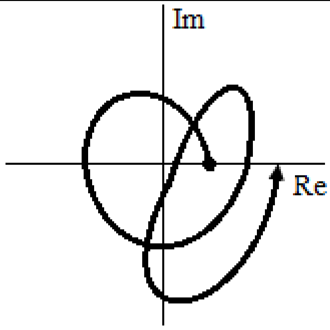

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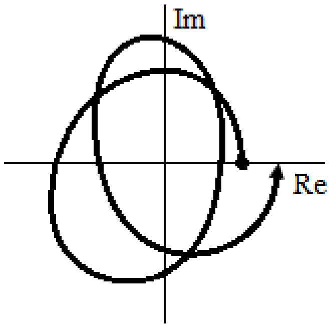

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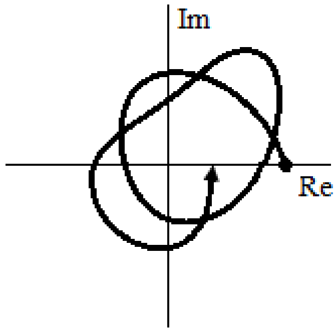

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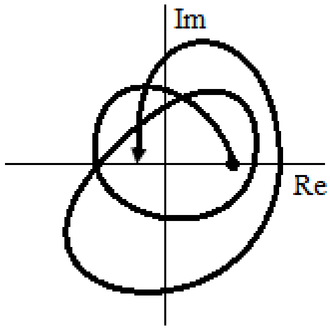

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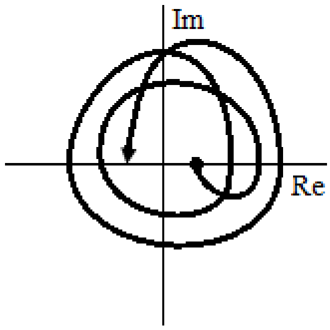

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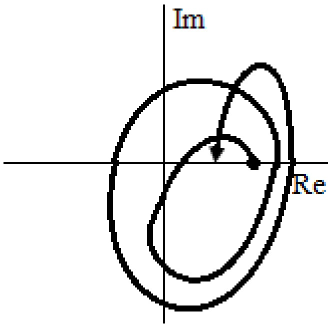

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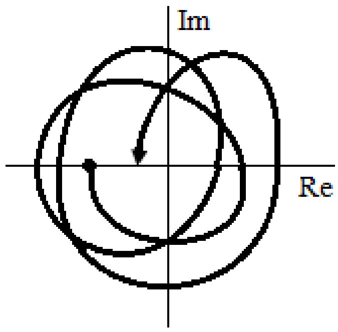


3.7.5 uzilgan tizimning uzatish funksiyasi berilgan. Naykvist qoidasi asosida yopiq tizimning turg`unligi haqida xulosa qilinsin.

$$a)\ \frac{1}{z-1}\ ;\quad b)\ \frac{z}{z-1}\ ;\quad c)\ \frac{1}{z^2+0{,}3z+1}\ ;\quad d)\ \frac{z-1}{z^2+0{,}3z+1}\ ;$$

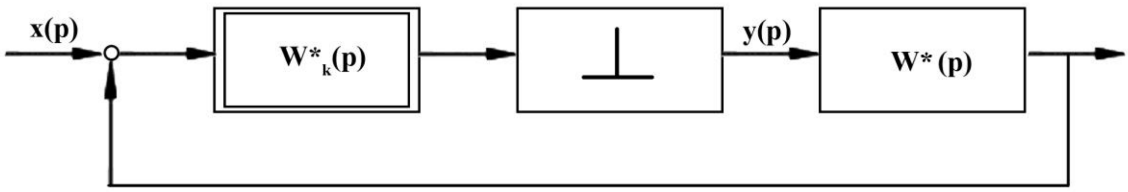
$$e)\ \frac{z^2+z+1}{z^2+0{,}3z+1}\ ;g)\ \frac{z}{z^3-0{,}5z^2+2z-1}\ ;h)\ \frac{z^2+0{,}3z+1}{z^3-0{,}5z^2+2z-1}.$$

3.7.6 Avvalgi 3/7/5-mashqda keltirilgan uzatish funksiyalar uchun yopiq tizim turg`un bo`ladigan statik kuchaytirish koeffitsientining qiymatlar oralig`I aniqlansin.

3.8 Chiziqli impuls tizimlarni korreksiyalash

Nazariy kirish

Impuls tizimlarda ketma-ket korreksiya 3.8.A-rasmdagidek kiritilishi mumkin.



3.8.A-rasm. Ketma-ket korreksiyalash.

Bu yerda :

$W^*(p)=\frac{P^*(p)}{Q^*(p)}-$ dastlabki qismning uzatish funksiyasi,

$W_k^*(p)-$ korreksiyalovchi bo`g`inning uzatish funksiyasi.

Yopiq tizimning dinamikasiga nisbatan qo`yilgan talablar $K_{is}^*(p)$ uzatish funksiya orqali berilgan bo`lsa, korreksiyalovchi bo`g`inning uzatish funksiyasi uchun quyidagi munosabat o`rinli:

$$W_k^*=\frac{1}{W^*(p)}\cdot\frac{K_{is}^*(p)}{1-K_{is}^*(p)}=\frac{Q^*(p)}{P^*(p)}\cdot\frac{K_{is}^*(p)}{1-K_{is}^*(p)}\,.$$

Korreksiyalovchi bo`g`in fizik ro`yobga oshirilanuvchan bo`lishi uchun

$$n - m \leq n_{is} - m_{is}$$

tengsizlikni ta'minlash zarur bo`ladi. Bu yerda n – dastlabki uzatish funksiyaning mahrajining tartibi, m – uning suratining tartibi; $n_{is} -$ istalgan yopiq tizim uzatish funksiyasining mahrajining tartibi, $m_{is} -$ uning suratining tartibi.

Sintez qilingan tizim dag`allik shartlariga javob berishi uchun:

istalgan yopiq tizimning uzatish funksiyasi $\boldsymbol{K_{is}^*(p)}$ no`llari tarkibida dastlabki qismning o`ng no`llari va $(\mathbf{1} - \boldsymbol{K_{is}^*(p)})$ uzatish funksiyaning no`llari tarkibida dastlabki qismning o`ng qutblari bo`lishi kerak.

Bundan:

Korreksiyalovchi bog`inning nollari orasida dastlabki qisimning o`ng qutblariga yaqin, qutblari orasida esa dastlabki qisimning o`ng no`llari yaqinlari bo`lmasligi kerak.

Mashqlar

3.8.1 Qurilanayotgan tizimning dastlabki (o`zgarmas) qismining uzatish funksiyasi berilgan

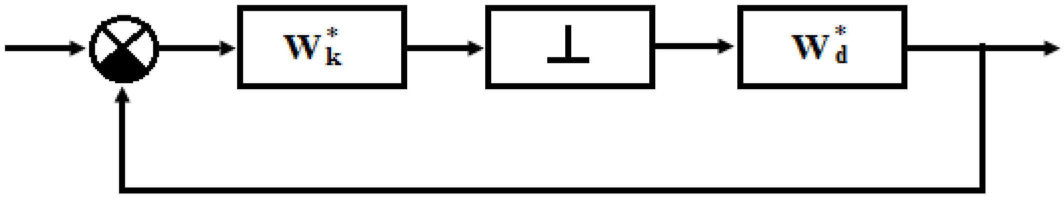
$$a)\ \frac{b_0}{a_0 + a_1 z};\ \ b)\ \frac{b_0}{a_0 + a_1 z + a_2 z^2};\ \ c)\ \frac{b_0 + b_1 z}{a_0 + a_1 z + a_2 z^2};$$

$$d)\ \frac{b_0 z}{(z-1)(a_0 + a_1 z)};\ \ e)\ \frac{b_0 z}{(z-1)(a_0 + a_1 z + a_2 z^2)};\ \ g)\ \frac{z(b_0 + b_1 z)}{(z-1)(a_0 + a_1 z + a_2 z^2)}.$$

Fizik ro`yobga oshirish shartini nazarda tutib, istalgan yopiq tizimning uzatish funksiyasining ko`rinishi taklif etilsin.

3.8.2 Avvalgi 3.8.1-mashqdagi o`zgarmas qisimlar uchun dag`allik shartini qanoatlantiradigan istalgan yopiq tizimning uzatish funksiyasining ko`rinishi taklif qilinsin.

3.8.3 Korreksiyalovchi bo`g`in 3.5-rasmdagidek kiritiladi deb qabul qilingan hol uchun, 3.8.1 va 3.8.2-mashqlardagi istalgan yopiq tizimning uzatish funksiyasi asosida diskret korreksiyalovchi bo`g`inning uzatish funksiyasi taklif etilsin.



3.5-rasm. Korreksiyalangan tizim

IV Nochiziq tizimlar

4.1 Statik nochiziq unsirlar tavsiflari

Nazariy kirish

Statik nochiziq unsirlarning tavsifi, umuman olganda, qandaydir nochiziq funksiyadir:

$$z = \Phi(u) .$$

Bu yerda u – unsirning kirishi; z – unsirning chiqishi. Funksuya $\Phi(\cdot)$ amalda yoki matematik ifoda, yoki grafik ko`rinishda berilishi mumkin. Bu tavsifni qo`llash jarayonida bir ko`rinishdan boshqasiga o`tish zarur bo`lishi mumkin.

Tavsif matematik ifoda ko`rinishida berilgan bo`lsa, undan grafik ko`rinishga o`tish axamiyatli qiyincvhiliklar yaratmaydi. Buning uchun erkin o`zgaruvchiga turli qiymatlar berib, funksiyaning mos qiymatlari hisoblab topiladi va ular koordinata tizimiga tushirilib, grafik hosil qilinadi.

Grafikdan matematik ifodaga o`tish uchun approksimatsiya usullaridan foydalaniladi. Buning uchun odatda funksiyaning aniqlanish sohasi Ω nimsohalarga $\Omega_i$,  $i = \overline{1,n}$ bo`linadi. Quyidagi shartlar bajarilishi lozim

$$\bigcap_{i=1}^{n} \Omega_i = \emptyset\,,$$

$$\bigcup_{i=1}^{n} \Omega_i = \Omega\,.$$

Umumiy matematik ifoda har bir nimsoha va yondosh nimsohalarning chegaralari uchun alohida matematik ifodalar ko`rinishida beriladi.

$$z = \Phi(u) = \begin{cases} f_1(u), & u \in \Omega_1 \\ f_2(u), & u \in \Omega_2 \\ \dots\dots\dots\dots\dots\dots\dots. & \\ f_n(u), & u \in \Omega_n \end{cases},$$

Indicator funksiyalardan foydalanilsa, keltirilgan ifodani quyidagi ko`rinishda yozish mumkin.

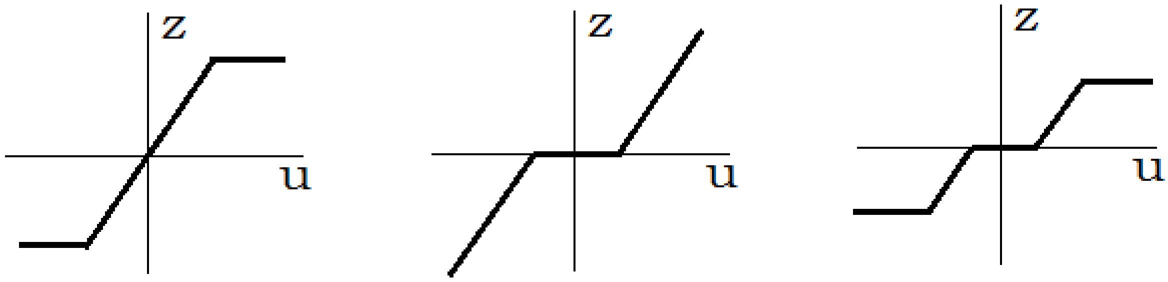
$$z = \Phi(u) = 1(\Omega_1)f_1(u) + 1(\Omega_2)f_2(u) + \cdots + 1(\Omega_n)f_n(u).$$

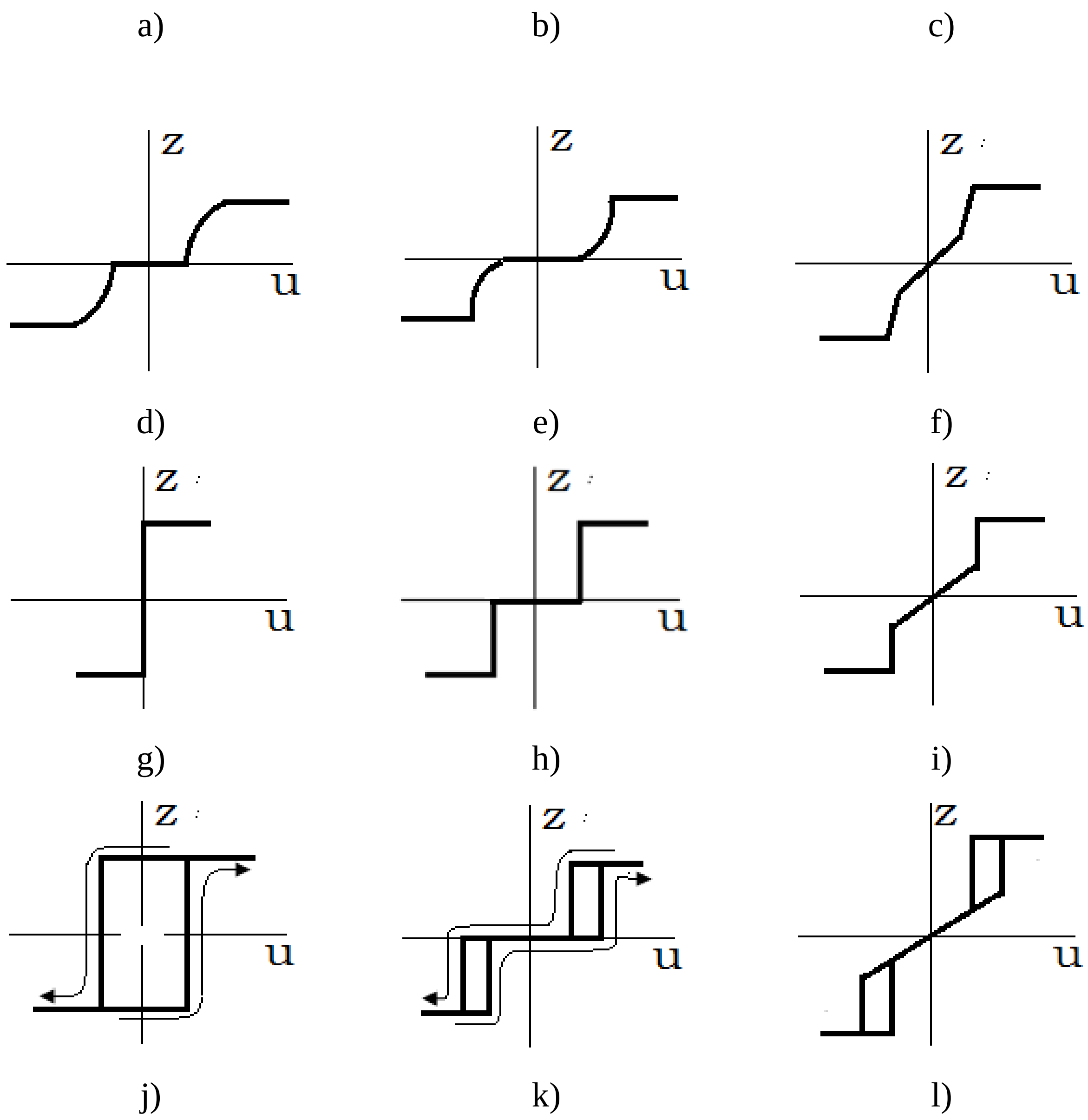
Bu yerda

$$1(\Omega_i) = \begin{cases} 1, & u \in \Omega_i \\ 0, & u \notin \Omega_i \end{cases}.$$

Mashqlar

4.1.1 Nochiziq unsirlarning toq simmetrik statik tavsiflari chizma k`rinishida 4.1-rasmda tasvirlangan. Lozim bo`lgan parametrlar kiritilsin va tavsiflar matematik bog`lanishlar orqali ifodalansin.





4.1-rasm. Nochiziq unsirlar tavsiflari.

4.1.2 Matematik munosabatlar orqali ifodalangan statik tavsiflarning chizmalari qurilsin.

a) $z = \begin{cases} |u|^q, & |u| \le a \\ a^q, & |u| > a; \end{cases} \quad q = 1,2,3, \ldots$

b) $z = \begin{cases} u^q\,, & 0 \le u \le a \\ -|u|^q\,, & 0 \le -u \le a \\ a^q\,, & u > a \\ -a^q\,, & u < -a \end{cases}$

c) $z = \begin{cases} 0, & |u| \le a_1 \\ ku - b, & a_1 < u \le a_2 \\ ku + b, & -a_1 > u \ge -a_2 \\ ka_2 + b, & u > a_2 \\ -ka_2 - b, & u < -a_2 \end{cases}$

d) $z = \begin{cases} 0, & |u| \le a_1 \\ |ku - b|\,, & a_1 < |u| \le a_2 \\ |ka_2 - b|\,, & |u| > a_2 \end{cases}$

e) $z = \begin{cases} 0, & |u| \le a \\ k\sqrt{|u| - a}, & |u| > a \end{cases}$

f) $z = \begin{cases} 0, & |u| \le a \\ k\sqrt{u - a}\,, & u > a \\ -k\sqrt{|u| - a}\,, & u < -a \end{cases}$

g) $z = \begin{cases} 0, & |u| \le a \\ \log(|u| - a + 1), & |u| > a \end{cases}$

h) $z = \begin{cases} 0, & |u| \le a \\ \log(u - a + 1)\,, & u > a \\ -\log(|u| - a + 1)\,, & u < -a \end{cases}$.

4.1.3 Har qanday o`lchash asbobi sezuvchansizlik oralig`iga hamda quyidan va yuqoridan cheklashga ega bo`lishi tushintirib va asoslab berilsin. Statik tavsifi sifat jihatdan tasvirlansin.

4.1.4 Avtomatik boshqarish tizimlarda qo`llaniladigan quyidagi unsirlarning statik tavsiflari asoslansin va sifat jihatdan tasvirlansin.

a) o`zgarmas tok motori;

b) qisqa ulangan rotorli asinxron motor;

c) ikki fazali asinxron motor;

d) silindrsimon gidrokuchaytirgich;

e) electron kuchaytirgich;

f) nasos;

g) kuchlanishni barqarorlashtiruvchi;

h) suv to`sqichlar.

4.2 Statik nochiziq unsirlarni o`zaro ulash

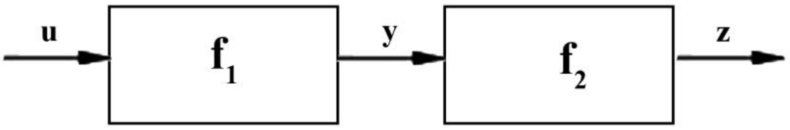
Nazariy kirish

Ikkita statik nochiziq unsirlar o`z tavsiflari

$$z_1 = f_1(u),$$
$$z_2 = f_2(u)$$

bilan berilgan.

1. **Ketma-ket ulash** (4.2.A-rasm)



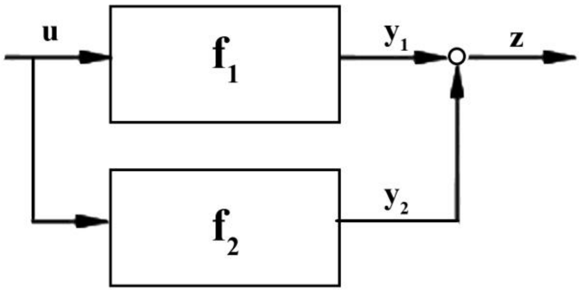
4.2.A-rasm. Ketma-ket ulangan unsirlar.

Natijaviy bog`lanish kompozitsiya ko`rinishiga ega

$$z = f_2(y) = f_2\big(f_1(u)\big).$$

Bir o`lchamli unsirlar uchun keltirilgan amal gravik usulda onson bajariladi.

2. **Parallel ulash** (4.2.B).

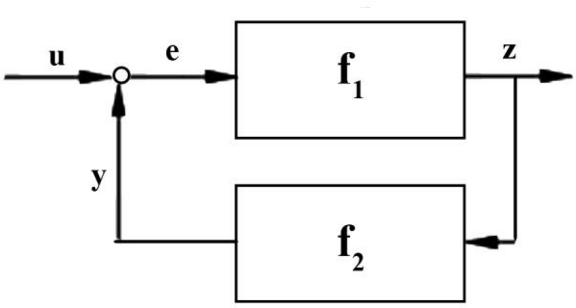


4.2.B-rasm. Parallel ulangan unsirlar.

Natijaviy bog`lanish analitik usulda ham, grafik usulda ham onson topiladi:

$$z = f_1(u) \pm f_2(u).$$

3. **Qayta aloqali (teskari parallel) ulash** (4.2.C-rasm).



4.2.C-rasm. Teskari parallel ulangan unsirlar.

Natijaviy bog`lanishni ikki yo`l bilan topish mumkin.

I. Parallel ulashga keltirish:

$$u = \Phi^{-1}(z) = f_1^{-1}(z) \pm f_2(z).$$

Bu ifodada minus ishora musbat qayta aloqaga, plyus ishora esa manfiy qayta aloqaga to`g`ri keladi. Chiqishni kirishdan bog`liqligini aniqlash uchun, topilgan funksiyani teskarisini chiqazish kerak.

II. Chiqishga nisbatan quyidagi bog`lanishlar o`rinli

$$z = f_1(e),$$

$$z = f_2^{-1}(y),$$

$$z = \Phi(u) = \Phi(e \pm y).$$

Bular orqali grafik usulda $\Phi(u)$ funksiyani qurish mumkin. Buning uchun chiqishga (z) har xil qiymatlar berib, ularga mos keladigan y va e qiymatlari aniqlanadi. Bulardan esa, chiqishning tanlangan qiymatiga mos keladigan kirish qiymati $u = e \pm y$ aniqlanadi. Bu yerda ham minus ishora musbat qayta aloqaga, plyus ishora esa manfiy qayta aloqaga mos keladi.

Mashqlar

4.2.1 Yuqorida, 4.1.1-mashqda, keltirilgan nochiziq unsirlardan juft-juft tanlab olib, ularni ketma-ket ulashdan hosil bo`ladigan nochiziq unsirning statik tavsifi topilsin. Unsirlarni ketma-ket ulanishi kommutativlik xossaga ega emasligiga ichonch hosil qilinsin.

4.2.2 Yuqorida, 4.1.1-mashqda, keltirilgan yagona qiymatli nochiziq unsirlarga nisbatan teskari bo`lgan unsirlarning tavsiflari qurilsin.

4.2.3 Yuqorida, 4.1.1-mashqda, keltirilgan nochiziq unsirlardan juft-juft tanlab olib, ularni musbat va manfiy parallel ulashdan hosil bo`ladigan unsirning statik tavsifi topilsin.

4.2.4 Yuqorida, 4.1.1-mashqda, keltirilgan nochiziq unsirlardan biri va chiziqli statik unsir teskari parallel (qayta alqali) ulanganidan hosil bo`ladigan unsirning statik tavsifi topilsin:

a) to`g`ri yo`lda nochiziq unsir, qayta aloqa yo`lida chiziqli unsir;

b) to`g`ri yo`lda chiziqli unsir, qayta aloqa yo`lida nochiziq unsir.

Har ikkala hol uchun manfiy va musbat qayta aloqa ko`rilsin.

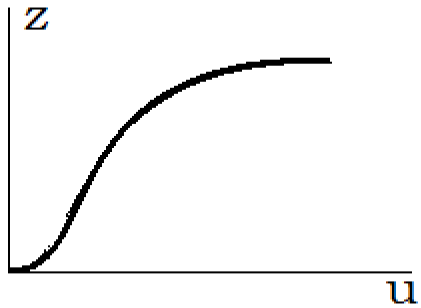
4.2.5 Yuqorida, 4.1.1-mashqda, keltirilgan nochiziq unsirlardan juft-juft tanlab olib, ulardan manfiy va musbat teskari parallel ulashdan hosil bo`ladigan unsirning statik tavsifi topilsin.

4.2.6 Funksional almashtirgichning nochiziq statik tavsifi berilgan (4.2-rasm). Kirish bilan chiqish orasidagi bog`lanish $y = ku$ ko`rinishga ega bo`lishi uchun qanday tavsifli unsirni berilgan bilan:

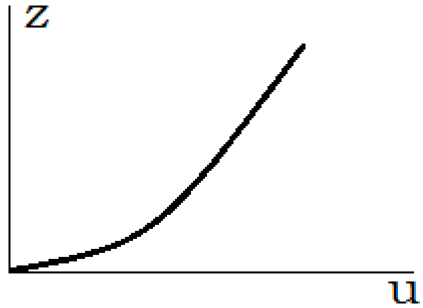
a) ketma-ket,

b) parallel

ulash lozim?



1)



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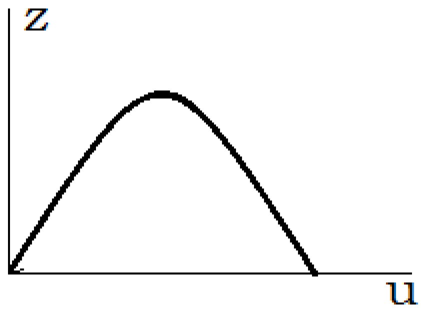

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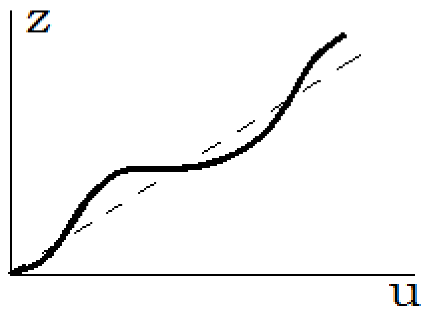


4)

4.2-rasm. Funksional almashtirgichlarning statik tavsiflari

4.3 Statik nochiziq unsirlarni chiziqlantirish

Nazariy kirish

Statik chiziqlantirish. Berilgan:

nochiziq bog`lanish $z = \Phi(u)$,

ish oralig`i $[u_1, u_2]$,

o`zgarish oralig`i $[z_1, z_2]$.

$$z_1 = \Phi(u_1); \quad z_2 = \Phi(u_2).$$

Chiziqli model

$$\overline{z} = k^{(s)}u + b^{(s)}$$

koeffitsientlari quyidagicha aniqlanadi.

$$k^{(s)} = \frac{z_1 - z_2}{u_1 - u_2} \;;$$

$$b^{(s)} = \frac{u_1 z_2 - u_2 z_1}{u_1 - u_2}.$$

Differensial chiziqlantirish. Berilgan:

nochiziq bog`lanish $z = \Phi(u)$,

ish nuqtasi $z_0 = \Phi(u_0)$.

Chiziqli model

$$\overline{z} = k^{(d)} u + b^{(d)}$$

koeffitsientlari quyidagicha aniqlanadi.

$$k^d = \left.\frac{d\Phi(u)}{du}\right|_{u=u_0} \equiv \frac{d\Phi(u_0)}{du}.$$

$$b^{(d)} = z_0 - \frac{d\Phi(u_0)}{du} \cdot \mathrm{u}_0 \,.$$

Garmonik chiziqlantirish. Berilgan:

nochiziq bog`lanish $z = \Phi(u)$,

garmonik kirish jarayoni $u(t) = A \sin \omega t$.

Chiziqli model

$$\overline{z} = A[k_s^{(g)} \sin \omega t + k_c^{(g)} \cos \omega t]$$

koeffitsientlari quyidagicha aniqlanadi.

$$k_s^{(g)} = \frac{1}{A\pi} \int_0^{2\pi} \Phi(A \sin \omega t) \sin \omega t d\omega t \;;$$

$$k_c^{(g)} = \frac{1}{A\pi} \int_0^{2\pi} \Phi(A \sin \omega t) \cos \mathrm{\omega t} \, d\omega t \,.$$

Agar chiqish modeli

$$\overline{z} = A k^{(g)} \sin(\omega t + \psi(A))$$

ko`rinishda bo`lsa, uning parametrlari oldingisi bilan

$$k^{(g)} = \sqrt{\left(k_s^{(g)}\right)^2 + \left(k_c^{(g)}\right)^2} \,,$$

$$\psi(A) = arctg\frac{k_c^{(g)}}{k_s^{(g)}}$$

ko`rinishda bog`langan.

Stoxastik (statistik) chiziqlantirish. Berilgan:

nochiziq bog`lanish $z = \Phi(u)$,

statsionar tasodifiy kirish jarayoni *u(t)*.

Chiziqli model

$$\bar{z} = k^{(st)}u + b^{(st)}$$

koeffitsientlari quyidagicha aniqlanadi.

$$k^{(st)} = \frac{\overline{yu} - \overline{y}\overline{u}}{\overline{u^2} - \overline{u}^2}\ ;$$

$$b^{(st)} = \frac{\overline{y}\overline{u^2} - \overline{yu}\overline{u}}{\overline{u^2} - \overline{u}^2}\,.$$

Bu yerda harflar ustidagi to`g`ri chiziqlar matematik kutilma amalining belgisidir.

Mashqlar

4.3.1 Nochiziq statik tavsif berilgan ush nuqtasi u_0 atrofida differensial chiziqlantirilsin va chiziqlantirishdan hosil bo`lgan xatolik baholansin.

$a)\ z = ku^2\ ;\qquad b)\ z = ku^3\ ;$

$c)\ z = k\sqrt{u}\ ;\qquad d)\ z = k\sqrt[3]{u}\ ;$

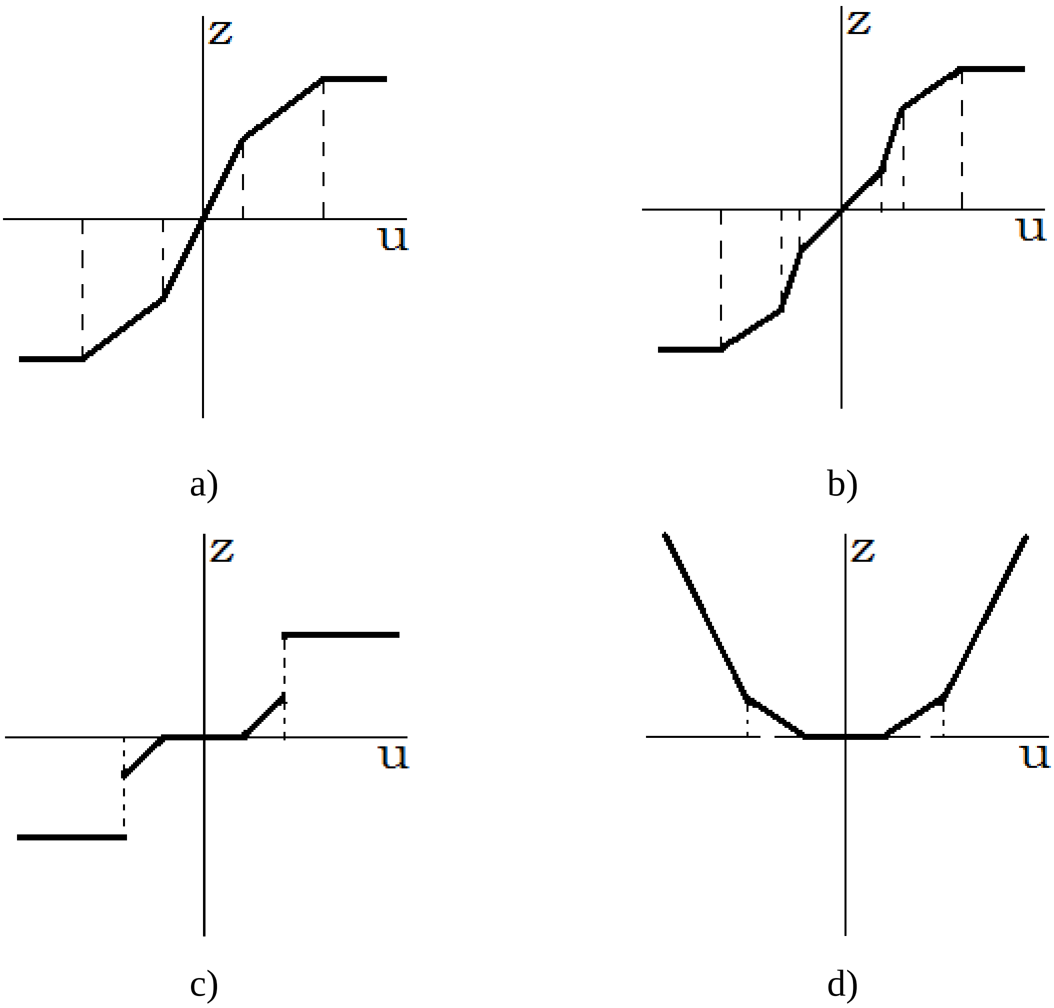
$e)\ z = ke^{\lambda u}\ ;\qquad f)\ z = klog_a(u + c);$

$g)\ z = kSin(au);\qquad h)\ z = ktg(au).$

4.3.2 Yuqoridagi, 4.3.1-mashqdagi, nochiziq unsirlarning statik tavsifi, berilgan ish oralig`i $(u_1, u_2)$ uchun statik chiziqlantirilsin. Chiziqlantirishdan hosil bo`lgan xatolikning ish oraligidagi maksimal qiymati topilsin.

4.3.3 Nochiziq statik tavsif (4.3-rasm) berilgan ish oralig`i $(u_1, u_2)$ uchun statik chiziqlantirilsin. Ish oralig`idagi maksimal xatolik topilsin.

Ilova. Ish oralig`ining chegaraviy qiymatlarini o`qituvchi beradi.



4.3-rasm. Statik tavsiflar

4.3.4 Ruxsat etilgan xatolik Δ ma’lum bo`lsa, 4.3.1-mashqdagi nochiziq unsirlar uchun differensial chiziqlantirish natijasida olingan modelning ruxsat etilgan ish oralig`i aniqlansin.

4.3.5 Ruxsat etilgan maksimal xatolik Δ va ish oralig`ining quyi chegarasi $u_1$ ma’lum bo`lsa, 4.3.2, 4.3.3-mashqlardagi nochiziq unsirlar uchun static chiziqlantirish natijasida olingan modelning ruxsat etilgan ish oralig`ining yuqori chegarasi aniqlansin.

4.3.6 Kirish signali

$$u(t) = a_0 + ASin\omega_0 t$$

Bo`lganida, 4.1.1-mashqda keltirilgan nochiziq unsirlar garmonik chiziqlantirilsin.

4.3.7 Avvalgi, 4.3.6-mashqda, chiziqlantirish natijasidan foydalanib, kompleks chiziqlantirish koeffitsienti $k_g(A)$ topilsin va $(-1/k_g(A))$ ifodaning godografi kompleks sirtda qurilsin.

4.3.8 Nochiziq unsirning kirishiga tasodifiy signal

$$u(t) = m_0 + \xi(t)$$

uzatilganida, 4.1.1-mashqda keltirilgan yagona qiymatli statik tavsifga ega bo`lgan nochiziq unsirlar stoxastik chiziqlantirilsin. Bu yerda: $m_0 -$ o`zgarmas son, $\xi(t) -$ markazlashtirilgan tasodifiy jarayon.

4.4 Nochiziq tizimlarning muvozanat holatlari

Nazariy kirish

Gammershteyn shaklida ifodalangan tizimdagi majburiy tafovutga nisbatan tenglama quyidagi ko`rinishga ega:

$$u^M(t) = x(t) - \int_0^\infty w(\tau)\Phi\big(u^M(t-\tau)\big)d\tau\,.$$

Bu yerda

$x(t) -$ yopiq tizimning kirish ta’siri;

$w(\tau) -$ chiziqli qismning vazn funksiyasi;

$\Phi(\cdot) -$ statik nochiziq unsirning tavsifi.

Kirish ta’siri no`lga teng bo`lganda tizimda hosil bo`ladigan majburiy harakatga avtonom harakat deyiladi, ya’ni

$$u^a(t) = -\int_0^\infty w(\tau)\Phi\big(u^a(t-\tau)\big)d\tau.$$

Bu tenglamaning o`zgarmas qiymatga, ya’ni $u^a(t) \equiv const$, teng bo`1gan yechimlariga tizimning muvozanat holatlari deyiladi. Bunday hol uchun tenglama quyidagi sodda ko`rinishga keladi:

$$\Phi(u^a) = -\frac{u^a}{W(0)}.$$

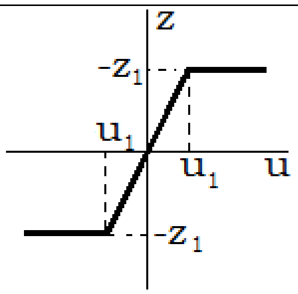
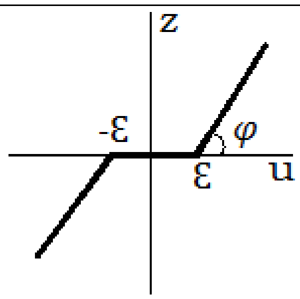
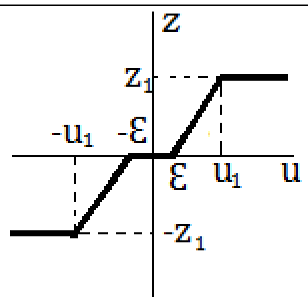
Bu yerda $W(0) = W(p)|_{p=0}$, ya’ni chiziqli qismning statik kuchaytirish koeffitsienti. Tizimda musbat qayta aloqa qo`llanilgan bo`lsa, tenglamadagi minus yo`qoladi.

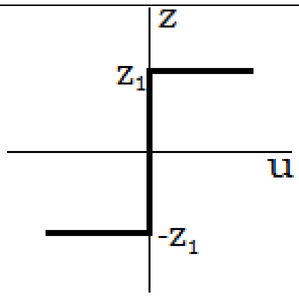
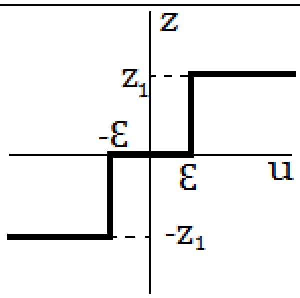
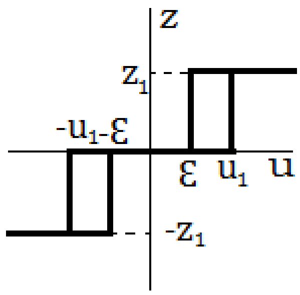
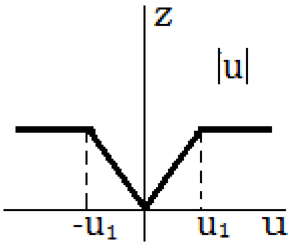
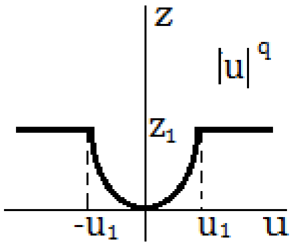
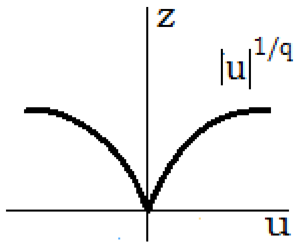
Keltirilgan tenglamaning yechimlarini, demak muvozanat holatlarini, grafo-analitik usulda topish ancha qulay.bu yechimlar tenglamaning chap tomonidagi nochiziq statik tavsifning grafigi bilan, o`ng tomonda ifodalangan to`g`ri chiziq kesishgan nuqtalar ko`rinishida aniqlanadi.

Mashqlar

4.4.1 Gammershteyn tuzilmasiga ega bo`lgan tizimning nochiziq unsirining tavsifi, hamda chiziqli qismining uzatish funksiyasi berilgan (4.1.-jadval). Manfiy va musbat qayta aloqa qo`llangan hollar uchun, kirish signali $x(t) \equiv 0$ bo`lganda, tizimdagi muvozanat holatlar topilsin.

4.1-jadval a) Nochiziq statik tavsiflar

t/r	Tavsif	t/r	Tavsif	t/r	Tavsif
1		2		3	

4		5		6	
7		8		9	

4.1-jadval b) Chiziqli qismning uzatish funksiyalari

t/r	Uzatish funksiya	t/r	Uzatish funksiya
1	$\dfrac{k}{1+Tp}$	2	$\dfrac{k}{p(1+Tp)}$
3	$\dfrac{k}{1+a_1p+a_2p^2}$	4	$\dfrac{k(1+Tp)}{1+a_1p+a_2p^2}$
5	$\dfrac{k(1+Tp)}{p(1+a_1p+a_2p^2)}$	6	$\dfrac{\sum_{i=0}^{m} b_ip^i}{\sum_{i=0}^{n} a_ip^i}$
7		8	$ke^{-p\tau}$

	$\dfrac{\sum_{i=0}^{m} b_i p^i}{p^\nu \sum_{i=0}^{n} a_i p^i}$		
9	$\dfrac{e^{-p\tau} \sum_{i=0}^{m} b_i p^i}{\sum_{i=0}^{n} a_i p^i}$	10	$\dfrac{e^{-p\tau} \sum_{i=0}^{m} b_i p^i}{\sum_{i=0}^{n} a_i p^i + e^{-p\tau} \sum_{i=0}^{m} b_i p^i}$

4.5 Nochiziq tizimdagi muvozanat holatga tashqi bezovtalovchining ta'siri

Nazariy kirish

Kirish ta'siri no`ldan farqli o`zgarmas qiymat qabul qilgan bo`lsa, $x(t) = c = const$, tizimning muvozanat holati quyidagi tenglamani qanoatlantiradi:

$$\Phi(u^a) = \frac{c - u^a}{W(0)}.$$

Agar tizimda musbat qayta aloqa qo`llanilgan bo`lsa, tenglamaning o`ng tomonidagi ifodada minus belgisi plyus belgiga almashtirilishi lozim.

Keltirilgan tenglamaning yechimlarini grafo-analitik usulda topish qo`l keladi.

Mashqlar

4.5.1 Yuqoridagi 4.4.1-mashq shartida kirish signali noldan farqli o`zgarmas songa, yani $x(t) = c$, teng bo`lganida tizimdagi muvozanat holatlari topilsin.

4.5.2 Yuqoridagi 4.1-jadval ma'lumotlariga mos keladigan Gammershteyn tuzilmasiga ega bo`lgan tizimda talab etilgan $(u_0, z_0)$ muvozanat holatini ta'minlaydigan kuchaytirish koeffitsienti $k$ va o`zgarmas kirish ta'siri c aniqlansin.

4.5.3 Avvalgi mashq shartlari bajarilganida talab etilgan (u_0, z_0) muvozanat holatidan boshqa muvozanat holatlar topilsin.

4.6 Avtotebranishlar

Nazariy kirish

Tizimda garmonik avtotebranish hosil bo`lish sharti

$$-\frac{1}{\overrightarrow{k^{(g)}}(A)} = W(j\omega)$$

garmonik balans munosabati deb nomlanadi. Bu yerda $W(j\omega) -$ chiziqli uzliksiz qismning chastota tavsifi. Kompleks garmonik chiziqlantirish koeffitsienti $\overrightarrow{k^{(g)}}(A)$ uchun quyidagi munosabatlar o`rinli:

$$\overrightarrow{k^{(g)}}(A) = k_s^{(g)}(A) - jk_c^{(g)}(A) = k^{(g)}(A)e^{-j\psi(A)} .$$

Alohida garmonik chiziqlantirish koeffitsientlari uchun quyidagi ifodalardan foydalaniladi:

$$k_s^{(g)} = \frac{1}{\pi A}\int_0^{2\pi} \Phi(A\sin\omega t)\sin\omega t d\omega t;$$

$$k_c^{(g)} = \frac{1}{\pi A}\int_0^{2\pi} \Phi(A\sin\omega t)\cos\omega t d\omega t;$$

$$k^{(g)}(A) = \sqrt{\left(k_s^{(g)}(A)\right)^2 + \left(k_c^{(g)}(A)\right)^2};$$

$$\psi(A) = arctg\frac{k_c^{(g)}}{k_s^{(g)}}.$$

Garmonik abtotebranish mavjudligini bilish uchun kompleks sirtga chiziqli qismning chastota tavsifi $W(j\omega)$ chiziladi. Undan alohida holda garmonik chiziqlantirish koeffitsientiga teskari bo`lgan miqdor manfiy ishora bilan chiziladi. Ikkala egri chiziq o`zaro kesishsa, shu nuqta avtotebranishning ω_a chastotasini chastota tavsifdan belgilaydi, A amplitudasini esa garmonik chiziqlantirish koeffitsientini manfiy teskari qiymati belgilaydi. Bu egri chiziqlar o`zaro kesishmasa qidirilgan shakldagi avtotebranish mavjud bo`lmaydi.

Avtotebranish mavjud bo`lsa, uning turg`un-noturg`unligi tegishli usullar yordamida alohida tekshirilishi mumkin.

Mashqlar

4.6.1 Statik tavsifi 4.1-jadvaldagi 1 – 6 ko`rinishga ega bo`lgan nochziq unsirlarning garmonik chiziqlantirish koeffitsientlari topilsin.

4.6.2 Nochiq unsiri bir qiymatli, uzliksiz qismi esa nodavriy bo`lgan tizimda avtotebtanish bo`lmasligi ko`rsatilsin va tushintirilsin.

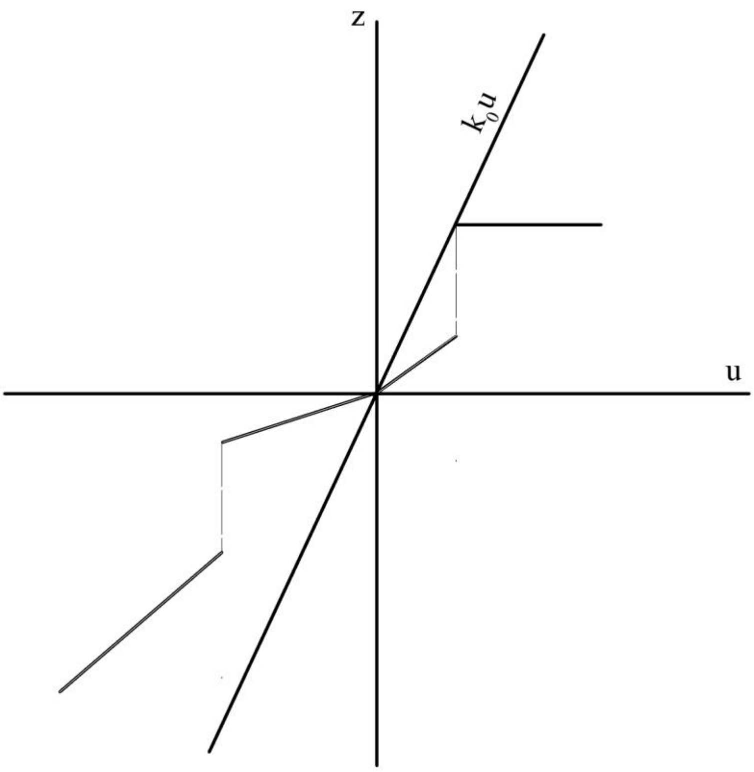
4.6.3 Yuqoridagi 4.4.1-mashqda keltirilgan 4.1-jadvaldagi 1 – 6-nochiziq unsirlar va uzliksiz qismlarning uzatish funksiyalar parametrlari tanlangan son qiymatlarga teng deb qabul qilingan holler uchun avtotebranishlar mavjudligi tekshirilsin. Avtotebranishlarning amplituda va chastotasi baholansin.

4.7 Statik tavsiflarning joylashish sektorlari

Nazariy kirish

Absolyut turg`unlik masalasi alohida nochiziq unsirga $\Phi(\cdot)$ nisbatan emas, nochiziq unsirlarning sinfiga $\Phi(\cdot) \in \mathcal{K}$ nisbatan qo`yiladi. Ko`p hollarda bunday sinf nochiziq tavsiflar sirtida koordinata boshidan o`tadigan to`g`ri chiziqlar orqali ajratiladi. Bu tog`ri chiziqlar bilan ajratilgan soha nochiziq tavsifni to`la-to`kis qamrab oladigan sektorlar ko`rinishiga ega.

Sektorlarni hosil qiladigan to`g`ri chiziqlarning og`ish koeffitsientlari nochiziq tavsifning o`zgarish sohasi, hosilalarining maksimal va minimal qiymatlari, sinish nuqtalari bilan belgilanadi.

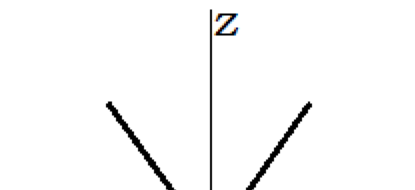


4.7.A-rasm. Nochiziq tavsifning joylashish sektorlari.

Mashqlar

4.7.1 Yuqoridgi 4.1.1-mashqda keltirilgan nochiziq unsirlar uchun minimal sektor aniqlansin.

4.7.2 Juft tavsifga ega bo`lgan quyidagi (4.4-rasm) nochiziq unsirlar uchun minimal sektor aniqlansin.



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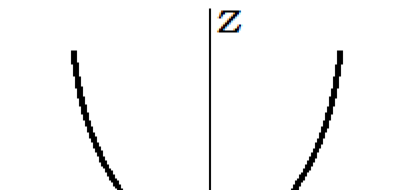

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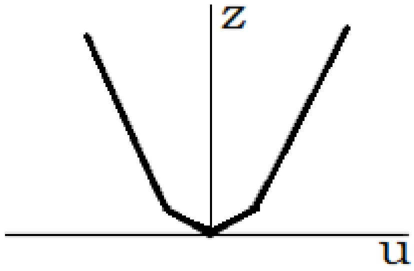
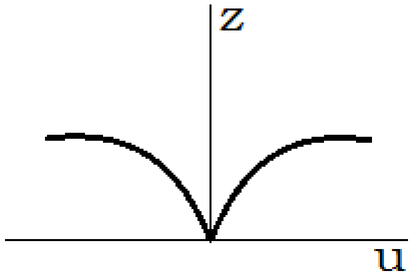


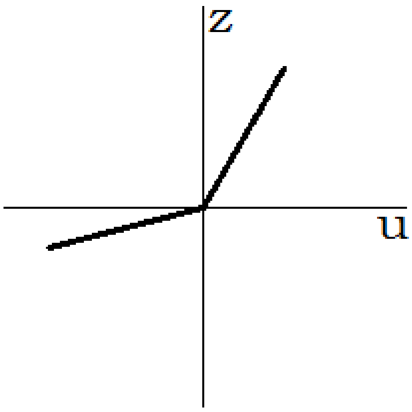
c)



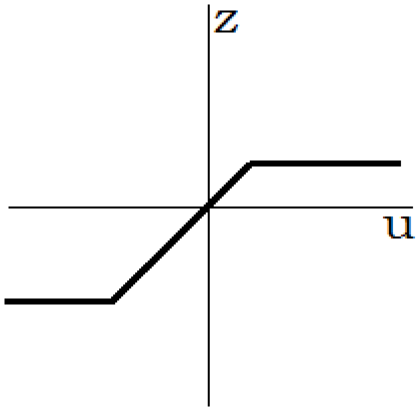
d)

4.4-rasm. Juft tavsifli unsirlar

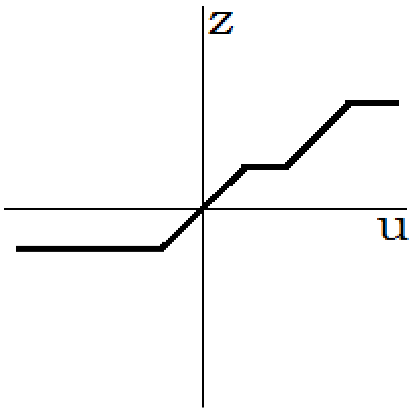
4.7.3 Statik tavsifi simmetrik bo`lmagan nochiziq unsirlar (4.5-rasm) uchun minimal sector aniqlansin.



a)



b)



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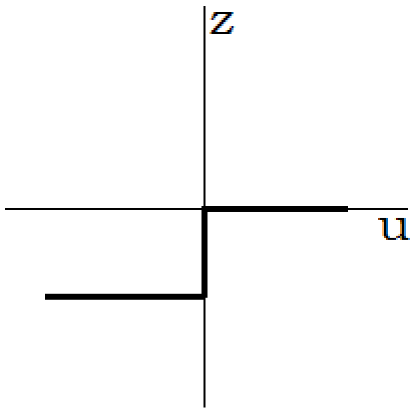
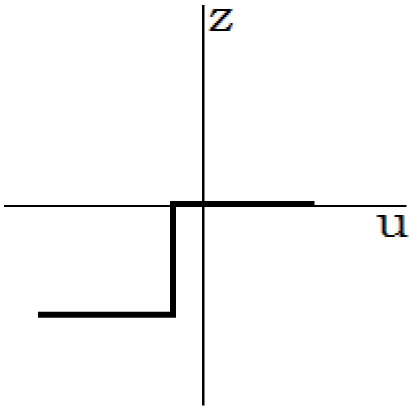
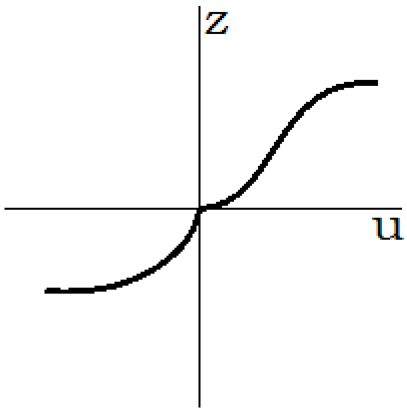


d)



e)



f)

4.5-rasm. Simmetrik bo`lmagan statik tavsiflar

4.8 Popovning to`g`ri chizig`i va modifikatsiyalangan chastota tavsif

Nazariy kirish

Chiziqli uzliksiz qism turg`un bo`lib, nochiziq statik tavsif $[0, k_0]$ sektorga tegishli bo`lsa, yopiq zanjirli tizimdagi muvozanat holatining turg`unligi Popovning to`g`ri chizig`i mavjudligi orqali tekshiriladi.

Popovning to`g`ri chizig`i $(U_M, jV_M)$ sirtida $(-\frac{1}{k_0}, j0)$ nuqtadan o`tadigan shunday to`g`ri chiziqki, dinamik qismning modifikatsiyalashgan chastota tavsifi $W_M(j\omega)$ to`la-to`kis shu chiziqdan o`ngda qoladi.

Dinamik qismning chastota tavsifi

$$W(j\omega) = U(\omega) + jV(\omega)$$

bo`lsin. Unda uning modifikatsiyalashgan haqiqiy va mavhum chastota tavsiflari quyidagilarga teng

$$U_M(\omega) = U(\omega), \quad V_M(\omega) = \omega V(\omega).$$

Popov chizig`ining (U_M, V_M) sirtda tenglamasi quyidagicha

$$U_M = qV_M - \frac{1}{k_0}.$$

Bu to`g`ri chiziqning og`ish koeffitsienti q quyidagi shartlarni qanoatlantirishi kerak:

a) $|q| < \infty$, agar $\Phi(\cdot)$ bir qiymatli bo`lsa,

b) $0 \le q < \infty$, agar $\Phi(\cdot)$ manfiy gisterezisli bo`lsa,

c) $-\infty < q \le 0$, agar $\Phi(\cdot)$ musbat gisterezisli bo`lsa,

d) $q = 0$, agar $\Phi(\cdot)$ nostatsionar bo`lsa.

Mashqlar

4.8.1 Chiziqli qismning berilgan uzatish funksiyalari uchun modifikatsiyalangan chastota tavsiflari aniqlansin va godogralar qurilsin.

$a)\ W(p) = p;$ $b)\ W(p) = \frac{1}{p};$

$c)\ W(p) = 1 + Tp;$ $d)\ W(p) = \frac{1}{1+Tp};$

$e)\ W(p) = \frac{1+T_1p}{1+T_2p};$ $f)\ W(p) = \frac{1}{p(1+Tp)};$

$g)\ W(p) = \frac{1+T_1p}{p(1+T_2p)};$ h) $W(p) = \frac{e^{-p\tau}}{1+Tp};$

$i)\ W(p) = \frac{e^{-p\tau}}{p(1+Tp)};$ $j)\ W(p) = \frac{1}{1+2\xi Tp+T^2p^2}.$

4.8.2 Avvalgi 4.8.1-mashqda keltirilgan chiziqli qism uchun nochiziq unsirning yo`l qo`yilishi mumkin bo`lgan maksimal sektor chegarasining og`ish koeffitsienti k_0 baholansin.

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